SOLVED EXAMPLES

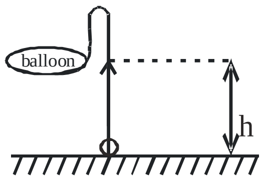
Example 1.

A balloon starting from the surface of earth has been ascending at a uniform velocity for 4.5 sec and a stone let fall from it reaches the ground in 7 second. Find the velocity of balloon and its height when stone was let fall

(1) $20.88\ ms^{-1}$; 90 m (2) $20\ ms^{-1}$; 84 m

(3) $20.88\ ms^{-1}$; 94 m (4) $17.66\ ms^{-1}$; 100 m

Solution : (3)



Let u be the velocity with which balloon is ascending which is constant

h = height of the balloon when the stone is dropped

For the stone (assuming direction of initial velocity of stone is taken to be positive)

As $\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$ hence

$$-h = 7u - \frac{1}{2}.g.7^2 \Rightarrow -4.5u = 7u - \frac{1}{2}.g \times 49$$

$\Rightarrow u = 20.88\, ms^{-1}$ $\therefore h = 4.5u = 94m$

Example 2.

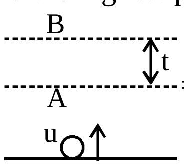
A paticle is projected with initial velocity u in the vertical upward direction then find the distance covered in the last t second while particle is going up.

(1) gt^2 (2) $2gt^2$

(3) $1/2\ gt^2$ (4) $3/4\ gt^2$

Solution : (3)

As B is the highest point upto which particle ascends



= time taken is the last part AB or in the last t second

velocity at $A = u - g\left(\frac{u}{g} - t\right)$

$\therefore$ Distance covered in last t seconds $= AB = (gt).t - \frac{1}{2}gt^2 = \frac{1}{2}gt^2$

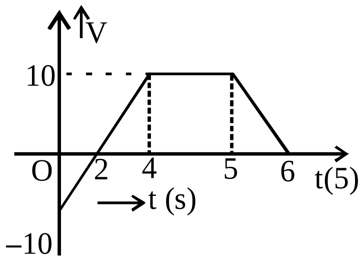
<u>2nd approach</u>

The distance covered in the last t seconds while going upwards = the distance covered in the first t seconds while going downwards $= \frac{1}{2}gt^2$

Example 3.	The displacement of a particle moving in a straight line is described by relation $s = 6 + 12t - 2t^2$. Here s is in metre and t is in seconds. The distance covered by particle in first 5 seconds. (1) 15 m (2) 20 m (3) 36 m (4) 26 m
Solution :	**(4)** $v = \frac{ds}{dt} = 12 - 4t$ $\therefore$ at t = 0, u = 12 ms^{-1} $a = \frac{dv}{dt} = -4ms^{-2}$ particle comes momentarily at rest at t = 3 seconds As t_0 = 3 sec < 5 second At t_0 velocity becomes zero, hence distance is greater than magnitude of displacment which is given by $d = \left\|\frac{u^2}{2}\right\| + \left\|\frac{1}{2}a\left(t - t_0\right)^2\right\|$ $= \frac{144}{2 \times 4} + \frac{1}{2} \times 4 \times 4 = 18 + 8 = 26m$
Example 4.	A juggler maintains four balls in motion, making each of them to rise up to height of 20 m from his hand. What time interval should he maintain, for maintaining proper distance between them (g = 10 m/s^2) (1) 2 sec (2) 1 sec (3) 3 sec (4) 4 sec
Solution :	(2) From $v^2 = u^2 - 2as$ we have $0 = u^2 - 2\,(10)\,(20)$ or u = 20 m/s Also v = u – at or 0 = 20 – 10 t or t = 2s So, the ball returns to the hand of the juggler after 4s. To maintain proper distance, the balls must be thrown up at an interval of $\frac{4}{4}$ or 1s.

Example 5.

From the v–t graph shown in the figure



Evaluate

(1) The average velocity for the first 6 second.

(2) The average speed for the first 6 second.

(3) The average acceleration from time t = 1 sec to 4 sec.

(4) The instantaneous acceleration at 3rd second.

(5) Plot a–t and x–t graph assuming x =0, t=0

Solution :

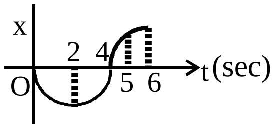
(1) $V_{av} = \frac{\text{Disp}}{\text{Time}} = \frac{\text{Area under curve}}{\Delta t} = \left[-\left(\frac{10\times 2}{2}\right)+\left(\frac{4+1}{2}\right)10\right]\frac{1}{6} = 25\text{ms}^{-1}$

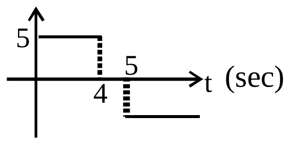
(2) $\text{Av.Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{|\text{Area}|}{\text{Time}} = \left[\frac{10\times 2}{2}+\frac{4+1}{2}\times 10\right]\frac{1}{6} = \frac{35}{6}\text{ms}^{-1}$

(3) $a_{av} = \frac{v_f - v_i}{\Delta t} = \frac{10-(-5)}{3} = 5\text{ms}^{-2}$

(4) a_{inst} = slope at t = 3 = 5ms^{-2}

(5)





Example 6.

