Lesson-1
SIMPLE HARMONIC MOTION

PERIODIC AND OSCILLATORY MOTION

Periodic Motion: A motion which repeats itself after equal intervals of time is called periodic motion.

Oscillatory Motion: A body is said to possess oscillatory or vibratory motion if it moves back and forth repeatedly about a mean position. For an oscillatory motion, a restoring force is required.

SIMPLE HARMONIC MOTION

Simple Harmonic Motion is a periodic motion in which a body moves to and fro about its mean or equilibrium position such that its restoring force or its acceleration is directly proportional to the displacement from its mean position and is directed towards its mean position. It can be expressed mathematically as,

$$F = m\frac{d^2x}{dt^2} = -kx.$$

where m is the mass on which a restoring force F acts to impart an acceleration $\frac{d^2x}{dt^2}$ along x-axis such that the restoring force F or acceleration is directly proportional to the displacement x along x-axis and k is a constant. The negative sign shows that the restoring force or acceleration is directed towards the mean position.

The differential equation of a simple hormonic motion is given by,

$$\frac{d^2x}{dt^2} + \left(\frac{K}{m}\right)x = 0$$

or $$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

where $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{\text{acceleration}}{\text{displacement}}}$

The time period, T, to complete one vibration by a body undergoing simple harmonic motion is given by

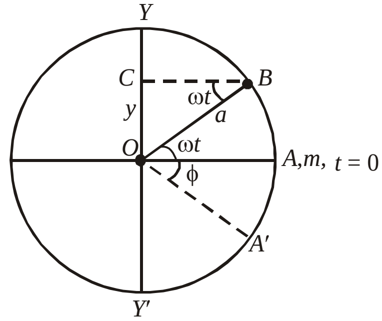
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{m}{K}}$$

REPRESENTATION OF SIMPLE HARMONIC MOTION :

If a point mass m is moving with uniform speed along a circular path of radius a, its projection on the diameter of the circle along y-axis represents its simple harmonic motion.

$$y = a \sin \omega t$$

where ω is the uniform angular velocity of the body of mass m along a circular path of radius a such that ωt is angle covered by the radius in time t from the initial position A at $t = 0$ to the position B. As $\angle AOB = \angle OBC = \omega t$, the foot of perpendicular from B to the diameter YOY' gives the projection at the point C such that $y = OC$ is the projection of this body on the diameter and represents the displacement of the body executing S.H.M. along y-axis.



If the body does not start its motion from the point A but at a point A' so that $\angle AOA'$ is the phase angle ϕ, then

$$y = a\sin(\omega t \pm \phi) \qquad \text{..... }(i)$$

where ϕ is the phase angle which may be positve or negative. The phase angle represents the fraction of the angle by which the motion of the body is out of step between the initial position of the body and the mean position of simple hormonic motion. The phase difference is the fraction of angle 2π or time period T of S.H.M. by which the body is out of step initially from the mean position of the body.

Differentiating equation (i),

$$\frac{dy}{dt} = v = a\omega\cos(\omega t \pm \phi)$$

$$\text{As } \sin(\omega t \pm \phi) = \frac{y}{a},\ \cos(\omega t \pm \phi) = \sqrt{1-\sin^2(\omega t \pm \phi)} = \sqrt{1 - \frac{y^2}{a^2}} = \frac{\sqrt{a^2 - y^2}}{a}$$

$$\therefore\quad v = \omega\sqrt{a^2 - y^2} \qquad \text{.....}(ii)$$

Differentiating (ii), the acceleration = $\dfrac{d^2y}{dt^2} = -a\omega^2\sin(\omega t \pm \phi)$

$$\therefore\quad \frac{d^2y}{dt^2} = -\omega^2 y$$

It represents the equation of simple harmonic motion where $\omega = \sqrt{\left(\dfrac{d^2y}{dt^2}\right)\Big/ y} = \sqrt{\dfrac{\text{acceleration}}{\text{displacement}}}$

Time period, $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{\text{displacement}}{\text{acceleration}}}$

EXAMPLES OF SHM

There are two types of SHM's

1. Linear simple harmonic motion
2. Angular simple harmonic motion

LINEAR SIMPLE HARMONIC MOTION

If a particle executes SHM on a straight line, then it is said to be linear simple harmonic motion.

In this case, to find out the time period, we find out the linear restoring force and hence linear acceleration.

i.e. $F = ma$

$\Rightarrow$ $-kx = ma$, where k is a constant.

$$\Rightarrow \quad a = \frac{-k}{m}x \qquad .....(i)$$

Also, we know that the equation of SHM is

$$a = -\omega^2 x \qquad .....(ii)$$

Comparing (*i*) and (*ii*), we get ω and hence the time period.

ANGULAR SIMPLE HARMONIC MOTION

When a particle executes S.H.M. on a curve path, then it is said to be angular SHM. For example simple pendulum.

In this case to find out the time period, we find out restoring torque and hence angular acceleration.

i.e. $\Gamma = -k\theta$ where k is a constant

$\Rightarrow$ $I\alpha = -k\theta$ where I moment of inertia

$$\Rightarrow \quad \alpha = \frac{-k}{I}\theta \qquad ....(i)$$

Also, the equation of SHM for angular SHM, is

$$\alpha = -\omega^2\theta \qquad ......(ii)$$

comparing (*i*) and (*ii*), we get ω, hence the time period.

SIMPLE PENDULUM

It is an example of angular simple harmonic motion. Let's calculate its time period.

Let us suppose that a bob of mass m is executing SHM. The length of the pendulum is l, which is the distance between the point of oscillation and the centre of mass of the bob. Torque acting on the bob about the point O.

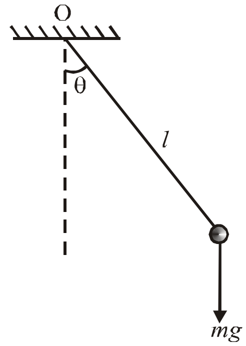
$$\Gamma = mg\,l\,\theta$$

$$\Rightarrow \quad I\alpha = -mgl\,\theta$$

$$\Rightarrow \quad \alpha = -\frac{mgl}{I}\theta$$ where α is angular acceleration

$$= -\frac{mgl}{ml^2}\theta$$

$$\alpha = -\frac{g}{l}\theta \qquad \text{...........}(i)$$



The equation of SHM is

$$\alpha = -\omega^2 \theta \qquad \text{...........}(ii)$$

Comparing (*i*) and (*ii*), we get

$$\omega^2 = \frac{g}{l}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{l}}$$

$$\Rightarrow \quad \frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

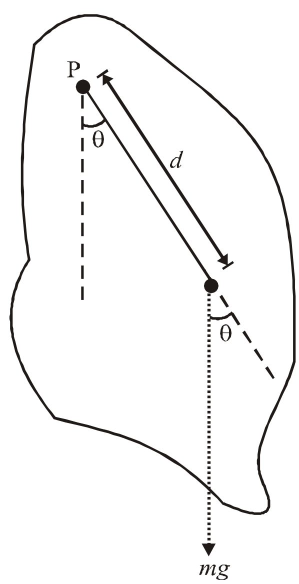
$$\Rightarrow \quad T = 2\pi\sqrt{\frac{l}{g}}$$

Notice that the period is independent of the mass of the suspended particle.

THE PHYSICAL PENDULUM

Any rigid body mounted so that it can swing in a vertical plane about some axis passing through it is called a physical pendulum.

A body of irregular shape is pivoted about a horizontal frictionless axis through P and displaced from the equilibrium position by an angle θ. (The equilibrium position is that in which the centre of mass C of the body lies vertically below P). The distance from the pivot to the centre of mass is *d*. The moment of inertia of the body about an axis through the pivot is I and the mass of the body is M.



The restoring torque about the point P,

$$I = Mgd \sin\theta$$

if θ be very small, $\sin\theta \approx \theta$

$$I = Mgd\theta,$$

$$\Rightarrow \quad I\alpha = -Mgd\,\theta,$$

$$\Rightarrow \quad \alpha = -\frac{Mgd}{I}\theta \qquad \text{............}(i)$$

comparing with the equation of SHM

$$\omega^2 = \frac{Mgd}{I}$$

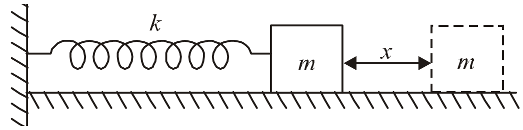
$$\Rightarrow \quad \omega = \sqrt{\frac{Mgd}{I}}$$

$$\Rightarrow \quad \frac{2\pi}{T} = \sqrt{\frac{Mgd}{I}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{Mgd}}$$

SPRING MASS SYSTEM

As shown in the figure a mass m is attached to a massless spring. It is displaced from its mean position to a distance x. The restoring force is given by



$F = -kx$ when k, is the force constant.

$\Rightarrow ma = -kx$

$\Rightarrow a = -\frac{k}{m}x$ ....(*i*)

$\because a \propto -x$, motion is SHM

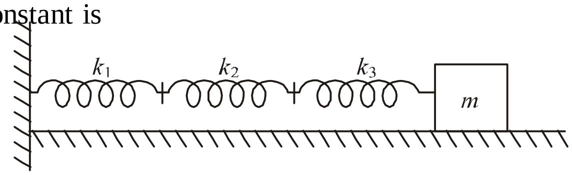
$\Rightarrow \omega^2 = \frac{k}{m}$ or $\omega = \sqrt{\frac{k}{m}}$

$\Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$

SERIES AND PARALLEL COMBINATION OF SPRINGS

Series combination of springs: Springs are connected in series if tension in springs is same while change in their length may be different. When springs are connected in series, having force constants k_1, k_2, k_3 then the equivalent force constant is

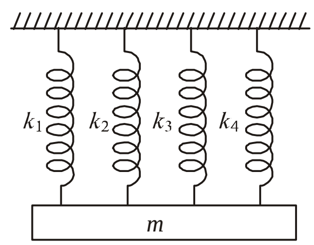
$$\frac{1}{k_{eff.}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + ...$$



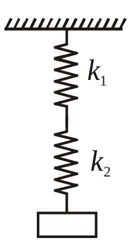
$$T = 2\pi\sqrt{\frac{m}{k_{eff}}}$$

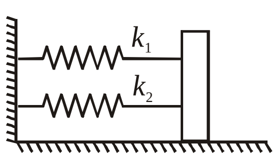
***Parallel combination of springs*:** If spring are connected in parallel, then the effective force constant is given by

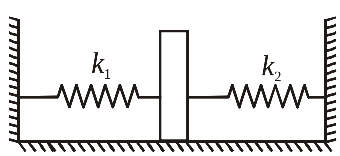
$$k_{eff.} = k_1 + k_2 + k_3 + ...$$

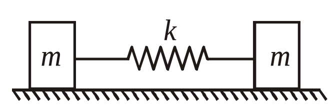


$$T = 2\pi\sqrt{\frac{m}{k_{eff}}}$$









$T = 2\pi\sqrt{\frac{m}{k_{eff}}}$ $T = 2\pi\sqrt{\frac{m}{k_{eff}}}$ $T = 2\pi\sqrt{\frac{m}{k_{eff}}}$ $T = 2\pi\sqrt{\frac{m_{eff}}{k}}$