The acceleration time graph of a particle moving in a straight line is as shown in figure. The velocity of particle at time t=0 is 2 ms^{-1}. The velocity after 2 seconds will be

(1) 4 m/s (2) 2 m/s

(3) 1 m/s (4) 6 m/s

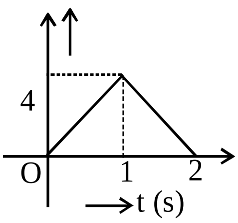
Solution :

(4) Area under the a-t curve is equal to the change in velocity

$$\Delta v = \frac{1}{2}\times 2\times 4 = 4\text{ms}^{-1}$$

$v - u = 4\text{ms}^{-1}$ since u = 2ms^{-1}. ; v= velocity at time t = 2 sec

u = velocity at time t = 0 sec



$\therefore$ velocity after 2 seconds

v = 6 ms^{-1}

Example 7.	The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. Then what will the acceleration be. (1) $-2av^3$ (2) $2av^2$ (3) $-v^3$ (4) v^3/a
Solution :	(1) Given relation is $t = ax^2+bx$ on diffrentiating w.r.t. time, $1 = 2ax.\frac{dx}{dt} + b.\frac{dx}{dt}$ (i) $v = \frac{dx}{dt} = \frac{1}{2ax+b}$ (ii) Diffrentiating (i) given w.r.t time $0 = 2a\left(\frac{dx}{dt}\right)^2 + 2ax.\frac{d^2x}{dt^2} + b.\frac{d^2x}{dt^2}$ $\frac{d^2x}{dt^2} = \frac{-2a.v^2}{b+2ax} = -2av^2\left(\frac{1}{b+2ax}\right)$ $= -2av^3$ (using (ii))
Example 8.	A particle located at x =0 at time t=0, starts moving along positive x-direction with a velocity v that varies on $v = \alpha\sqrt{x}$. Displacment of particle does vary with time as (1) $x \alpha t^2$ (2) $x \alpha t^3$ (3) $x \alpha t^4$ (4) $x \alpha t$
Solution :	(1) As $v = \alpha\sqrt{x}$ i.e. v is a function of displacement $\frac{dx}{dt} = \alpha\sqrt{x}$; α is a positive constant or $\frac{dx}{\alpha\sqrt{x}} = dt \Rightarrow 0$ on integrating from x = 0 to x for time t $\frac{1}{\alpha}\left[2\left(x^{1/2}\right)\right] = t$ $\therefore$ $x = \frac{\alpha^2}{4}.t^2$ $\therefore$ $x \alpha t^2$

Example 9.

A particle is projected vertically upwards and it attains maximum height H. If the ratio of times to attain height h (h < H) is $\frac{1}{3}$, then find h

(1) $\frac{2}{3}H$ (2) $\frac{3}{4}H$ (3) $\frac{4}{5}H$ (4) $\frac{5}{6}H$

Solution :

(2)

Let u be velocity of projection then if t is the time when projectile is at height h then

$$h = ut - \frac{1}{2}gt^2 \quad \text{or} \quad \frac{1}{2}gt^2 - ut + h = 0$$

or $t^2 - \frac{2u}{g}t + \frac{2h}{g} = 0$

Sum of roots, $t_1 + t_2 = \frac{2u}{g}$

where t_1 and t_2 are times at which projectile attains height h

Product of roots, $t_1 + t_2 = \frac{2h}{g}$

Given $\frac{t_1}{t_2} = \frac{1}{3}$ or $t_2 = 3t_1$

So $4t_1 = \frac{2u}{g}$ or $t_1 = \frac{4}{2g}$

or $3t_1^2 = \frac{2h}{g}$ or $3\frac{u^2}{4g^2} = \frac{2h}{g}$

or $\frac{u^2}{g} = \frac{8h}{3}$ ...(1)

Also $\frac{u^2}{2g} = H$ ...(2) (max. height)

$$\frac{(1)}{(2)}; \frac{\frac{u^2}{g}}{\frac{u^2}{2g}} = \frac{\frac{8h}{3}}{H} \quad \text{or} \quad 2 = \frac{8h}{3H}$$

$$h = \frac{3H}{4}$$

Example 10.	A ball is thrown straight upward with a velocity v_o from a height h above the ground. Then find time taken for the ball to strike the ground (1) $\frac{v_0}{g}\left(1+\frac{2gh}{v_0^2}\right)$ (2) $\frac{v_0}{g}\left(\sqrt{1+\frac{2gh}{v_0^2}}\right)$ (3) $\frac{v_0}{g}\left(1-\sqrt{1+\frac{2gh}{v_0^2}}\right)$ (4) $\frac{v_0}{g}\left(1+\sqrt{1+\frac{2gh}{v_0^2}}\right)$
Solution :	(4) $x(t)=x(0)+v_x(0)t+\frac{1}{2}a_x t^2$; Assuming origin at the point of projection $\therefore \quad -h=0+v_0t-\frac{1}{2}gt^2$ or $\frac{1}{2}gt^2-v_0t-h=0$ $t^2-\frac{2v_0}{g}t-\frac{2h}{g}=0$ $t=\frac{2v_0}{g}\pm\frac{\sqrt{\frac{4v_0^2}{g}+\frac{8h}{g}}}{2}=\frac{v_0}{g}\pm\frac{v_0}{g}\sqrt{1+\frac{2hg}{v_0^2}}=\frac{v_0}{g}\left[1+\sqrt{1+\frac{2hg}{v_0^2}}\right]$ Choice (4) is correct.
Example 11.	A body is dropped from a height h. If t_1 and t_2 be the times in covering first half and the next half distance respectively, then find the relation between t_1 and t_2 (1) $t_1 = t_2$ (2) $t_1 = 2t_2$ (3) t_1=2t_2 (4) $t_1=\frac{t_2}{\left(\sqrt{2}-1\right)}$
Solution :	(4) $\frac{h}{2}=\frac{1}{2}gt_1^2$; $h=\frac{1}{2}gt_3^2$, t_3 = time taken to cover h distance $t_1=\sqrt{\frac{h}{g}}$; $t_3=\sqrt{\frac{2h}{g}}$ $t_2=t_3-t_1=\sqrt{\frac{2h}{g}}-\sqrt{\frac{h}{g}}$ $\frac{t_2}{t_1}=\frac{\sqrt{\frac{2h}{g}}-\sqrt{\frac{h}{g}}}{\sqrt{\frac{h}{g}}}=\frac{\sqrt{2}-1}{1}$ or $t_1=\frac{t_2}{\sqrt{2}-1}$

Example 12.	Three particles start from the origin at he same time, one with a velocity v_1 along the x-axis, the second along the y-axis with a velocity v_2 and the third particle moves alon the line x = y line. The velocity of the third particle, so that the three may always lie on the same line, is

(1) $\frac{v_1 + v_2}{2}$ (2) $\sqrt{v_1 + v_2}$

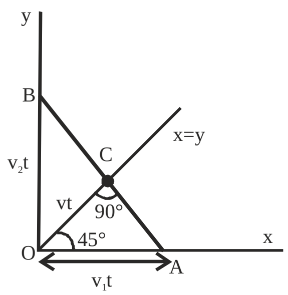
(3) $\frac{v_1 v_2}{v_1 + v_2}$ (4) $\frac{\sqrt{2} v_1 v_2}{(v_1 + v_2)}$

Solution :

Let v be the velocity of 3rd particle along the time x = y

Then the first particle A covers distance OA = $x = v_1 t$, the second paticle B covers distance $OB = y = v_2 t$.

Where 3rd particles C covers distance OC= v_1 point A, B, C must lie on a straight time ACB.



The equation of line, joining positions of three particles of time t may be represented by $\frac{x}{a} + \frac{y}{b} = 1$

here $a = v_1 t$, $b = v_2 t$

Slope of line x = y is tan 45° = 1

If the coordinates of C (x, y), then x = vt cos 45°

y = u sin 45°

This point lie on the straight line

$$\therefore \quad \frac{v_1 t \sin 45^o}{v_1 t} + \frac{v_2 t \cos 45^o}{v_2 t} = 1$$

or $$v_1 \frac{v}{\sqrt{2}} + \frac{v}{v_2 \sqrt{2}} = 1$$

$$v = \left[\frac{v_2 + v_1}{\sqrt{2} v_1 v_2}\right] = 1$$

$$v = \frac{\sqrt{2} v_1 v_2}{v_1 + v_2}$$

Example 13.

A particle moves along a straight line OX. At a time t (seconds) (he distance x (m)) of the particle from o is given by

$x = 40 + 12t - t^3$

How long would the particle travel before coming to rest ?

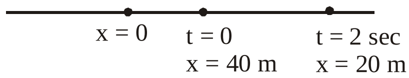
(1) 16 m (2) 24 m

(3) 40 m (4) 56 m

Solution :

$x = 40 + 12t - t^3$

$= v = 12 - 3t^2 = 0$



$t = +\,2$ sec

$\therefore$ Distance covered till comes to resp

$= 40 + 24 - 8 = 64 - 8$

$= 56$ m

$\therefore$ Distance covered $= 56 - 40$

$= 16$ m

Example 14.

A car runs at a constant speed on a circular track of a radius 100m, taking 62.8 seconds for every circular lap. The average velocity and average speed for each circular lap by respectively as

(1) $10\ ms^{-1}, 0$ (2) 0, 0

(3) $0, 10\ ms^{-1}$ (4) $10\ ms^{-1}, 10\ ms^{-1}$

Solution :

r = 100 m

average speed $= \dfrac{2\pi r}{\text{time}} = \dfrac{2 \times 3.14 \times 100}{62.80} = 10 ms^{-1}$

average velocity = 0

Example 15.

A car moves from x to y with a uniform speed v_u and returns to y with a uniform speed v_d. The average speed from this round trip is

(1) $\sqrt{v_u . v_d}$ (2) $\dfrac{v_u . v_d}{v_d + v_u}$

(3) $\dfrac{v_u + v_d}{2}$ (4) $\dfrac{2v_u . v_d}{v_u + v_d}$

Solution :

average speed $= \dfrac{2s}{\dfrac{s}{v_u} + \dfrac{s}{v_d}} = \dfrac{2v_u . v_d}{v_d + v_u}$

Example 16.	A particle moving along x-axis has acceleration f at a time t is given by $f = f_0 \left(1 - \frac{t}{T}\right)$, where f_0 and T are constant. The particle at t = 0 has zero velocity. In the time interval between, t = 0 and instant, when f = 0 the particles velocity v_x is (1) $\frac{1}{2} f_0 T^2$ (2) $f_0 T^2$ (3) $\frac{1}{2} f_0 T$ (4) $f_0 T^2$
Solution :	given acceleration $f = f_0\left(1 - \frac{t}{T}\right)$ ...... (1) when f = 0 , t = T (1) $\Rightarrow \int dv = f_0\left(1 - \frac{t}{T}\right)dt$ $v = f_0\ t - \frac{f_0 t^2}{2T}$ ...... (2) $\therefore$ At time t = T $v = f_0\ T - \frac{f_0 T^2}{2T} = f_0\ T - \frac{1}{2} f_0\ T$ $= \frac{1}{2} f_0\ T$
Example 17.	A particle starting from origin (0, 0) moves in a straight line in the (x, y) plane. Its co-ordinates at a later time ax $\left(\sqrt{3}, 3\right)$. The path of particle makes with x-axis at an angle of (1) 45° (2) 60° (3) 0° (4) 30°
Solution :	slope of line along which particle is moving $= \frac{3 - 0}{\sqrt{3} - 0} = \sqrt{3} = \tan\theta$ $\Rightarrow \theta = 60°$

Example 18. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is

(1) $v_0 + \frac{g}{2} + f$

(2) $v_0 + 2g + 3f$

(3) $v_0 + \frac{g}{2} + \frac{f}{3}$

(4) $v_0 + g + f$

Solution :

$v = v_0 + gt + ft^2$

$\Rightarrow \quad dx = v_0\,dt + gt\,dt + ft^2\,dt$

$$\Rightarrow \quad x = v_0 t + g\frac{t^2}{2} + f\frac{t^3}{3} + c$$

as $c = 0$ since $x = 0$, $t = 0$

$$x(t) = v_0 t + g\frac{t^2}{2} + f\frac{t^3}{3}$$

$$\therefore \quad x(1) = v_0 + \frac{g}{2} + \frac{f}{3}$$

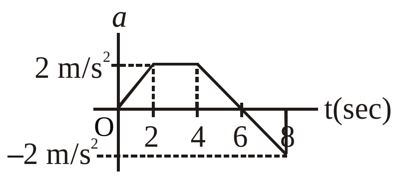
❑❑

MULTIPLE CHOICE QUESTIONS

LEVEL - I

1. A ball is dropped on to the floor from a height of 5 m. It rebounds to a height of 1.25m. If the ball is in contact with floor for 0.01 seconds, what is the average acceleration during contact?
   (1) 700 m/s^2 (2) 1400 m/s^2
   (3) 1500 m/s^2 (4) 2800 m/s^2

2. The acceleration time graph of a particle O moving along x-axis is shown. Find its average acceleration between time 0 to 8 sec.