ENERGY OF A SIMPLE HARMONIC MOTION

When a body of point mass m undergoes simple harmonic motion, its kinetic energy at a time, when its instantaneous velocity is v, is given by,

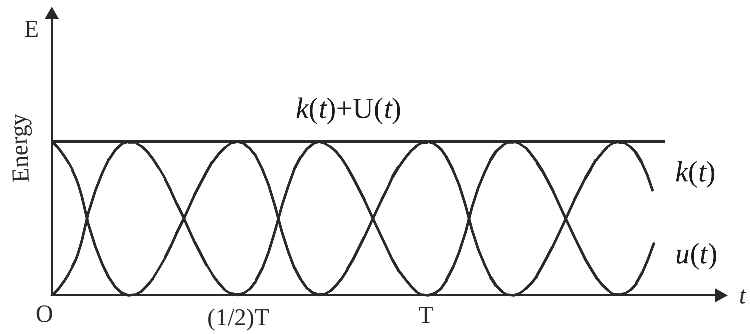
$$\text{kinetic energy} = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(a^2 - y^2)$$

$$\text{Its potential energy} = \frac{1}{2}ky^2 = \frac{1}{2}m\omega^2y^2$$

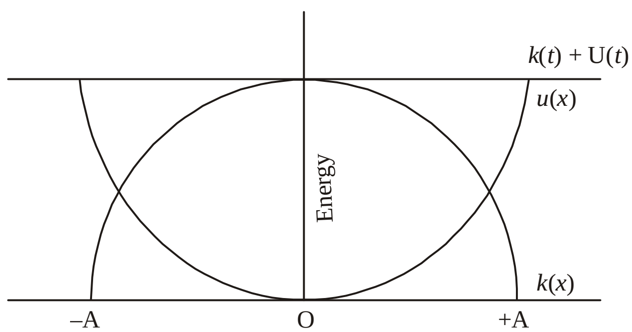
$$\text{Total instantaneous energy} = \frac{1}{2}m\omega^2(a^2 - y^2) + \frac{1}{2}m\omega^2y^2 = \frac{1}{2}m\omega^2a^2.$$

The kinetic energy is maximum and potential energy is minimum at the mean position in a simple harmonic motion whereas the kinetic energy is minimum and potential energy is maximum at the extreme position. The sum of instantaneous kinetic energy and potential energy is constant at all positions of the body.

GRAPH FOR ENERGY OF SHM



Variation of energy w.r.t. to time



x (displacement for mean position)

COMBINATION OF SHM

Case-I: ***Same frequency and same direction:***

Let the simple harmonic motion be

$x_1 = A_1 \cos(\omega t + \delta_1)$ & $x_2 = A_2 \cos(\omega t + \delta_2)$

The resultant displacement,

$x = x_1 + x_2$

$\Rightarrow \quad x = A_1(\cos\omega t \cos\delta_1 - \sin\omega t \sin\delta_1) + A_2(\cos\omega t \,.\, \cos\delta_2 - \sin\omega t \sin\delta_2)$

Let the resultant amplitude be A,

$$x = A\cos(\omega t + \delta_R)$$

where $A = A_1^2 + A_2^2 + 2A_1A_2(\cos\delta_1\cos\delta_2 + \sin\delta_1\sin\delta_2)$

$$= A_1^2 + A_2^2 + 2A_1A_2\cos(\delta_1 - \delta_2)$$

or $A = [A_1^2 + A_2^2 + 2A_1A_2\cos(\delta_1 - \delta_2)]^{1/2}$

If the phase difference $\delta_1 - \delta_2 = 0$

$$A_{max} = A_1 + A_2$$

& if $\delta_1 - \delta_2 = \pi$

$$A_{min} = A_1 - A_2$$

Case-II: ***Composition of two simple harmonic motion of the same frequency in mutually perpendicular direction:*** Let the two rectangular vibrations be represented by

$$x = a\sin(\omega t + \theta) \qquad \text{......}(i)$$

$$y = b\sin\omega t \qquad \text{......}(ii)$$

from (*i*) equation,

$$\frac{x}{a} = \sin(\omega t + \theta)$$

$$\Rightarrow \quad \frac{x}{a} = \sin\omega t\cos\theta + \cos\omega t\sin\theta \qquad \text{......}(iii)$$

Now, from (*ii*)

$$\frac{y}{b} = \sin\omega t$$

$$\Rightarrow \quad \sqrt{1 - \frac{y^2}{b^2}} = \cos\omega t$$

$$\text{or} \quad \sqrt{\frac{b^2 - y^2}{b^2}} = \cos\omega t$$

Now, from equation (*iii*)

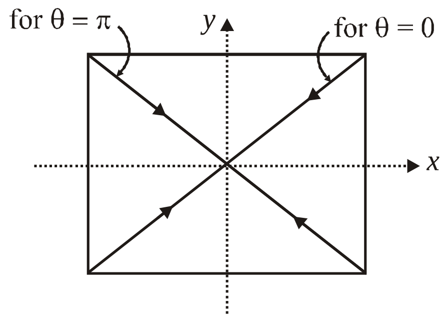
$$\frac{x}{a} = \frac{y}{b}\cos\theta + \sqrt{\frac{1 - y^2}{b^2}}\sin\theta$$

$$\Rightarrow \quad \frac{x}{a} - \frac{y}{b}\cos\theta = \sqrt{\frac{1 - y^2}{b^2}}\sin\theta$$

Squaring on both sides, we get

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\theta = \sin^2\theta}$$

Let us discuss few particular cases,

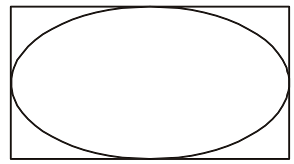


Case-I: $\theta = 0$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

$$\Rightarrow \quad \pm\left(\frac{x}{a} - \frac{y}{b}\right) = 0$$

This represents a pair of coincident straight lines lying in the first and third quadrants.

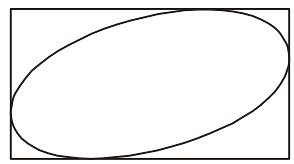


Case-II: $\theta = \frac{\pi}{2}$

$\sin\theta = 1, \cos\theta = 0$

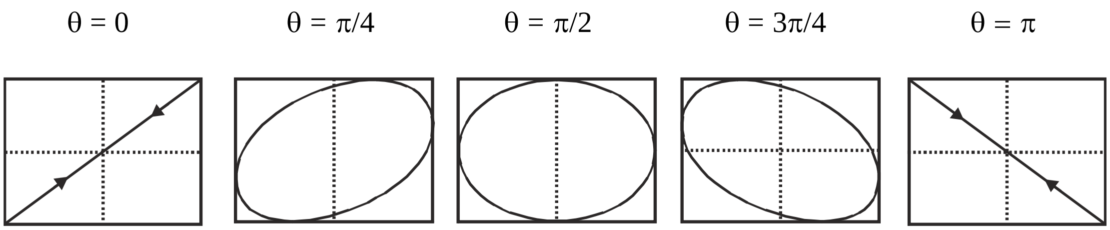
$\therefore \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$ which represents an ellipse.

Case-III: If $\theta = \frac{\pi}{4}$,



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{\sqrt{2}\,xy}{ab} = \frac{1}{2}$$

Which is an oblique ellipse



Varying phases, periods $\boldsymbol{\omega_1 : \omega_2 = 1 : 1}$

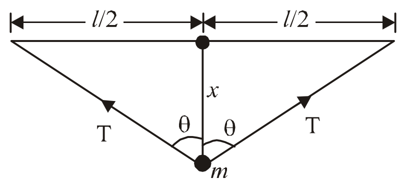
SOLVED EXAMPLES

Ex.1. Find the time period of small oscillations in a horizontal plane performed by a ball of mass 40 g fixed at the middle of a horizontally stretched string 1.0 m in length. The tension of the string is assumed to be constant and equal to 10 N.

Sol.: Consider a ball of mas m placed at the middle of a string of length l and tension T. The components of tension T towards mean position is T cosθ. The force acting on the ball = 2 T cosθ

$$\therefore \quad ma = -\frac{2\,\mathrm{T}\,x}{\sqrt{(l^2/4 + x^2)}} \qquad \left(\because\ \mathrm{T} = \mathrm{F} \text{ and } \cos\theta = \frac{x}{\sqrt{(l^2/4+x^2)}}\right)$$

As x is small, x^2 can be neglected in the denominator.



$$\therefore \quad a = -\frac{2\,\mathrm{T}\,x}{m(l/2)} = -\left(\frac{4\,\mathrm{T}}{ml}\right)x = -\omega^2 x$$

The acceleration is directly proportional to the negative of displacement x and is directed towards the mean position. Hence the motion is S.H.M.

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{(4\,\mathrm{T}/ml}} = \pi\sqrt{\left(\frac{ml}{\mathrm{T}}\right)}$$

Substituting the given values, we get

$$T = 3.14 \times \sqrt{\left(\frac{(4\times10^{-2})(1.0)}{10}\right)}\ \text{sec.}$$

$$= 0.2 \text{ sec.}$$

Ex.2. A if tunnel is dug through the earth from one side to the other side along a diameter. Show that the motion of a particle dropped into the tunnel is simple harmonic motion. Find the time period. Neglect all the frictional forces and assume that the earth has a uniform density.

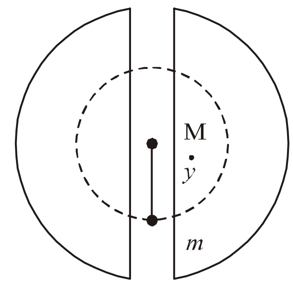
$G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2};

density of earth = 5.51×10^3 kg m^{-3}

Sol.: Consider a tunnel dug along the diameter of the earth.A particle of mass m is placed at a distance y from the centre of the earth. There will be a gravitational attraction of the earth experienced by this particle due to the mass of matter contained in a sphere of radius y. Force acting on particle at distance y from centre

$$\mathrm{F} = \frac{\mathrm{GM}\,m}{\mathrm{R}^3}y$$

$$\Rightarrow \quad ma = -\frac{GMm}{R^3}y$$



$$\Rightarrow \quad a = -\frac{GM}{R^3}y$$

$$= -\frac{G \times d \times \frac{4}{3}\pi R^3}{R^3}y$$

$$= -\frac{4\pi G}{3}dy$$

As the force is directly proportional to the displacement and is directed towards the mean position, the motion is simple harmonic.

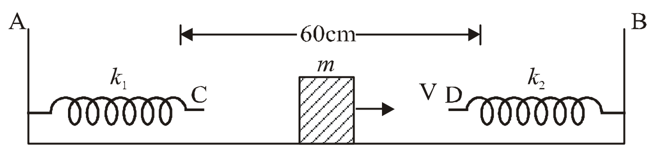
$$\Rightarrow \quad \omega^2 = \frac{4}{3}\pi d G.$$

and $T = 2\pi\sqrt{\left(\frac{3}{4\pi d G}\right)}$

$$= \sqrt{\left(\frac{3\pi}{dG}\right)} = \sqrt{\left(\frac{3 \times 3.14}{5.51 \times 10^3 \times 6.67 \times 10^{-11}}\right)}$$

= 5062 second = 84.4 minutes

Ex.3. Two light springs of force constant k_1 and k_2 and a block of mass m are in one line AB on a smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in figure.



The distance CD between the free ends of the springs is 60 cm. If the block moves along AB with a velocity 120 cm/sec in between the springs, calculate the period of oscillation of the block.
(k_1 = 1.8 N/m, k_2 = 3.2 N/m, m = 200 gm)

Sol.: If initially block is mid way of CD, then the time period T is equal to sum of time to travel 30 cm to right, time in contact with spring k_2, time to travel 60 cm to left, time in contact with spring k_1 and time to travel 30 cm to right.

$$\therefore \quad T = \frac{30}{120} + \frac{1}{2}\left[2\pi\sqrt{\left(\frac{m}{k_2}\right)}\right] + \frac{60}{120} + \frac{1}{2}\left[2\pi\sqrt{\left(\frac{m}{k_1}\right)}\right] + \frac{30}{120}$$

$$= 0.25 + \pi\sqrt{\left(\frac{0.2}{3.2}\right)} + 0.5 + \pi\sqrt{\left(\frac{0.2}{1.8}\right)} + 0.25$$

= 2.83 second.

Ex.4. Two identical balls A and B, each of mass 0.1 kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius 0.06 m. Each spring has a natural length 0.06π m and spring constant 0.1 N/m. Initially both the balls are displaced by angle $\pi/6$ radian with respect to the diameter PQ of the circle and released from rest.

(a) Calculate the frequency of oscillation of ball B.

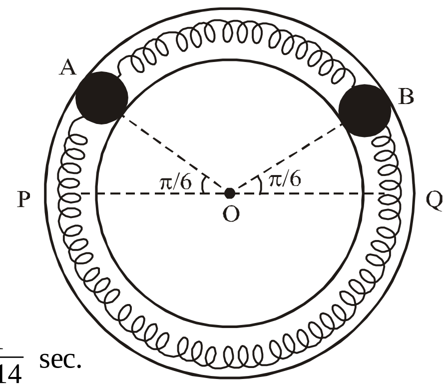
(b) Find the speed of the ball A when A and B are at the two ends of diameter PQ

(c) What is the total energy of the system?

Sol.: (a) Restoring force on A or $B = k\,\Delta x + k\,\Delta x = 2\,k\,\Delta x$.

Where Δx is compression in the spring at one end.

Effective force constant = $2k$



Frequency $\quad v = \dfrac{1}{2\pi}\sqrt{\dfrac{2k}{\mu}}$

where μ is reduced mass of system.

reduced mass, $\mu = \dfrac{mm}{m+m} = \dfrac{m}{2}$

$$v = \frac{1}{2\pi}\sqrt{\frac{2k}{m/2}} = \frac{1}{3.14}\sqrt{\frac{0.1}{0.1}} = \frac{1}{3.14}\ \text{sec.}$$

(b) P and Q are equilibrium positions. Balls A and B at P and Q have only kinetic energy and it is equal to the potential energy at extreme positions.

Potential energy at extreme positions $= \dfrac{1}{2}\,k\,(2\Delta x)^2 + \dfrac{1}{2}k\,(2\Delta x)^2$

$= 4k\,(\Delta x)^2$

where $\Delta x = R \times \dfrac{\pi}{6}$

$$\Rightarrow \text{P.E.} = \frac{\pi^2 k R^2}{36} = \frac{(3.14)^2 \times 0.1 \times (0.06)^2}{36} \approx 3.94 \times 10^{-4}\ \text{J}$$

When the balls A and B are at points P and Q respectively,

$KE_{(A)} + KE_{(B)} = \text{P.E.}$

$2\,KE_{(A)} = \text{P.E.}$

$2 \times \dfrac{1}{2}mv^2 = 3.94 \times 10^{-4}$

$$v = \left(\frac{3.94}{0.1}\right)^{\frac{1}{2}} \times 10^{-2} = 6.28 \times 10^{-2} = 0.0628\ \text{ms}^{-1}$$

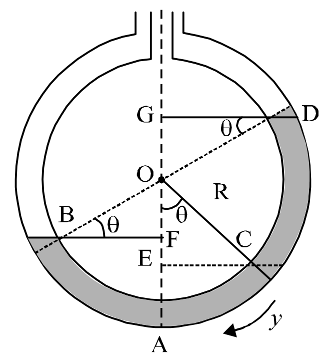
(c) Total potential and kinetic energy of the system is equal to total potential energy at the extreme position = 3.94×10^{-4} J.

Ex.5. Two non-viscous, incompressible and immiscible liquids of density ρ and 1.5 ρ are poured into two limbs of a circular tube of radius *R* and small cross-section kept fixed in a vertical plane as shown in the figure. Each liquid occupies one fourth the circumference of the tube.

(a) Find the angle that the radius vector to the interface makes with the vertical in the equilibrium position.

(b) If the whole liquid is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations.

Sol.: (a) Density of liquid column $BC = 1.5\,\rho$



Density of liquid column $CD = \rho$

Pressure at A due to liquid column $BA = p_{AB}$

$= AF \times 1.5\,\rho \times g$

$= (AO - OF)\,1.5\,\rho g \times g\rho$

$= (R - R\sin\theta)\,1.5\,g\rho$

Pressure at A due to liquid column $AD = p_{AD}$

$= AE \times 1.5\,\rho \times g + EG\,\rho g$

$\therefore\ \ p_{AD} = (AO - OE)\,1.5\,\rho g + (EO + OG)\,\rho g$

$= (R - R\cos\theta)\,1.5\,\rho g + R(\cos\theta + \sin\theta)\,\rho g$

In equilibrium $p_{AB} = p_{AD}$

$R(1 - \sin\theta)\,1.5\,\rho g = R(1 - \cos\theta)\,1.5\,\rho g + R(\cos\theta + \sin\theta)\,\rho g$

$1.5 - 1.5\sin\theta = 1.5 - 1.5\cos\theta + \cos\theta + \sin\theta$

$-2.5\sin\theta = -0.5\cos\theta$

$$\tan\theta = \frac{0.5}{2.5} = \frac{1}{5}, \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{1}{5}\right)$$

(b) If a is area of cross-section, length of each column $= \dfrac{2\pi R}{4} = \dfrac{\pi R}{2}$

Volume of each column $= \dfrac{\pi Ra}{2}$

Mass of column $BC = \dfrac{\pi Ra}{2} \times 1.5\,\rho$

Mass of column $CD = \dfrac{\pi Ra}{2} \times \rho$

M.I. of whole liquid about $O = \left(\dfrac{\pi Ra\rho}{2}\right)(1.5 + 1)R^2$ or $I = \dfrac{2.5\pi R^3 a\rho}{2}$

Let y be small displacement towards left and θ be the angular displacement,

$$\theta = \frac{y}{R} \quad \text{or} \quad y = R\theta$$

Angular acceleration = $\frac{d^2\theta}{dt^2}$

Torque about $A = I\frac{d^2\theta}{dt^2} = \frac{2.5\pi R^3 a\rho}{2}\left(\frac{d^2\theta}{dt^2}\right)$

Restoring torque due to displaced liquid,

$$\tau_{rest} = -[ay \times 1.5\ \rho g + ay\ \rho g] \times R\cos\theta$$

$$= -2.5\ ay\ \rho g \times R\cos\theta$$

$$= -2.5\ a\ \rho g\ R^2 \cos\theta.\theta$$

[$R\cos\theta$ is perpendicular distance of gravitational force from axis of rotation]

Equating $\left(\frac{2.5\pi R^3 a\rho}{2}\right)\frac{d^2\theta}{dt^2} = -(2.5\ a\ \rho\ gR^2\cos\theta)\ \theta$

$$\frac{d^2\theta}{dt^2} = -\left(\frac{2g\cos\theta}{\pi R}\right)\theta = -\omega^2\theta$$

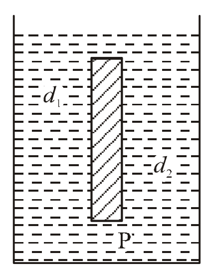
As $\frac{2g\cos\theta}{\pi R}$ is a constant, angular acceleration is proportional to angular displacement and is directed towards mean position, the liquid undergoes S.H.M.

$$T = \frac{2\pi}{\omega} = 2\pi \times \sqrt{\frac{\pi R}{2g\cos\theta}}$$

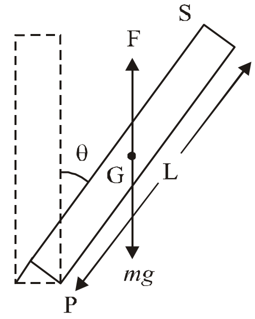
As $\tan\theta = \frac{1}{5}$, $\cos\theta = \frac{5}{\sqrt{26}}$, $T = 2\pi\sqrt{\frac{\pi R}{2 \times g \times \frac{5}{\sqrt{26}}}} = 2(\pi)^{\frac{3}{2}}\sqrt{\frac{R}{\left(\frac{10g}{\sqrt{26}}\right)}}$

Ex.6. A thin rod of length L and area of cross section S is pivoted at its lowest point P inside a stationary, homogeneous and non-viscous liquid as shown in the figure.

The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the material of the rod is smaller than the density d_2 of the liquid. The rod is displaced by a small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters.



Sol.: Consider the rod be displaced through an angle θ. The different forces on the rod are shown in the figure.



Weight of the rod acting downward = $S\,L\,d_1\,g = mg$

Buoyant force acting upwards = $S\,L\,d_2\,g$

Net thrust acting on the rod upwards ; $F = S\,L\,(d_2 - d_1)\,g$

Restoring torque $\tau = F \times \frac{L}{2}\sin\theta$

$$= S\,L\,(d_2 - d_1)\,g\,\frac{L}{2}\sin\theta$$

$\therefore\quad \tau = S\,L\,(d_2 - d_1)\,g\,\frac{L}{2}\theta \qquad (\because \text{ for small } \theta;\ \sin\theta = \theta)$

If θ is in clockwise direction, then τ will be in anticlockwise direction.

$$\therefore\quad \tau = -\frac{1}{2}\,S\,L^2\,(d_2 - d_1)\,g\,\theta$$

$$\tau = I\,\alpha = \left(\frac{M\,L^2}{3}\right)\frac{d^2\theta}{dt^2} = \left(\frac{S\,L\,d_1 \times L^2}{3}\right)\frac{d^2\theta}{dt^2}$$

$$\therefore\quad \frac{d^2\theta}{dt^2} = -\frac{3}{S\,L^3\,d_1} \times \frac{1}{2}\,S\,L^2\,(d_2 - d_1)\,g\,\theta$$

or $\frac{d^2\theta}{dt^2} = \frac{3g}{2L}\left(\frac{d_2 - d_1}{d_1}\right)\theta = 0$, so motion is S.H.M; comparing with differential equation of S.H.M.

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0;\quad \omega = \sqrt{\frac{3g}{2L}\left(\frac{d_2 - d_1}{d_1}\right)}$$

Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2L\,d_1}{3g\,(d_2 - d_1)}}$

OBJECTIVE QUESTIONS

1. A particle in simple harmonic motion has the same displacements, equal to half the amplitude, but opposite velocities at two instants. The minimum time interval between the two instants is

(a) T/2 (b) T/3 (c) T/4 (d) T/6

2. The motion of a particle is given by $x = A \sin \omega t + B \cos \omega t$. The motion of the particle is

(a) not simple harmonic
(b) simple harmonic with amplitude A + B
(c) simple harmonic with amplitude (A + B)/2
(d) simple harmonic with amplitude $\sqrt{A^2+B^2}$.