   (1) 1 m/s^2 (2) 2 m/s^2
   (3) 3/4 m/s^2 (4) 5/4 m/s^2

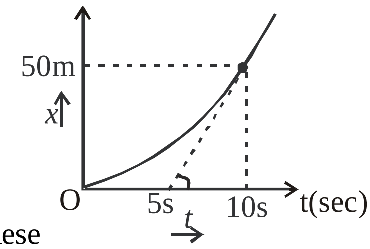
3. A stone is dropped from the 25th storey of a multistoreyed building and it reaches the ground in 5 sec. In the first second, it passes through how many storeys of the building? (g = 10 m/s^2)
   (1) 1 (2) 2
   (3) 3 (4) None of these

4. The velocity of a body at the end of 4 seconds is 26 ms^{-1} and at the end of 12 seconds is 58 ms^{-1} and at the end of 22 seconds is 98 ms^{-1}. The body is moving with
   (1) Uniform velocity
   (2) Uniform speed
   (3) Uniform acceleration
   (4) Uniform displacement

5. A truck and a car are moving with equal velocity. On applying the brakes both will stop after certain distance; then (assume that both brakes have equal retarding force)
   (1) Truck will cover less distance before rest
   (2) Car will cover less distance before rest
   (3) Both will cover equal distance
   (4) None of these

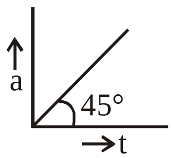
6. A car accelerates from rest at constant rate for t seconds and covers a distance x. The distance covered by it in next t seconds will be
   (1) x (2) $2x$
   (3) $3x$ (4) $4x$

7. A particle starts from rest and moves with uniform acceleration 'a'. The $x - t$ graph is as shown in figure. The acceleration of particle is



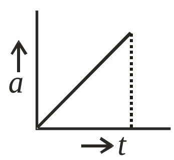
   (1) 2 m/s^2
   (2) 3 m/s^2
   (3) 1 m/s^2
   (4) None of these

8. A particle is moving along a straight line path with non-uniform acceleration. The a-t graph is drawn as shown in figure. At $t = 0$, $v = 10$ m/s. The average velocity of the particle during time interval $t = 0$ to $t = 10$ s is

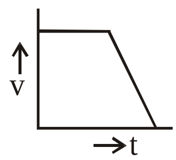


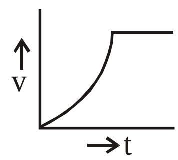
   (1) 26.7 m/s (2) 30 m/s
   (3) 25 m/s (4) None of these

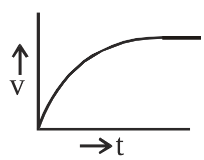
9. The acceleration-time graph for a body is shown in figure



   The corresponding velocity-time graph is

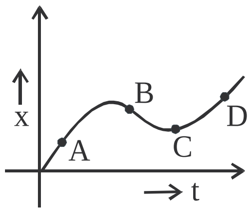
   (1) 

   (2) 

   (3) 

   (4) None of these

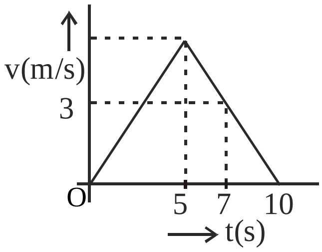
10. The displacement time graph of a moving particle is as shown in the figure. The instantaneous velocity of particle is negative at point



(1) C (2) D
(3) A (4) B

11. If a particle has different average velocities in different intervals then motion is said to be
(1) Uniform Motion
(2) Non-Uniform Motion
(3) Uniform or Non-uniform Motion
(4) None

12. The velocity-time graph of a body moving in a straight line is as shown in figure. The average velocity of a particle during time interval $t = 0$ to $t = 7$s is



(1) 3 m/s (2) 2.95 m/s
(3) 3.05 m/s^{-1} (4) None of these

13. The direction of the acceleration ($\vec{a}$) of a moving body is same as
(1) The direction of velocity
(2) The direction of change in displacement
(3) The direction of change in velocity
(4) The direction of initial velocity

14. A balloon is ascending with constant acceleration $a = 2m/s^2$. A stone is dropped from the balloon after some time. The acceleration of stone soon after, it is released from the balloon is
(1) 2 m/s^2 (2) –2 m/s^2
(3) g (4) Can not be calculated

15. A body is thrown upward and reaches its maximum height. At that position
(1) Its velocity is zero and its acceleration is also zero
(2) Its velocity is zero but its acceleration is maximum
(3) Its acceleration is minimum
(4) Its velocity is zero and its acceleration is the acceleration due to gravity

16. Two balls are dropped from heights h and 2h respectively above the earth surface. The ratio of time of these balls to reach the earth is
(1) $1:\sqrt{2}$ (2) $\sqrt{2}:1$
(3) 2 : 1 (4) 1 : 4

17. From the top of a tower, a particle is thrown vertically downwards with a velocity of 10 m/s. The ratio of the distances, covered by it in the 3rd and 2nd seconds of the motion is (Take g = 10 m/s^2)
(1) 5 : 7 (2) 7 : 5
(3) 3 : 6 (4) 6 : 3

18. A ball is projected vertically up with an initial speed of 20 m/s on a planet where acceleration due to gravity is 10 m/s^2.
The time taken to reach the maximum height and the maximum height attained will be respectively
(1) 2 s, 20 m (2) 4 s, 10 m
(3) 2 s, 10 m (4) 4 s, 20 m

19. A car moving with a speed of 40 km/hr can be stopped by applying brakes after at least 2 m. If the same car is moving with a speed of 80 km/hr, what is the minimum stopping distance?
(1) 2 m (2) 4 m
(3) 6 m (4) 8 m

20. A ball is released from the top of a tower of height h metres. It takes T seconds to reach the ground. What is the position of the ball in T/3 seconds?
(1) $\frac{h}{9}$ metres from the ground
(2) $\frac{7h}{9}$ metres from the ground
(3) $\frac{8h}{9}$ metres from the ground
(4) $\frac{17h}{18}$ metres from the ground

LEVEL - II

1. A particle starts with initial velocity 2.5 m/s along the x direction and accelerates uniformly at the rate 50 cm/s^2. Find time taken to increase the velocity to 7.5 m/s.
(1) 5 sec (2) 10 sec
(3) 15 sec (4) 20 sec

2. The displacement of a particle is given by y = a + bt + ct^2 + dt^4. Find the acceleration of a particle.
(1) 2d + 12 ct^2 (2) 2c + 6 ct^2
(3) 2c + 12 dt^2 (4) 2d + 6 ct^2

3. A particle starts with a constant acceleration. At a time t second speed is found to be 100 m/s and one second later speed becomes 150 m/s. Find acceleration of the particle.
(1) 20 m/s^2 (2) 25 m/s^2
(3) 100 m/s^2 (4) 50 m/s^2

4. A body travels a distance of 2 m in 2 seconds and 2.2 m in next 4 seconds. What will be the velocity of the body at the end of 7th second from the start.
(1) 0.1 m/s (2) 0.2 m/s
(3) 0.3 m/s (4) 0.4 m/s

5. The velocity acquired by a body moving with uniform acceleration is 20 m/s in first 2 sec and 40 m/s in first 4 sec. Calculate initial velocity.
(1) 0 m/s (2) 5 m/s
(3) 10 m/s (4) 20 m/s

6. A truck starts from rest with an acceleration of 1.5 ms^{-2} while a car 150 metre behind starts from rest with an acceleration of 2 ms^{-2}. How much distance is travelled by the car, before it overtakes the truck
(1) 200 m (2) 500 m
(3) 600 m (4) 100 m

7. A body is released from a height and falls freely towards the earth. After one second another body is released. What is the distance between the two bodies after 2 sec after release of the second body, if g = 9.8 m/s^2.
(1) 24 m (2) 24.5 m
(3) 25 m (4) 25.5 m

8. The position of a particle moving along a stright line path is given by $x = 12 + 18t + 9t^2$ metre. Its acceleration at any instant is
(1) $18\,ms^{-2}$ (2) $45\,ms^{-2}$
(3) $9\,ms^{-2}$ (4) $12\,ms^{-2}$

9. Displacement (x) of a particle is related to time (t) as $x = at + bt^2 - ct^3$ where a, b and c constants of motion. The velocity of the particle when its acceleration is zero is given by
(1) $a + \frac{b^2}{c}$ (2) $a + \frac{b^2}{2c}$
(3) $a + \frac{b^2}{3c}$ (4) $a + \frac{b^2}{4c}$

10. A person walks up a stationary escalator in time t_1. He remains stationary on the escalator, then it can take him up in time t_2. How much time would it take him to walk up the moving escalator.
(1) $\frac{t_1 + t_2}{2}$ (2) $\sqrt{t_1 t_2}$
(3) $\frac{t_1 t_2}{t_1 + t_2}$ (4) $t_1 + t_2$

11. A body starts from rest with uniform acceleration a, its velocity after n seconds is v. The displacement of the body in last 3 seconds is:
(1) $\frac{v(6n-9)}{2n}$ (2) $\frac{2v(6n-9)}{n}$
(3) $\frac{2v(2n+1)}{n}$ (4) $\frac{2v(n-1)}{n}$

12. When a ball is thrown up vertically with velocity V_0, it reaches a maximum height of 'h'. If one wishes to triple the maximum height then the ball should be thrown with velocity
(1) $\sqrt{3}V_0$ (2) $3V_0$
(3) $9V_0$ (4) $3/2V_0$

13. A small block slides without friction down an inclined plane starting from rest. Let S_n be the distance travelled from time t = n–1 to t = n.

Then $\frac{S_n}{S_{n+1}}$ is

(1) $\frac{2n-1}{2n}$ (2) $\frac{2n+1}{2n-1}$

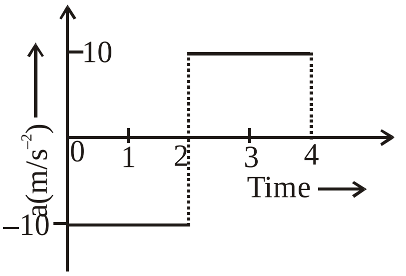
(3) $\frac{2n-1}{2n+1}$ (4) $\frac{2n}{2n+1}$

14. Two particle are released from the same height at an interval of 1s. How long after the first particle begins to fall will the two particles be 10 m apart. (g = 10 m/s^2)

(1) 1.5 s (2) 2 s

(3) 1.25 s (4) 2.5 s

15. A particle starts from rest at t = 0 and moves on a straight line with acceleration as shown graphically. The speed will be maximum after



(1) 1 sec (2) 2 sec

(3) 3 sec (4) 4 sec

16. The position of a particle along *x*-axis at time *t* is given by $x = 1 + t - t^2$. The distance travelled by the particle in first 2*s* is

(1) 1 m (2) 2 m

(3) 2.5 m (4) 3 m

17. The position x of a particle varies with time (t) as $x = at^2 - bt^3$. The acceleration at time t of the particle will be equal to zero, where t is equal to

(1) $\frac{2a}{3b}$ (2) $\frac{a}{b}$

(3) $\frac{a}{3b}$ (4) zero

18. For a particle displacement-time relation is $t = \sqrt{x} + 3$. Its displacement, when its velocity is zero, is

(1) 2m (2) 4m

(3) 0 (4) None of these

19. A body, thrown upwards with some velocity reaches the maximum height of 50 m. Another body with double the mass thrown up with double the initial velocity will reach a maximum height of

(1) 100 m (2) 200 m

(3) 300 m (4) 400 m

20. A ball is projected upwards from the foot of a tower. The ball crosses the top of the tower twice after an interval of 6s and the ball reaches the ground after 12s. The height of the tower is (g =10m/s^2)

(1) 120 m (2) 135 m

(3) 175 m (4) 80 m

LEVEL - III

1. The relation between time *t* and distance *x* is

$$t = \alpha x^2 + \beta x$$

where α and β are constants. The retardation is

(1) $2\alpha v^3$ (2) $2\beta v^3$

(3) $2\alpha\beta v^3$ (4) $2\beta^2 v^3$

2. A ball is thrown vertically upwards. It was observed at a height h twice with a time interval Δt. The initial velocity of the ball is