3. The displacement of a particle is given by $\vec{r} = a(\vec{i} \cos \omega t + \vec{j} \sin \omega t)$. The motion of the particle is

(a) simple harmonic
(b) on a straight line
(c) on a circle
(d) with constant acceleration.

4. A particle moves on the x-axis according to the equation $x = A + B \sin \omega t$. The motion is simple harmonic with amplitude

(a) A (b) B (c) A+B (d) $\sqrt{A^2+B^2}$

5. The period of vibration of a mass suspended by a spring is T. The spring is cut into n equal parts and the body is again suspended by one of the pieces. The time period of oscillation of the mass is

(a) $\sqrt{n}T$ (b) nT (c) $T/\sqrt{n}$ (d) n^2T

6. The maximum velocity of a body undergoing SHM is 0.2 m s^{-1} and its acceleration at 0.1 m from the mean position is 0.4 m s^{-2}. The amplitude of the SHM is

(a) 0.25 m (b) 0.3 m (c) 0.1 m (d) 0.05 m

7. Which of the following quantities are always negative in a simple harmonic motion?

(a) $\vec{F}.\vec{a}$. (b) $\vec{v}.\vec{r}$. (c) $\vec{a}.\vec{r}$. (d) $\vec{F}.\vec{r}$.

8. Which of the following quantities are always positive in a simple harmonic motion?

(a) $\vec{F}.\vec{a}$. (b) $\vec{v}.\vec{r}$. (c) $\vec{a}.\vec{r}$. (d) $\vec{F}.\vec{r}$.

9. The time period of the simple harmonic motion represented by $f + 4\pi^2 x = 0$, where f and x are the instantaneous acceleration and displacement, is

(a) 1/2 s (b) 4s (c) 1 s (d) 2 s

10. A particle moves on the X-axis according to the equation $x = x_0 \sin^2 \omega t$. The motion is simple harmonic

(a) with amplitude x_0 (b) with amplitude $2x_0$ (c) with time period $\frac{2\pi}{\omega}$ (d) with time period $\frac{\pi}{\omega}$

11. A coin is placed on a horizontal platform, which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time.

(a) at the highest position of the platform
(b) at the mean position of the platform
(c) for an amplitude of g/ω^2
(d) for an amplitude of $\sqrt{g}/\omega$

12. A coin is placed on a horizontal platform, which undergoes horizontal simple harmonic motion about a mean position O. The coin does not slip on the platform. The force of friction acting on the coin is F.

(a) F is always directed towards O.

(b) F is directed towards O when the coin is moving away from O, and away from O when the coin moves towards O.

(c) F = 0 when the coin and platform come to rest momentarily at the extreme position of the harmonic motion.

(d) F is maximum when the coin and platform come to rest momentarily at the extreme position of the harmonic motion.

13. In the previous question, the angular frequency of the simple harmonic motion is ω. The coefficient of friction between the coin and the platform is μ. The amplitude of oscillation is gradually increased. The coin will begin to slip on the platform for the first time

(a) at the extreme positions of oscillations (b) at the mean position

(c) for an amplitude of $\mu g/\omega^2$ (d) for an amplitude of $g/\mu\omega^2$

14. When the displacement is one-fourth of the amplitude, the fraction of the total energy that is kinetic is

(a) 1/2 (b) 3/4 (c) 12/16 (d) 15/16

15. In a simple harmonic motion the displacement and velocity differ in phase by

(a) 0 (b) $\pi/2$ (c) π (d) $\pi/4$

16. The displacement x of a particle in motion is given as a function of time by $x(x-4) = 1 - 5\cos\omega t$ then :

(a) particle execute SHM

(b) particle execute oscillatory motion which is not SHM.

(c) the motion of particle is not oscillatory

(d) the particle is not acted upon by force when $x = 4$

17. Two particles P and Q describe SHM with the same amplitude A and the same frequency f. The maximum distance separating the particles is observed to be A. The phase difference between the particles is –

(a) zero (b) $\pi/2$ (c) $\pi/3$ (d) $2\pi/3$

18. The displacements y of a particle executing a certain periodic motion is given by $y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$.

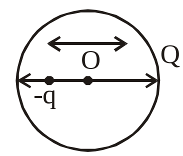
This expression may be considered to be the superposition of n independent harmonic motions. Then, n is equal to

(a) 2 (b) 3 (c) 4 (d) 5

19. A particle executes S. H. M. starting from its mean position at t = 0. If its velocity is $\sqrt{3}b\omega$, when it is at a distance b from the mean position, when $\omega = 2\pi/T$, the time taken by the particle to move from b to the extreme position on the same side is :

(a) $5\pi/6\omega$ (b) $\pi/3\omega$ (c) $\pi/2\omega$ (d) $\pi/4\omega$

20. A particle of mass *m* and charge –q moves along a diameter of uniformly charged sphere of radius R and carrying a total charge +Q. The frequency of the SHM of the particle, if the amplitude does not exceed , is :



(a) $\frac{1}{2\pi}\sqrt{\frac{qQ}{4\pi\varepsilon_0 R^2 m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{qQ}{4\pi\varepsilon_0 m R^3}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{qQ}{4\pi\varepsilon_0 m R}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{qQm}{4\pi\varepsilon_0 R^2}}$

21. The angular velocity and the amplitude of a simple pendulum are ω and A respectively. At a displacement x from the mean position if its kinetic energy is T and potential energy is U, then the ratio of T to U is :

(a) $\frac{x^2\omega^2}{(A^2 - x^2\omega^2)}$ (b) $\frac{x^2}{(A^2 - x^2)}$ (c) $\frac{A^2 - x^2\omega^2}{(A^2 - x^2\omega^2)}$ (d) $\frac{A^2 - x^2}{x^2}$

22. What will be the percentage change in the time period of a simple pendulum if its length is increased by 6% -

(a) 3% (b) 9% (c) 6% (d) 1/9%

23. The height of liquid column in a U tube is 0.3 meter. If the liquid in one of the limbs is depressed and then released, then the time period of liquid column will be-

(a) 1.1 sec (b) 19 sec (c) 0.11 sec (d) 2 sec

24. A 5 kg. weight is suspended from a spring. The spring stretches by 2 cm/kg. If the spring is stretched and released, its time period will be- ($g = 10\ m/s^2$)

(a) 0.628 sec (b) 6.28 sec (c) 62.8 sec (d) 0.0628 sec

25. A particle is executing simple harmonic motion along a straight line 8 cm long. While passing through mean position its velocity is 16 cm/s. Its time period will be-

(a) 0.157 sec. (b) 1.57 sec (c) 15.7 sec (d) 0.0157 sec.

26. A simple pendulum with length L and mass M of the bob is vibrating with amplitude a. Then the maximum tension in the string is :

(a) Mg (b) $Mg\left[1+\left(\frac{a}{L}\right)^2\right]$ (c) $Mg\left[1+\frac{a}{L}\right]^2$ (d) $Mg\left[1+\frac{a}{2L}\right]^2$

27. A tunnel has been dug through the centre of the earth and a ball is released in it. It executes S.H.M. with time period :

(a) 42 minutes (b) 1 day (c) 1 hour (d) 84.6 minutes.

28. The potential energy of a particle executing SHM changes from maximum to minimum in 5s. Then the time period of SHM is –

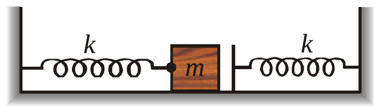
(a) 5s (b) 10s (c) 15s (d) 20s

29. A body of mass 0.1 kg is attached to two springs of force constants 6 N/m and 4 N/m and supported by two rigid supports. If the body is displaced along the length of the springs, the frequency of vibrations will be-

(a) 5 vibrations/sec (b) 10 vibrations/sec (c) $5/\pi$ vibrations/sec (d) $\pi/5$ vibrations/sec

30. The time period of oscillation of a block of mass m attached to a light spring of spring constant k is $2\pi\sqrt{\frac{m}{k}}$ in the absence of all other forces. Suppose that a block of mass m is attached to a fixed spring on one side and rests (i.e. is not attached to) against an identical spring on the other side. It is now allowed to oscillate. The frequency of small oscillations is :

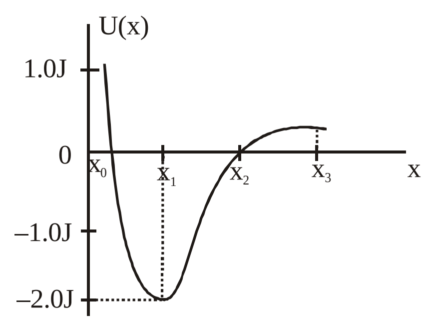
(a) $\frac{1}{\pi}\sqrt{\frac{k}{m}}$ (b) $\frac{1}{\pi}(2-\sqrt{2})\sqrt{\frac{k}{m}}$



(c) $\frac{(4-2\sqrt{2})}{\pi}\sqrt{\frac{k}{m}}$ (d) $\frac{(2+\sqrt{2})}{\pi}\sqrt{\frac{k}{m}}$

MORE THAN ONE CORRECT CHOICE

31. A conservative force has the potential energy function U (x) as shown by the graph. A particle moving in one dimension under the influence of this force has kinetic energy 1.0 J when it is at position x_1. Which of the following is/are correct statement(s) about the motion of the particle –



(a) It oscillates (b) It moves to the right of x_3 and never returns

(c) It comes to rest at either x_0 or x_2 (d) It cannot reach either x_0 or x_2

32. A 20gm. particle is subjected to two simple harmonic motions $x_1 = 2 \sin 10t$, $x_2 = 4\sin\left(10t+\frac{\pi}{3}\right)$

(a) The displacement of the particle at t = 0 will be $2\sqrt{3}$m

(b) Maximum speed of the particle will be $20\sqrt{7}$ m/s

(c) Magnitude of the maximum acceleration of the particle will be $200\sqrt{7}$ m/s^2

(d) Energy of the resultant motion will be 28 J

33. Three simple harmonic motions in the same direction having the same amplitude ‘a’ and same period are superposed. IF each differs in phase from the next by 45°, then –

(a) the resultant amplitude is $(1+\sqrt{2})a$

(b) the phase of the resultant motion relative to the first is 90°

(c) the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion.

(d) the resulting motion is not simple harmonic

34. A particle is executing SHM between points $-X_m$ and X_m, as shown in figure-I. The velocity of V (t) of the particle is partially graphed and shown in figure-II. Two points A and B corresponding to time t_1 and time t_2 respectively are marked on the V(t) curve

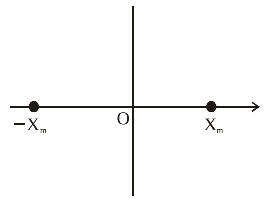


Figure-I

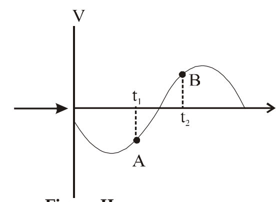


Figure-II

(a) At time t_2, its position lies in between $-X_m$ and O.

(b) The phase difference $\Delta\phi$ between points A and B must be expressed as $90° < \Delta\phi < 180°$.

(c) At time t_1, its speed is decreasing

(d) At time t_1, it is going towards X_m.