(1) $\sqrt{8gh + g^2(\Delta t)^2}$

(2) $\sqrt{8gh + \left(\frac{g^2 \Delta t}{2}\right)^2}$

(3) $\frac{1}{2}\sqrt{8gh + g^2(\Delta t)^2}$

(4) $\sqrt{8gh + 4g^2(\Delta t)^2}$

3. A point mass starts moving in straight line with constant acceleration from rest at t=0. At time t =2s, the acceleration changes the sign, remaining the same in magnitude. The mass returns to the initial position at time t=t_0 after starts of motion. Here t_0 is

(1) 4s (2) $\left(4+2\sqrt{2}\right)$s

(3) $\left(2+2\sqrt{2}\right)$s (4) $\left(4+4\sqrt{2}\right)$s

4. The displacement x of a particle varies with time according to the relation

$x = \frac{a}{b}\left(1 - e^{-bt}\right)$. Then

(1) At t = 1/b, the displacement of the particle in nearly (2/3) (a/b)

(2) The velocity and acceleration of the particle at t = 0 are a and $-ab$ respectively

(3) The particle cannot reach a point at a distance x' from its starting position if $x' > a/b$

(4) All the above

5. Between the two stations, a train accelerates uniformly at first, then moves with constant velocity and finally retards uniformly. If the ratio of the time, taken by is 1 : 8 :1 and the maximum speed attained be 60 km/h, then what is the average speed over the whole journey?

(1) 52 km/h (2) 48 km/h

(3) 54 km/h (4) 56 km/h

6. A car, starting from rest, accelerates at the rate *f* through a distance S, then continues at constant speed for time *t* and then decelerates at the rate f/2 to come to rest. If the total distance traversed is 15 S, then

(1) $S = \frac{1}{2}ft^2$ (2) $S = \frac{1}{4}ft^2$

(3) S = ft (4) $S = \frac{1}{72}ft^2$

7. A parachutist after bailing out falls 50 m without friction when parachute opens, it decelerates at 2 m/s^2. He reaches the ground with a speed of 3 m/s.At what height, did he bail out?

(1) 293 m (2) 111 m

(3) 91 m (4) 182 m

8. A point traversed half of the distance with a valocity v_0. The half of remaining part of the distance was convered with velocity v_1 for the first half time and with v_2 for the next half time. The mean velocity of the point, averaged over the whole time of motion is

(1) $\frac{v_0 + v_1 + v_2}{3}$ (2) $\frac{2v_0 + v_1 + v_2}{3}$

(3) $\frac{v_0 + 2v_1 + 2v_2}{3}$ (4) $\frac{2v_0(v_1 + v_2)}{(2v_0 + v_1 + v_2)}$

9. A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6.0 m/s^2, find the distance travelled by the car after he finds the need to put the brakes on

(1) 3 m (2) 18.75 m

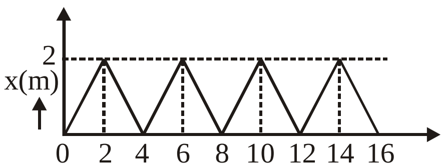
(3) 21.75 m (4) 22.75 m

10. A juggler throws balls into air. He throws one whenever the previous one is at its highest point. How high do the balls rise if he throws n balls each sec. Acceleration due to gravity is g.

(1) $2gn^2$ (2) $g/2n^2$

(3) $2n^2/g$ (4) $2g/n^2$

11. The figure shows the position –time (x – t) graph of one–dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



(1) 0.2 Ns (2) 0.4 Ns

(3) 0.8 Ns (4) 1.6 Ns.

12. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P (R, θ) on the circle of radius R is (Here θ is measured from the x–axis)

(1) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

(2) $-\frac{v^2}{R}\cos\theta\hat{i} + \frac{v^2}{R}\sin\theta\hat{j}$

(3) $-\frac{v^2}{R}\sin\theta\hat{i} + \frac{v^2}{R}\cos\theta\hat{j}$

(4) $-\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$.

ANSWERS (KINEMATICS - ID)

LEVEL - I

1. (3)	5. (2)	9. (2)	13. (3)	17. (2)
2. (3)	6. (3)	10. (4)	14. (3)	18. (1)
3. (1)	7. (3)	11. (2)	15. (4)	19. (4)
4. (3)	8. (1)	12. (2)	16. (1)	20. (3)

LEVEL - II

1. (2)	5. (1)	9. (3)	13. (3)	17. (3)
2. (3)	6. (3)	10. (3)	14. (1)	18. (3)
3. (4)	7. (2)	11. (1)	15. (2)	19. (2)
4. (1)	8. (1)	12. (1)	16. (3)	20. (2)

LEVEL - III

1. (1)	3. (2)	5. (3)	7. (1)	10. (2)
2. (3)	4. (4)	6. (4)	8. (4)	11. (3)
			9. (3)	12. (4)

HINT & SOLUTIONS (LEVEL - I)

1. $a_{ar} = \frac{v-u}{\Delta t} = \frac{5+10}{0.01} = 1500\ m/s^2$

2. ar.ace = area under art graph/time interval

3. height of the building = $\frac{1}{2}\times10\times25$ = 125 m, height of each storey = 5m.

   heigh in 1st second = $\frac{1}{2} \times 10 \times 1^2 = 5m$

4. $\left.\begin{array}{l} u+4a=26 \\ u+12a=58 \\ u+22a=98 \end{array}\right\} a = ym/s^2$

5. The body with lesser acceleration will travel more distance before coming to rest.

6. $\frac{1}{2}.a.(2t)^2 - \frac{1}{2}at^2 = \frac{1}{2}.3a.t^2 = 3.\frac{1}{2}at^2 = 3x$

7. acc. = $\frac{\text{Slope at } 10s - 0}{10s} = \frac{\frac{50}{5}-0}{10} = 1\ m/s^2$

8. $a = t$

   Average velocity = $\frac{\text{displacement}}{\text{time}}$

10. v = Slope of x ~ t graph.