35. A block is placed on a horizontal plank. The plank is performing SHM along a vertical line with amplitude of 40 cm. The block just loses contact with the plank when the plank is momentarily at rest. Then:

(a) the period of its oscillation is $2\pi/5$ sec.

(b) the weight of the block on the plank is double its actual weight, when the plank is at one of the positions of momentary rest.

(c) the block weighs 1.5 times its weight on the plank halfway down from the mean position

(d) the block weights its true weight on the plank, when velocity of the plank is maximum.

36. A particle is subjected to two simple harmonic motions along x and y directions according to, $x = 3 \sin 100\pi t$; $Y = 4 \sin 100\pi t$.

(a) Motion of particle will be on ellipse traversing it in clockwise direction.

(b) Motion of particle will be on a straight line with slope 4/3.

(c) Motion will be a simple harmonic motion with amplitude 5.

(d) Phase difference between two motions is $\pi/2$.

37. A particle is executing SHM with amplitude A, time period T, maximum acceleration a_0 and maximum velocity v_0. It starts from mean position at t = 0 and at time t, it has the displacement A/2, acceleration a and velocity v. Then :

(a) $t = T/12$ (b) $a = a_0/2$ (c) $v = v_0/2$ (d) $t = T/8$

38. From the differential equation $\frac{d^2x}{dt^2} + 16x = 0$

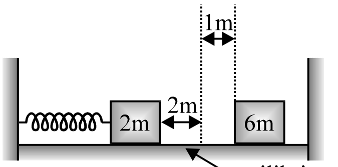
(a) We can find time period of oscillation

(b) We cannot find time period of oscillation

(c) We cannot find initial phase of oscillation

(d) We cannot find frequency of oscillation.

39. Two blocks of masses 3 kg and 6 kg rest on a horizontal smooth surface. The 3 kg blocks is attached to a spring with a force constant k = 900 N/m which is compressed 2 m from beyond the equilibrium position. The 6 kg mass is at rest at 1m from mean position. 3 kg mass strikes the 6 kg mass and the two stick together.



(a) velocity of the combined masses immediately after the collision is 10 m/s

(b) velocity of the combined masses immediately after the collision is 5 m/s

(c) Amplitude of the resulting oscillation is $\sqrt{2}$ m

(d) Amplitude of the resulting oscillation is $\sqrt{5/2}$ m

40. The speed v of a particle moving along a straight line, when it is at a distance (x) from a fixed point of the line is given by $v^2 = 108 - 9x^2$ (all quantities are in cgs units) :

(a) the motion is uniformly accelerated along the straight line

(b) the magnitude of the acceleration at a distance 3cm from the point is 27 cm/sec^2

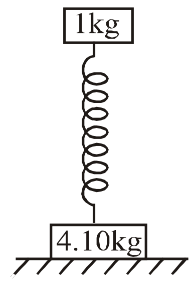
(c) the motion is simple harmonic about the given fixed point.

(d) the maximum displacement from the fixed point is 4 cm.

MISCELLANEOUS ASSIGNMENT

Comprehension-1

When force acting on the particle is of nature $F = -kx$, motion of particle is S.H.M. Velocity at extreme is zero while at mean position it is maximum. In case of acceleration situation is just reverse. Maximum displacement of particle from mean position on both sides is same and is known as amplitude.Refer to figure. One kg block performs vertical harmonic oscillations with amplitude 1.6 cm and frequency 25 rad s^{-1}.



1. The maximum value of the force that the system exerts on the surface is

(a) 20 N (b) 30 N
(c) 40 N (d) 60 N

2. The minimum force is

(a) 20 N (b) 30 N (c) 40 N (d) 60 N

Comprehension-2

A block attached to an elastic spring performs SHM. Consider such a spring block system lying on a smooth horizontal table with a block of mass m = 2kg attached at one end of a spring (k = 200 N/m) whose other end is fixed. The block is pulled so that the spring is extended by 0.05 m. If at this moment (t = 0), the block is projected with a speed of 1 m/s in the direction of increasing extension of the spring.

3. The angular frequency (ω) of motion is :

(a) 100/sec (b) 10/sec (c) 20/sec (d) 0

4. If the displacement (x) of the block is measured from the equilibrium position, it can be written as a function of time as $x = A\sin(\omega t + \delta)$. The constants A and δ have values :

(a) 0.112 m, $\cos^{-1}(0.446)$ (b) 0.0025 m, $\sin^{-1}(0.446)$
(c) 0.112 m, $\sin^{-1}(0.446)$ (d) 0.1 m, $\sin^{-1}(0.446)$

5. First time (t) when spring attains maximum extension is ($\sin^{-1}(0.446) = 26.5^{o}$) :

(a) 0.11 s (b) 0.22 s (c) 0.33 s (d) 0.44 s

6. First time (t) when velocity of block is maximum is :

(a) 0.268 s (b) 0.36 s (c) 0.47 s (d) 0.58 s

7. For a particle under going linear SHM about x = 0, choose the correct possible combination. Symbols have their usual meaning.

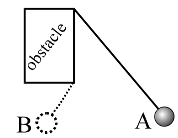
	Column I		**Column II**
A.	$\vec{v}.\vec{a} > 0$	(p)	extreme position
B.	velocity is negative	(q)	mean position
C.	acceleration is negative	(r)	$0 < x < A$
D.	$\vec{v} \times \vec{a} = 0$	(s)	$-A < x < 0$

8. The radius of ear ring and disc are 'r' length of rod is 'r' and side of square plate is r. Then match the time period of different objects executing S.H.M about an axis perpendicular to its plane and passing through edge

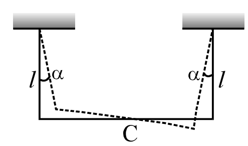
Column – I		Column – II	
A.	Ear ring	(p)	$2\pi\sqrt{3r/2g}$
B.	uniform rod	(q)	$4\pi\sqrt{r/3\sqrt{2}g}$
C.	circular disc	(r)	$2\pi\sqrt{2r/g}$
D.	Square plate	(s)	$2\pi\sqrt{r/2g}$

INTEGER TYPE QUESTIONS

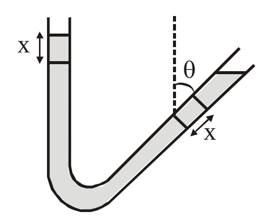
9. A simple pendulum has a time period T = 2 sec when it swings freely. The pendulum is hung as shown in figure, so that only one-fourth of its total length is free to swing to the left of obstacle. It is displaced to position A and released. It takes $n/2$ seconcds to swing to extreme displacement B and return to A? Assume that displacement angle is always small. Find the value of n



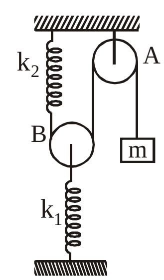
10. A uniform rod of mass m = 1.5 kg suspended by two identical threads l = 90 cm in length (see figure) was turned through a small angle about the vertical axis passing through its middle point C. The threads deviated in the process through an angle α = 5.0°. The rod was released to start performing small oscillations. The rod's oscillations energy is $n \times 10^{-2}$ J. Find the value of n



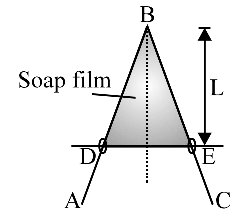
11. The period of oscillations of mercury of mass m = 200g is $n \times 10^{-1}$ second which is poured into a bent tube whose right arm forms an angle θ = 30º with the vertical. The cross sectional area of the tube S = 0.50 cm^2. The viscosity of mercury is to be neglected. Find the value of n



12. A block of mass m is tied to one end of a string which passes over a smooth fixed pulley A and under a light smooth movable pulley B. The other end of the string is attached to the lower end of a spring of spring constant k_2. The time period of small oscillations of mass m about its equilibrium position is $T = n\pi\sqrt{\dfrac{m(k_1+4k_2)}{k_1k_2}}$. Find the value of n.

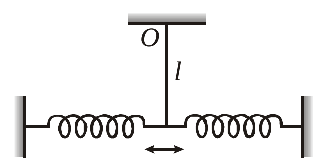


13. An A-shaped wire is made of bent wire ABC and a straight wire DE. Rings are provided which can slide over the bent wire as well as on straight wire. The arrangement is dipped into a soap solution and is then taken out and is placed in a vertical plane. The wire DE hangs in equilibrium at a distance L = 4.9 cm from the bent B as shown. When DE is displaced vertically parallel to itself it starts oscillating. Its period of oscillation in milli-seconds is 110 *n* milliseconds. Surface tension of soap solution is 0.07 N/m. Find the value of *n*.(g = 10 m/s^2)



14. A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, time period of oscillation is $T = 2\pi\sqrt{\dfrac{n}{\sqrt{g^2 + v^4/R^2}}}$. Find the value of *n*

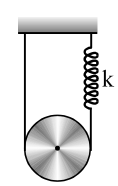
15. The frequency of small oscillations of a thin uniform vertical rod of mass m and length ℓ hinged at the point O is $\omega = \sqrt{\dfrac{3g}{2\ell} + \dfrac{nk}{M}}$ rad/sec. The combined stiffness of the springs is k. Find the value of *n*



16. A spring of force constant k = 300 N/m connects two blocks having masses 2 kg and 3 kg, lying on a smooth horizontal plane. If the spring block system is released from a stretched position the number of complete oscillations in 1 minute are 50 *n*. Take $\pi = \sqrt{10}$. Find the value of *n*

17. What will be the time period of a simple pendulum in hours if its length is equal to radius of earth (=6400 km)?

18. The pulley of radius r shown in figure has a moment of inertia I about its axis and mass m. The time period of vertical oscillations of its centre of mass is $2\pi\sqrt{\dfrac{\left(\dfrac{I}{r^2} + m\right)}{nk}}$. The spring has spring constant k and the string does not slip over pulley. Find the value of *n*



19. SHM is given by equation $y = \sin^2 \omega t$. Its time period is $n\pi/\omega$. Find the value of *n*.

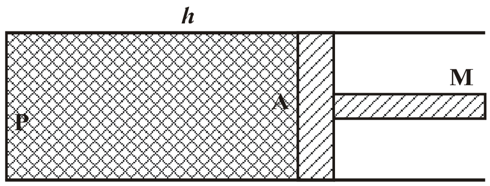
PREVIOUS YEAR QUESTIONS

IIT-JEE/JEE-ADVANCE QUESTIONS

1. The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be (if it is a second's pendulum on earth)

(a) $1/\sqrt{2}$second (b) $\sqrt{2}$ second (c) 2 second (d) (1/2) second

2. A cylindrical piston of mass M slides smoothly inside a long cylinder closed at one end, enclosing a certain mass of gas. The cylinder is kept with its axis horizontal. If the piston is disturbed from its equilibrium position, it oscillates simple harmonically. The period of oscillation will be (assuming temperature of gas remains constant.)



(a) $T = 2\pi\sqrt{\left(\frac{Mh}{PA}\right)}$ (b) $T = 2\pi\sqrt{\left(\frac{MA}{Ph}\right)}$

(c) $T = 2\pi\sqrt{\left(\frac{M}{PAh}\right)}$ (d) $T = 2\pi\sqrt{(MPhA)}$

3. A simple pendulum of length L and mass (bob) M is oscillating in a plane about a vertical line between angular limits $-\phi$ and $+\phi$. For an angular displacement $\theta(\sqrt{|\theta| < \phi})$ the tension in the string and the velocity of the bob are T and v respectively. The following relations hold good under the above conditions.