12. Average velocity = $\frac{\text{Displacement}}{\text{time}}$

    $= \frac{\text{area}(v \sim t)\text{graph}}{\text{time interval}}$

16. $t = \sqrt{\frac{2h}{g}}$; $t \propto \sqrt{h}$

17. $S_n = u + \frac{a}{2}(2nd)$

19. In first case, initial velocity u_1 = 40 km/hr and stopping distance s_1 = 2m

In second case, Initial velocity u_2 = 80 km/hr

We know that $v^2 = u^2 + 2as$

or, $2as = -u^2$

$\therefore$ $s \propto -u^2$

because acceleration in both cases should be same.

Hense, $\frac{s_1}{s_2} = \left(\frac{u_1}{u_2}\right)^2$

or, $\frac{2}{s_2} = \frac{(40)^2}{(80)^2}$

or, $s_2 = 8$ m

20. We have $h = \frac{1}{2}gT^2$

In $\frac{T}{3}$ sec, the distance fallen $= \frac{1}{2}g\left(\frac{T}{3}\right)^2 = \frac{h}{9}$

$\therefore$ Position of the ball from the ground

$= h - \frac{h}{9} = \frac{8h}{9}$ m

HINT & SOLUTIONS (LEVEL - II)

1. $v = u + at$

$7.5 = 2.5 + 0.5t$

$5.0 = 0.5t$

$t = \frac{50}{5} = 10\,\text{sec}$

2. $v = \frac{dy}{dt} = \frac{d}{dt}\left(a + bt + ct^2 + dt^4\right) = b + 2ct + 4dt^3$

$a = \frac{dv}{dt} = 2c + 12dt^2$

3. From equation (1) of motion v = u + at

$\Rightarrow$ 100 = 0 + at

100 = at ... (1)

Now consider velocity one second later -

$v' = 0 + a(t+1)$

$\Rightarrow$ $150 = a(t+1)$ ... (2)

On subtracting equation (1) from equation (2)

$a = 50\,m/s^2$

4. Here, case (i) s = 2m, t = 2s

case (ii) t = 2 + 4 = 6s

Let u and a be the initial velocity and uniform acceleration of the body.

we know that, $s = ut + \frac{1}{2}at^2$

Case (i) $2 = u \times 2 + \frac{1}{2}a \times 2^2$

or 1 = u + a .... (1)

Case (ii) $4.2 = u \times 6 + \frac{1}{2}a \times 6^2$

or $0.7 = u + 3a$ ... (2)

Subtracting (2) from (1), we get

$0.3 = 0 - 2a = -2a$

or $a = -0.3/2 = -0.15\,ms^{-2}$

From (i), $u = 1 - a = 1 + 0.15$

$u = 1.15\,ms^{-1}$

For the velocity of body at the end of 7th second, we have

$u = 1.15\,ms^{-1}; a = -0.15\,ms^{-2}$,

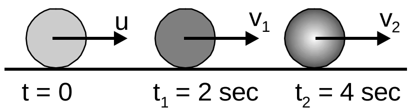
v = ?, t = 7s

As, v = u + at

$\therefore$ v = 1.15 + (–0.15) x 7

v = 0.1 m/s

5. $a = \frac{v_2 - v_1}{t_2 - t_1}$



$$a = \frac{40-20}{4-2} = \frac{20}{2} = 10\,m/s^2$$

Now, $v = u + at$

$$v_1 = u + at_1$$

$\Rightarrow \quad 20 = u + 10 \times 2$

$\Rightarrow \quad 20 = u + 20 \quad \Rightarrow \quad u = 0\,m/s$

6. **(a)** $s_T = \frac{1}{2}at^2$

$$s_T = \frac{1}{2}(1.5)t^2 \quad .... (1)$$

Distance covered by car when car one overtakes the truck

$$s_c = \frac{1}{2}(2)t^2$$

$$(s_T + 150) = \frac{1}{2}(2)t^2 \quad .... (2)$$

Divide equation (2) by equation (1)

$$\frac{s_T + 150}{s_T} = \frac{2}{1.5}$$

$\Rightarrow \quad 1 + \frac{150}{s_T} = \frac{20}{15} = \frac{4}{3}$

$\Rightarrow \quad \frac{150}{s_T} = \frac{4}{3} - 1 = \frac{1}{3}$ or $s_T = 450$

Distance travelled by car = 450 + 150 = 600 metre

(b) Now by equation (1) $s_T = \frac{1}{2}at^2$

$$450 = \frac{1}{2} \times 1.5 \times t^2$$

$$t^2 = \frac{450 \times 2}{1.5} \Rightarrow t = \sqrt{300 \times 2} = 24.5\,sec$$

Therefore car will overtake the truck after 24.5 second.

7. The 2nd body falls for 2s, so

$$h_2 = \frac{1}{2}g(2)^2 \quad ... \quad (1)$$

While 1st has fallen for 2 + 1 = 3 sec so

$$h_1 = \frac{1}{2}g(3)^2 \quad ... \quad (2)$$

$\therefore$ Separation between two bodies after 2 sec the release of 2nd body,

$$d = h_1 - h_2$$

$$= \frac{1}{2}g(3^2 - 2^2) = 4.9 \times 5 = 24.5\,m$$

8. $a = \frac{d^2x}{dt^2} = 18\ m/s^2$

9. Acceleration = 2b – 6ct = 0

$\Rightarrow \quad t = \frac{2b}{6c} = \frac{b}{3c}$

Velocity = a + 2bt – $3ct^2$

10. $t = \frac{d}{v_1 + v_2}$

$$= \frac{d}{\frac{d}{t_1} + \frac{d}{t_2}} = \frac{t_1 t_2}{t_1 + t_2}$$

11. The displacement is last 3 second = displacement in *n* second – displacement in *n* – 3 second.

$$= \frac{1}{2}a[n^2 - (n-3)^2]\frac{1}{2}a.(2n-3)(3)\Big\}$$

12. $h \propto u^2$

13. $S_n = g\sin\theta(2n - 1)$

$S_{n+1} = g\sin\theta[2(n + 1) - 1]$

$= g\sin\theta(2n + 1)$

$$\frac{S_n}{S_{n+1}} = \frac{2n-1}{2n+1}$$

14. Let the tune be *t s*.

$$\frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2 = 10$$

$\Rightarrow \quad \frac{1}{2} \times (2t - 1)(1) = 10$

$\Rightarrow \quad t = \frac{3}{2}\ s$

15. v = u + area under (a ~ t) graph.

16. $x = 1 + t - t^2$

$v = 1 - 2t$

$v = 0$ at $t = \frac{1}{2}s$

$\Delta s_1 = |x_{1/2} - x_0| = \frac{1}{4}$

$\Delta s_2 = |x_2 - x_{1/2}| = \left|-1-\left(1+\frac{1}{4}\right)\right|$

$\Delta s = \Delta s_1 + \Delta s_2 = 2.5$ m

17. $f = \frac{d^2x}{dt^2} = 2a - 6bt$

$f = 0$

$\Rightarrow \quad 6 = \frac{2a}{6b} = \frac{a}{3b}$

18. $x = (t - 3)2$

$v = 0$ at $t = 3s$

at 3s, $x = 0$

19. $h = \frac{u^2}{2g}$ and acceleration for both the masses is same.

$\frac{h_2}{h_1} = \frac{u_2^2}{u_1^2} = 4 \Rightarrow h_2 = 50 \times \times 4 = 200$ m

20. The height of the tower = $\frac{1}{2}g(6^2) - \frac{1}{2}g(3)^2$

$= 5(36 - 9)$

$= 27 \times 5 = 135$ m

HINT & SOLUTIONS (LEVEL - III)

1. $t = \alpha x^2 + \beta x$

$1 = 2\alpha x(1) + bv$

$v = \frac{1}{2\alpha x + \beta}$

$a = \frac{dv}{dt} = -\frac{1}{(2\alpha x + \beta)^2}.2\alpha v$

$= -2\alpha v^3$

2. $\frac{1}{2}gt^2 = \frac{u^2}{2g} - h \quad \Rightarrow t = \sqrt{\frac{u^2}{g^2} - \frac{2h}{g}}$

$\Delta t = 2\sqrt{\frac{u^2}{g^2} - \frac{2h}{g}}$

$\Rightarrow \quad u = \frac{1}{2}\sqrt{8gh + g^2\Delta t^2}$

3. The body will come to rest after 2 × 2 = 4 s.

The displacement of the body

$= \frac{1}{2} \times 4 \times 2a = 4a$

The time to cover the total distance with acceleration *a*

$= \sqrt{\frac{2 \times 4a}{a}} = 2\sqrt{2}\,s$

Thus the total time to reach the starting point

$= 4s + 2\sqrt{2}\,s$

4. at $t = \frac{1}{b}$, $x = \frac{a}{b}(1 - e^{-1}) = \frac{2a}{3b}$

Maximum displacement = $\frac{a}{b}$

$v = \frac{a}{b}.b e^{-bt} = ae^{-bt}$, and

Acceleration $= \frac{dv}{dt} = -ab e^{-bt}$

5. The time interval be t, 8t and t. Also in this case |acceleration| = |retardation|.

$$\text{Average speed} = \frac{\frac{1}{2}ft^2 + (ft)(8t) + \frac{(ft)^2}{2f}}{10t}$$

$$= \frac{f^2t^2 + 16f^2t^2 + f^2t^2}{20ft}$$

$$= \frac{18}{20}ft = 54 \text{ km/hr}$$

6. $S_1 = \frac{v^2}{2f}$ ...(i)

$S_2 = vt$ ...(ii)

$S_3 = \frac{v^2}{2\frac{f}{2}} = \frac{v^2}{f}$ ...(iii)

From (i), (ii) and (iii)

$$S = \frac{1}{72}ft^2$$

7. First he covers a distance of 50 m.

Velocity at the end of 50 m = $\sqrt{2gh}$

$= \sqrt{2\times10\times50} = \sqrt{1000}$ m/s

The distance travelled between a speed of $\sqrt{1000}$ m/s and 3 m/s with retardation of

$$2 \text{ m/s}^2 = \frac{1000-9}{2\times2}$$

Total distance = height = $\frac{991}{3} + 50$ = 293 m

8. The mean velocity = $\frac{2v_0v'}{v_0+v'}$

Where v' is the average velocity over next half distance

$$v' = \frac{v_1+v_2}{2}$$

9. Distance covered by the car during the application of brakes by driver-

$u = 54$ km/h

$= 54\times\frac{5}{18}$ m/s = 15 m/s

$s_1 = ut$

or $s_1 = 15 \times 0.2 = 3.0$ meter

After applying the brakes;

$v = 0$ $u = 15$ m/s, $a = 6$ m/s^2 $s_2 = ?$

Using $v^2 = u^2 - 2as$

$0 = (15)^2 - 2 \times 6 \times s_2$

$12 s_2 = -225$

$$\Rightarrow s_2 = \frac{225}{12} = 18.75 \text{ metre}$$

Distance travelled by the car after driver sees the need for it

$s = s_1 + s_2$

$s = 3 + 18.75 = 21.75$ metre.

10. Since the juggler is throwing n balls each second and he throws second ball when the first ball is at the highest point, so time taken by each ball to reach the highest point is t = 1/n

Taking vertical upward motion of ball up to the highest point, we have

$v = 0$, $a = -g$, $t = 1/n$, $u = ?$

As $v = u + at$

so $0 = u + (-g)\,1/n$

or $u = g/n$

Also $v^2 = u^2 + 2as$,

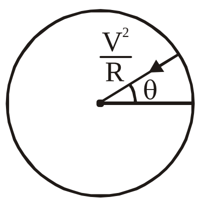
so $0 = u^2 - 2gh$

i.e., $h = (u^2/2g) = g/(2n^2)$

11. $\Delta V = (1) - (-1) = 2$ m/s

Impulse = (0.4) (2) = 0.8 kg – m/s.

12.



$$\vec{a} = -\frac{V^2}{R}\cos\theta\,\hat{i} - \frac{V^2}{R}\sin\theta\,\hat{j}.$$

❑❑