(a) $T\cos\theta = Mg$

(b) $T - Mg\cos\theta = Mv^2/L$

(c) the magnitude of the tangential acceleration of the bob $|a_T| = g\sin\theta$.

(d) $T = Mg\cos\theta$.

4. Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of M to that of N is

(a) k_1/k_2 (b) $\sqrt{(k_1/k_2)}$ (c) k_2/k_1 (d) $\sqrt{(k_2/k_1)}$

5. A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 joule. Its

(a) maximum potential energy is 100 J

(b) maximum kinetic energy is 100 J

(c) maximum potential energy is 160 J

(d) minimum potential energy is zero.

6. A uniform cylinder of length L and mass M having cross sectional area A is suspended with it vertical length, from a fixed point by a massless spring, such that it is half submerged in a liquid of density d at equilibrium position when the cylinder is given a small downward push and released. It starts oscillating vertically with a small amplitude. If the force constant of the spring is k, the frequency of oscillation of the cylinder is

(a) $\frac{1}{2\pi}\left(\frac{k - \mathrm{A}dg}{\mathrm{M}}\right)^{1/2}$ (b) $\frac{1}{2\pi}\left(\frac{k + \mathrm{A}dg}{\mathrm{M}}\right)^{1/2}$ (c) $\frac{1}{2\pi}\left(\frac{k + dg\mathrm{L}}{\mathrm{M}}\right)^{1/2}$ (d) $\frac{1}{2\pi}\left(\frac{k + \mathrm{A}dg}{\mathrm{A}dg}\right)^{1/2}$

7. A particle free to move along the X-axis has potential energy given by $U(x) = k[1 - \exp(-x^2)]$ for $-\infty \le x \le +\infty$, where k is a positive constant of appropriate dimensions. Then

(a) at points away from the origin, the particle is in unstable equilibrium

(b) for any finite non-zero value of x, there is a force directed away from the origin.

(c) if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin.

(d) for small displacements from $x = 0$, the motion is simple harmonic.

8. A spring of force-constant k is cut into two pieces such that one piece is double the length of the other.

(a) $(2/3)\,k$ (b) $(3/2)\,k$ (c) $3k$ (d) $6\,k$

9. The period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α, is given by

(a) $2\pi\sqrt{\left(\frac{L}{g\cos\alpha}\right)}$ (b) $2\pi\sqrt{\left(\frac{L}{g\sin\alpha}\right)}$ (c) $2\pi\sqrt{\left(\frac{L}{g}\right)}$ (d) $2\pi\sqrt{\left(\frac{L}{g\tan\alpha}\right)}$

10. A particle executes simple harmonic motion between $x = -A$ and $x + A$. The time taken for it to go from 0 to (A/2) is T_1 and to go from (A/2) to A is T_2. Then

(a) $T_1 < T_2$ (b) $T_1 > T_2$ (c) $T_1 = T_2$ (d) $T_1 = 2T_2$

11. An ideal spring with spring constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is

(a) $4Mg/k$ (b) $2Mg/k$ (c) Mg/k (d) $Mg/2k$

12. The displacement y of a particle executing periodic motion is given by

$$y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000\,t)$$

This expression may be considered to be a result of the superposition of

(a) two (b) three (c) four (d) five

13. A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is radius of earth. The value of T_2/T_1 is

(a) 1 (b) $\sqrt{2}$ (c) 4 (d) 2

14. Function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represent SHM

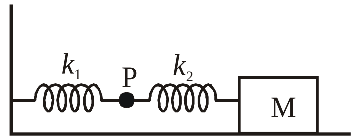
(a) for any value of A, B and C (except C = 0) (b) if A = –B; C = 2B, amplitude = $|B\sqrt{2}|$

(c) if A = B; C = 0 (d) if A = B; C = 2B, amplitude = $|B|$

15. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is

(a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_1}$

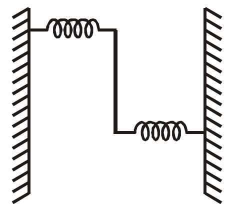
(c) $\frac{k_1 A}{k_1 + k_2}$ (d) $\frac{k_2 A}{k_1 + k_2}$



16. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is

(a) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$



17. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1 m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is

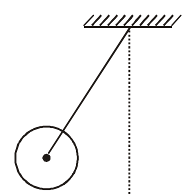
18. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (< L) is attached at its centre to the free end to the rod. Consider two ways the disc is attached: (case A) The disc is not free to rotate about its centre and (case B) the disc is free to rotate about its centre. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement (s) is (are) true?

(a) Restoring torque in case A = Restoring torque in case B

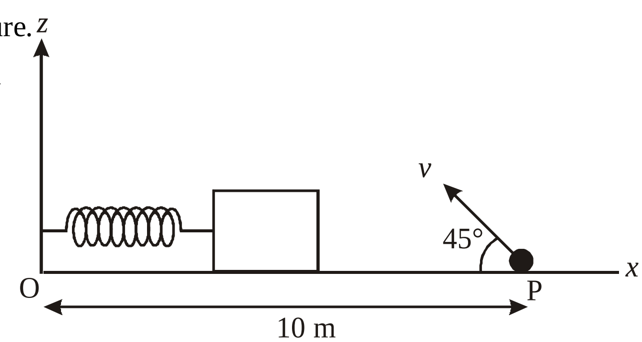
(b) Restoring torque in case A < Restoring torque in case B

(c) Angular frequency for case A > Angular frequency for case B

(d) Angular frequency for case A < Angular frequency for case B



19. A small block is connected to one end of a massless spring to un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at t = 0, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at t =1s, the value of v is (take g = 10 m/s^2)



(a) $\sqrt{50}$ m/s (b) $\sqrt{51}$ m/s

(c) $\sqrt{52}$ m/s (d) $\sqrt{53}$ m/s

20. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time $t = 0$ with an initial velocity u_0. When the speed of the particle is 0.5 u_0, it collides elastically with a rigid wall. After this collision

(a) the speed of the particle when it returns to its equilibrium position is u_0

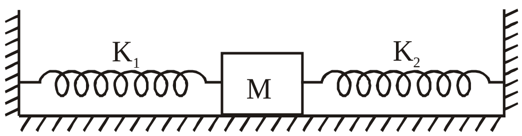
(b) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$

(c) the time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

(d) the time at which the particle passes through the equilirbium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

DCE QUESTIONS

1. If both spring constant K_1 and K_2 are increased 4 times, what will be the new frequency in terms of its original frequency?



(a) f (b) $2f$ (c) $f/2$ (d) $4f$

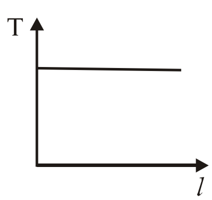
2. The displacements of a particle in SHM when KE = PE is (amplitude = $3\sqrt{2}$ cm) [

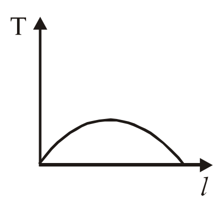
(a) 3 cm (b) 2 cm (c) $\frac{1}{\sqrt{2}}$ cm (d) $\sqrt{2}$ cm

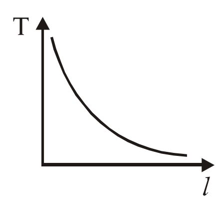
3. In the frequency of an oscillating particle is n, then the frequency of oscillations of its potential energy is

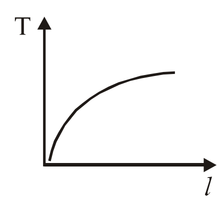
(a) n (b) $2n$ (c) $n/2$ (d) $4n$

4. In case of a simple pendulum, time period versus length is depicted by

(a) 

(b) 

(c) 

(d) 

5. If the length of a simple pendulum is tripled, what will be its new time period? (T = original time period)

(a) 0.7 T (b) 1.7 T (c) T/2 (d) T

6. A lift is falling freely under gravity, what is the time period of a pendulum attached to its ceiling?

(a) zero (b) infinity (c) one second (d) two second

7. If the frequency of oscillations of a particle doing SHM is n, the frequency of K.E. is

(a) $2n$ (b) n (c) $\frac{n}{2}$ (d) none of these

8. Energy of a particle executing SHM depends upon

(a) amplitude only (b) amplitude and frequency

(c) velocity only (d) frequency only

9. Two particles are executing SHMs. The equations of their motion are

$y_1 = 10 \sin(\omega t + \pi/4)$; $y_2 = 25 \sin(\omega t + 3\pi/4)$.

What is the ratio of their amplitudes?

(a) 1 : 1 (b) 2 : 5 (c) 1 : 2 (d) none of these

10. A particle executing S.H.M. at mid point of mean position and extremity. What is potential energy in terms of total energy (E)

(a) $\frac{E}{4}$ (b) $\frac{E}{16}$ (c) $\frac{E}{2}$ (d) $\frac{E}{8}$

11. What is time period of pendulum hanged in satellite? (T is time period on Earth)

(a) Zero (b) T (c) Infinite (d) $T/\sqrt{6}$

12. A spring of spring constant K is cut into two equal parts. A block of mass m is attached with one part of spring. What is the frequency of the system if α is frequency of block with original spring

(a) $\sqrt{2}\alpha$ (b) $\frac{\alpha}{2}$ (c) 2α (d) α

13. When the maximum K.E. of a simple pendulum is K, then what is its displacement (in terms of amplitude a) when its K.E. is $K/2$?

(a) $a/\sqrt{2}$ (b) $a/2$ (c) $a/\sqrt{3}$ (d) $a/3$

14. A spring (spring constant = k) is cut into 4 equal parts and two parts are connected in parallel. What is the effective spring constant ?

(a) 4 K (b) 16 K (c) 8 K (d) 6 K

MAINS QUESTIONS

1. The bob of simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $\left(\frac{4}{3}\right) \times 1000$ kg/m^3. What relationship between t and t_0 is true?

(a) $t = 2t_0$ (b) $t = t_0/2$ (c) $t = t_0$ (d) $t = 4t_0$

2. A particle at the end of a spring executes S.H.M. with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then

(a) $T^{-1} = t_1^{-1} + t_2^{-1}$ (b) $T^2 = t_1^2 + t_2^2$ (c) $T = t_1 + t_2$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

3. The total energy of a particle, executing simple harmonic motion is [

(a) independent of x (b) $\propto x^2$ (c) $\propto x$ (d) $\propto x^{1/2}$

where x is the displacement from the mean position.

4. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force F(t) proportional to cos ωt ($\omega \neq \omega_0$) is applied to the oscillator. The time displacement of the oscillator will be proportional to

(a) $\frac{1}{m(\omega_0^2 + \omega^2)}$ (b) $\frac{1}{m(\omega_0^2 - \omega^2)}$ (c) $\frac{m}{(\omega_0^2 - \omega^2)}$ (d) $\frac{m}{(\omega_0^2 + \omega^2)}$

5. In forced oscillation of particle the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force; then

(a) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large

(b) $\omega_1 > \omega_2$

(c) $\omega_1 = \omega_2$ (d) $\omega_1 < \omega_2$

6. The function $\sin^2(\omega t)$ represents

(a) a simple harmonic motion with a period $2\pi/\omega$

(b) a simple harmonic motion with a period π/ω

(c) a periodic, but not simple harmonic motion with a period $2\pi/\omega$

(d) a periodic, but not simple harmonic motion with a period π/ω

7. Two simple harmonic motions are represented by the equation $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos \pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is

(a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $-\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

8. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is

(a) $2\pi\alpha$ (b) $2\pi\sqrt{\alpha}$ (c) $\frac{2\pi}{\alpha}$ (d) $\frac{2\pi}{\sqrt{\alpha}}$

9. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would

(a) remain unchanged

(b) increase towards a saturation value

(c) first increase and then decrease to the original value

(d) first decrease and then increase to the original value

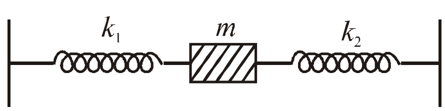
10. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is

(a) 0.01 s (b) 10 s (c) 0.1 s (d) 100 s

11. Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time

(a) $\frac{1}{6}$ s (b) $\frac{1}{4}$ s (c) $\frac{1}{3}$ s (d) $\frac{1}{12}$ s

12. Two springs, of force constants k_1 and k_2 are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



(a) $4f$ (b) $2f$ (c) $f/2$ (d) $f/4$

13. A particle of mass m executes simple harmonic motion with amplitude a and frequency ν. The average kinetic energy during its motion from the position of equilibrium to the end is

(a) $4\pi^2 ma^2\nu^2$ (b) $2\pi^2 ma^2\nu^2$ (c) $\pi^2 ma^2\nu^2$ (d) $1/4 ma^2\nu^2$

14. The displacement of an object attached to a spring and executing simple harmonic motion is given by

(a) 0.125 s (b) 0.25 s (c) 0.5 s (d) 0.75 s

15. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as : $a = A \cos(\omega t + \delta)$, then

(a) $A = x_0\,\omega^2$, $\delta = -\pi/4$

(b) $A = x_0\,\omega^2$, $\delta = 3\pi/4$

(c) $A = x_0$, $\delta = -\pi/4$

(d) $A = x_0\,\omega^2$, $\delta = \pi/4$

16. A mass M, attached to a horizontal spring, executes S.H.M with amplitude A_1. When the mass M passes through its mean position then a smaller mass *m* is placed over it and both to them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is

(a) $\frac{M}{M+m}$

(b) $\frac{M+m}{M}$

(c) $\left(\frac{M}{M+m}\right)^{1/2}$

(d) $\left(\frac{M+m}{M}\right)^{1/2}$

17. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance X_0 ($X_0 > A$). If the maximum separation between them is (X_0 + A), the phase difference between their motion is

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{6}$

18. If a simple pendulum has significant amplitude (up to a factor of $1/e$ of original) only in the period between $t = 0$ s to $t = \tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds:

(a) $\frac{0.693}{b}$

(b) b

(c) 1/b

(d) 2/b

19. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to α times its original magnitude, where α equals:

20. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance *a* and in next τ s it travels 2*a*, in same direction, then

(a) amplitude of motion is 4*a*

(b) time period of oscillations is 6τ

(c) amplitude of motion is 3*a*

(d) time period of oscillations is 8τ

BASIC LEVEL ASSIGNMENT

1. Consider a particle moving in simple harmonic motion according to the equation

$x = 2.0 \cos (50\, \pi t + \tan^{-1} 0.75)$

where x is in centimeter and t in second. The motion is started at $t = 0$.

(a) When does the particle come to rest for the first time?

(b) When does the acceleration has its maximum magnitude for the first time?

(c) When does the particle come to rest for the second time?

2. The equation of motion of a particle started at $t = 0$ is given by $x = 5 \sin (20\, t + \pi/3)$ where x is in centimetre and t in second. When does the particle

(a) first come to rest

(b) first have zero acceleration

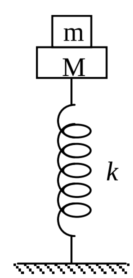
(c) first have maximum speed?

3. A particle having mass 10 g oscillates according to the equation $x = (2.0) \sin[(100)t + \pi/6]$ where x is in cm and t is in sec. Find

(a) the amplitude, the time period and the spring constant

(b) the position, the velocity and the acceleration at $t = 0$

4. A small block of mass m is kept on a bigger block of mass M which is attached to a vertical spring of spring constant k as shown in the figure. The system oscillates vertically.



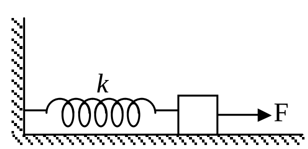
(a) Find the resultant force on the smaller block when it is displaced through a distance x above its equilibrium position.

(b) Find the normal force on the smaller block at this position. When is this force smallest in magnitude?

(c) What can be the maximum amplitude with which the two blocks may oscillate together?

5. A cubical body (side 0.1 m and mass 0.002 kg) floats in water. It is pressed slightly and then released so that it oscillates vertically. Find the time period.

6. The spring shown in figure is unstretched when a man starts pulling the block. The mass of the block is M. If the man exerts a constant force F, find

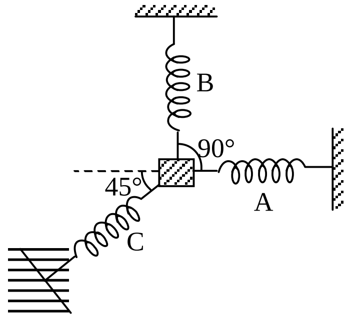


(a) the amplitude and the time period of the motion of the block,

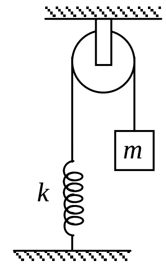
(b) the energy stored in the spring when the block passes through the equilibrium position and

(c) the kinetic energy of the block at equilibrium position.

7. A particle of mass m is attached to three springs A, B and C of equal force constants k as shown in figure. If the particle is pushed slightly against the spring C and released, find the time period of oscillation.

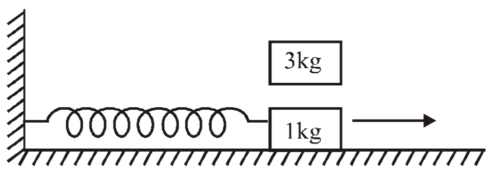


8. The string, the spring and the pulley shown in figure are light. Find the time period of the mass m if it is displaced slightly from equilibrium position

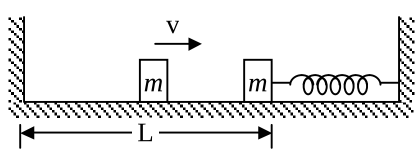


9. Solve the previous problem if the pulley has a moment of inertia I about its axis and the string does not slip over it.

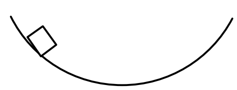
10. A 1 kg block is executing simple harmonic motion of amplitude 0.1 m on a smooth horizontal surface under the restoring force of a spring of spring constant 100 N/m. A block of mass 3 kg is gently placed on it at the instant it passes through the mean position. Assuming that the two blocks move together, find the frequency and the amplitude of the motion.



11. The left block in figure moves at a speed v towards the right block placed in equilibrium. All collisions to take place are elastic and the surfaces are frictionless. Show that the motions of the two blocks are periodic. Find the time period of these periodic motions. Neglect the widths of the blocks.

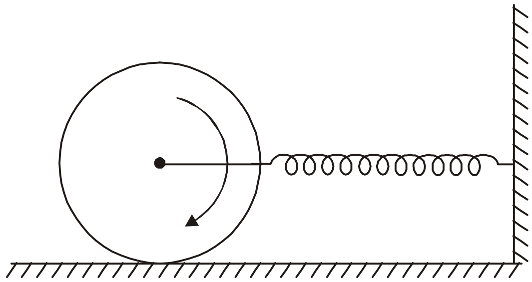


12. A small block oscillates back and forth on a smooth concave surface of radius R as shown in figure. Find the time period of small oscillation.



13. A spherical ball of mass m and radius r rolls without slipping on a rough concave surface of large radius R. It makes small oscillations about the lowest point. Find the time period.

14. A solid cylinder of mass M is attached to a massless spring of force constant k so that the cylinder can roll without slipping along the horizontal surface. Show that the centre of mass of the cylinder executes S.H.M. with a period given by $T = 2\pi\sqrt{\frac{3M}{2k}}$



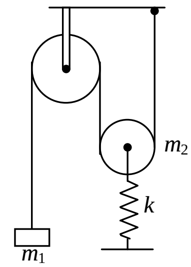
15. A horizontal spring block system of mass m executes simple harmonic motion. When the block is passing through its equilibrium position, an object mass m is put on it and the two move together. Find the new amplitude and frequency of vibration.

16. A particle of mass 0.1 kg is executing SHM of amplitude of 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} J. Obtain the equation of motion of this particle if initial phase of oscillation is 45°.

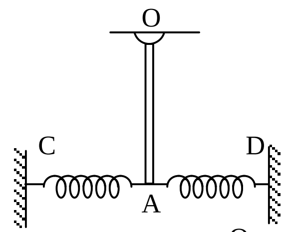
17. Two linear simple harmonic motion of equal amplitude and frequency ω and 2ω are impressed on a particle along the axes of x and y respectively. If the initial phase difference between them is $\pi/2$, find the resultant path following by the particle.

ADVANCED LEVEL ASSIGNMENT

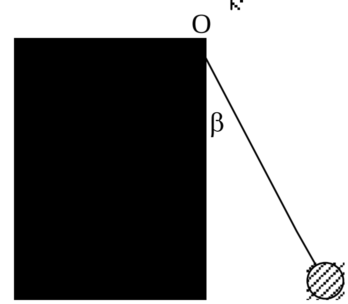
1. Calculate the period of free oscillations of the system shown in the figures if the mass m_1 is pulled down a little. Force constant of the spring is k and the masses of the fixed pulleys are negligible.



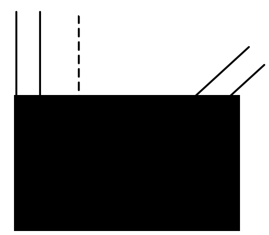
2. Calculate the frequency of oscillations of thin uniform rod of mass m and length l hinged at the upper end as shown in figure. The force constants of the springs are k_1 and k_2. The mass of the springs are negligible.



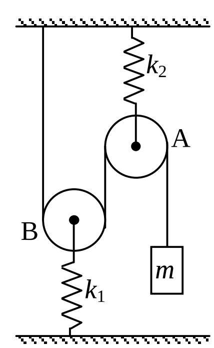
3. A ball is suspended by a thread of length l from a point O of a leaning wall, forming an angle α with the vertical. Thread is drawn through $\beta (\beta > \alpha)$ away from the wall and released. Assuming elastic collision of the ball with the wall, find the period of oscillations. Also consider the case $\alpha = \beta$.



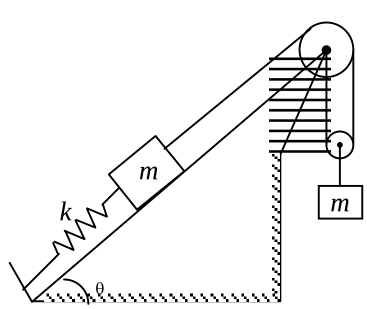
4. Determine the period of oscillations of a liquid of mass m = 200 g and density $\rho = 13.6 \times 10^3$ kg m^{-3} poured into a bent tube whose right arm forms an angle θ =60° with the vertical. The cross-sectional area of the tube is s = 0.5 cm².



5. A block of mass m is tied to one end of a string which passes over a smooth fixed pulley A and under a light smooth movable pulley B. The end of the string is attached to the lower end of a spring of spring constant k_2. The motion of the pulley B is controlled by a spring of spring constant k_1. Find the period of small oscillations.

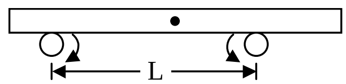


6. Calculate the natural frequency ω of the system shown in figure. There is no friction anywhere and the threads, spring and the pulleys are massless.



7. A pendulum clock is mounted in an elevator car which starts going up with a constant acceleration $a(a < g)$. At a height h the acceleration of the car reverses, its magnitude remaining constant. How soon after the start of the motion will the clock show the right time again?

8. A uniform plate of mass M stays horizontally and symmetrically on two wheels rotating in opposite directions. The separation between the wheels is L. The friction coefficient between each wheel and the plate is μ. Find the time period of oscillation of the plate if it is slightly displaced along its length and released.



9. Three simple harmonic motions of equal amplitudes A and equal time periods in the same direction combine. The phase of the second motion is 60° ahead of the first and the phase of the third motion is 60° ahead of the second. Find the amplitude of the resultant motion.

10. A particle is subjected to two simple harmonic motions given by

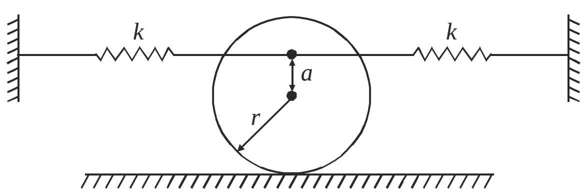
$x_1 = 2.0 \sin(100\,\pi t)$ and $x_2 = 2.0 \sin(120\,\pi t + \pi/3)$

where x is in centimeter and t in second. Find the displacement of the particle at

(a) $t = 0.012$ s (b) $t = 0.02$ s

11. A simple pendulum of length l oscillates with an amplitude of 30° on either side of the vertical. Show that the time period lies between $2\pi\sqrt{\frac{l}{g}}$ and $2\pi\sqrt{\frac{l}{g}}\sqrt{\frac{\pi}{3}}$. .

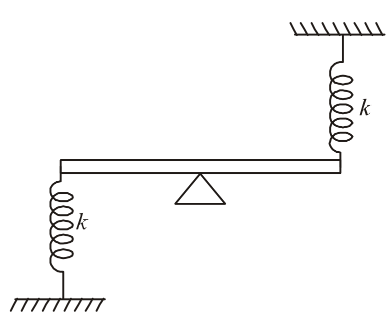
12. Use the energy method to find the natural frequency of the homogeneous cylinder as shown in the figure. Assume cylinder to be in pure rolling.



13. A uniform rod of length L and mass M is pivoted at its centre. It is held in position by a system of springs as shown in the figure. Show that when turned through a small angle θ_o and released the rod undergoes SHM with a frequency,

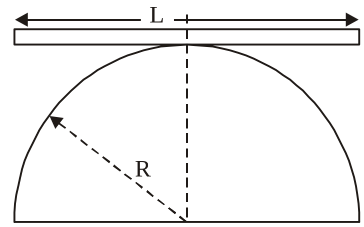
$$f = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$$

What will be the maximum speed of the tip of the rod?



14. A bob of mass M is attached to the lower end of a vertical string of length L and cross-sectional area A. The young's modulus of the material of the string is y. If the bob executes SHM in the vertical direction, find the frequency of these oscillations.

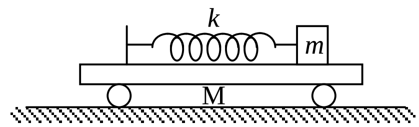
15. A uniform rod of length L and weight W is balanced on a fixed semi-circular cylinder of radius R as shown in the figure. If the plank is tilted slightly from its equilibrium position, then show that time period of its oscillations is given by $T = \frac{\pi L}{\sqrt{3gR}}$.



16. All the surfaces shown in figure are frictionless. The mass of the car is M, that of the block is m and the spring has spring constant k. Initially, the car and the block are at rest and the spring is stretched through a length x_0 when the system is released.

(a) Find the amplitudes of the simple harmonic motion of the block and of the car as seen from the road.

(b) Find the time period (s) of the two simple harmonic motions.

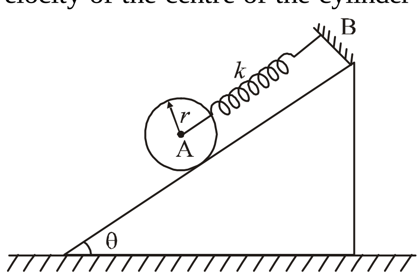


17. A body of mass m falls from a height h into a pan of a spring balance. The mass of the pan is M and that of the spring is negligible. The force constant of the spring is k. Having stuck the pan, the body starts performing harmonic oscillations in the vertical direction. Find the period, amplitude and energy of these oscillations.

18. A 7 kg uniform cylinder can roll without sliding on an incline and is attached to a spring AB as shown. If the centre of the cylinder is moved 10 mm drawn the incline and released. Find

(a) the period of oscillation.

(b) the maximum velocity of the centre of the cylinder (g = 10 m/s²)



given:
k = 800 N/m
r = 100 mm
θ = 15°

ANSWERS

Objective Questions

1.	(d)	2.	(d)	3.	(c)	4.	(b)	5.	(c)
6.	(c)	7.	(c, d)	8.	(a)	9.	(c)	10.	(d)
11.	(a, c)	12.	(a, d)	13.	(a, c)	14.	(d)	15.	(b)
16.	(a)	17.	(c)	18.	(b)	19.	(b)	20.	(b)
21.	(d)	22.	(a)	23.	(a)	24.	(a)	25.	(b)
26.	(b)	27.	(d)	28.	(d)	29.	(c)	30.	(b)
31.	(a,d)	32.	(a,b,c,d)	33.	(a,c)	34.	(a,c)	35.	(a,b,c,d)
36.	(b,c)	37.	(a,b)	38.	(a,c)	39.	(a,c)	40.	(b,c)

Miscellaneous Assignment

1. (d) 2. (c) 3. (b) 4. (c) 5. (a)

6. (a) 7. A-(r),(s); B-(r),(q),(s); C-(p),(r); D-(p),(q),(r),(s)

8. A-(r); B-(s); C-(p); D-(q) 9. (3) 10. (5)

11. (8) 12. (2) 13. (4) 14. (1) 15. (3)

16. (3) 17. (1) 18. (4) 19. (1)

Previous Year Questions

IIT-JEE/JEE-ADVANCE QUESTIONS

1.	(b)	2.	(a)	3.	(c, d)	4.	(d)	5.	(c)
6.	(b)	7.	(d)	8.	(b)	9.	(a)	10.	(a)
11.	(b)	12.	(b)	13.	(d)	14.	(a,b,d)	15.	(d)
16.	(c)	17.	(4)	18.	(a,d)	19.	(a)	20.	(a,d)

DCE QUESTIONS

1.	(b)	2.	(a)	3.	(b)	4.	(d)	5.	(b)
6.	(b)	7.	(a)	8.	(b)	9.	(c)	10.	(a)
11.	(c)	12.	(a)	13.	(a)	14.	(c)		

MAINS QUESTIONS

1.	(a)	2.	(b)	3.	(a)	4.	(b)	5.	(c)
6.	(b)	7.	(c)	8.	(d)	9.	(c)	10.	(a)
11.	(a)	12.	(b)	13.	(c)	14.	(c)	15.	(b)
16.	(d)	17.	(a)	18.	(d)	19.	(a)	20.	(b)

Basic Level Assignment

1. (a) 16 ms (b) 16 ms (c) 36 ms

2. (a) $\frac{\pi}{120}s$ (b) $\frac{\pi}{30}s$ (c) $\frac{\pi}{30}s$

3. (a) 2 cm, 0.0635, 100 N/m (b) 1 cm, 1.73 m/s, 100 m/s²

4. (a) $\frac{mkx}{M+m}$ (b) $mg - \frac{mkx}{M+m}$, at the highest point (c) $g\frac{(M+m)}{k}$

5. 0.028 sec.

6. (a) $\frac{F}{k}, 2\pi\sqrt{\frac{M}{k}}$ (b) $\frac{F^2}{2k}$ (c) $\frac{F^2}{2k}$

7. $2\pi\sqrt{\frac{m}{2k}}$

8. $2\pi\sqrt{\frac{m}{k}}$

9. $2\pi\sqrt{\frac{(m+I/r^2)}{k}}$

10. $\frac{5}{2\pi}$ Hz, 5 cm

11. $\left(\pi\sqrt{\frac{m}{k}}+\frac{2L}{v}\right)$

12. $2\pi\sqrt{\frac{R}{g}}$

13. $2\pi\sqrt{\frac{7}{5}\frac{(R-r)}{g}}$

15. $A' = A\left(\frac{M}{m+M}\right)^{1/2}, \ \omega' = \omega\sqrt{\frac{M}{M/m}}$

16. $y = 0.1 \sin(\pm 4t + \pi/4)$

17. $y^2 = 4x^2\left(1-\frac{x^2}{a^2}\right)$

Advanced Level Assignment

1. $2\pi\sqrt{\frac{m_2+4m_1}{k}}$

2. $\frac{1}{2\pi}\sqrt{\frac{3(k_1+k_2)}{m}+\frac{3g}{2l}}$

3. $2\sqrt{\frac{l}{g}}\left(\frac{\pi}{2}+\sin^{-1}\frac{\alpha}{\beta}\right), 2\pi\sqrt{\frac{l}{g}}$

4. 0.89 s

5. $4\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$

6. $\sqrt{\frac{4k}{5m}}$

7. $\sqrt{\frac{2h}{a}}\left[\frac{\sqrt{g+a}-\sqrt{g-a}}{\sqrt{g}-\sqrt{g-a}}\right]$

9. 2 A

10. (a) –2.41 cm (b) 0.27 cm

12. $f=\frac{1}{2\pi}\sqrt{\frac{4k(r+a)^2}{3\,mr^2}}$

13. L θ_0 $\sqrt{\frac{3k}{2\mathrm{M}}}$

14. $f=\frac{1}{2\pi}\sqrt{\frac{y\mathrm{A}}{\mathrm{ML}}}$

16. (a) $\frac{mx_0}{\mathrm{M}+m},\frac{mx_0}{\mathrm{M}+m}$ (b) $2\pi\sqrt{\left\{\frac{m\mathrm{M}}{k(\mathrm{M}+m)}\right\}}$

17. $\mathrm{T}=2\pi\sqrt{\frac{\mathrm{M}+m}{k}},\ a=\frac{(\mathrm{M}+m)g}{k}\sqrt{1+\frac{2m^2kh}{(\mathrm{M}+m)^3g}}\ ,\ \mathrm{E}=\frac{(\mathrm{M}+m)^2g^2}{2k}\left(1+\frac{2m^2kh}{(\mathrm{M}+m)^3g}\right)$

18. (a) 0.72 sec (b) 87.3 mm/s