Lesson-1
INDEFINITE INTEGRALS

Indefinite Integration as reverse of differentiation

If f and F are functions of x such that $F'(x) = f(x)$, then $F'(x) = f(x) \Rightarrow (F(x) + C)' = f(x)$ $\Rightarrow \int f(x)\, dx = F(x) + C$ so the function $F(x)$ is called as indefinite integral of $f(x)$ w.r.t. "x".

Here "C" is constant of integration; $f(x)$ is called the integrand and $F(x) + C$ is called its indefinite integral with respect to x.

so if $\frac{d}{dx}(f(x)) = g(x)$

then $\int g(x)\, dx = f(x) + c$

e.g., $\frac{d}{dx}(\sin x) = \cos x \Rightarrow \int \cos x\, dx = \sin x + c$

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n \Rightarrow \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Standard Results

- $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$
- $\int \frac{1}{x} dx = \ln|x| + c$
- $\int e^x dx = e^x + c$
- $\int a^x dx = \frac{a^x}{\ln a} + c;\ a \neq 1, a > 0$
- $\int \sin x\, dx = -\cos x + c$
- $\int \cos x\, dx = \sin x + c$
- $\int \sec^2 x\, dx = \tan x + c$
- $\int \text{cosec}^2 x\, dx = -\cot x + c$
- $\int \sec x \tan x\, dx = \sec x + c$
- $\int \text{cosec}\, x \cot x\, dx = -\text{cosec}\, x + c$

Properties of Indefinite Integration

- $\int [f_1(x) \pm f_2(x)]\, dx = \int f_1(x)\, dx \pm \int f_2(x) dx$
- $\int kf(x)\, dx = k\int f(x) dx$, k is a constant.

Integration has 3 methods

1. Substitution 2. Integration by parts 3. Partial fractions

Method 1: Integration by substitution :

Given an integral $\int f(x)dx,$ we usually reduce integrand $f(x)$ to a form which can be evaluated by using standard results. For this purpose, we use method of substitution.

For evaluating

$$\int g(f(x))\, f'(x)dx$$

Substitute $f(x) = t \Rightarrow dt = f'(x)dx$

Integral reduces to $\int g(f(x))\, f'(x)dx = \int g(t)\, dt$ which is or should be or supposed to be a standard result.

Illustration: $I = \int \tan x\, dx$

$$= \int \frac{\sin x}{\cos x} dx \qquad \text{Put } \cos x = t \Rightarrow dt = -\sin x dx$$

$$I = -\int \frac{dt}{t} = -\ln|t| + C = -\ln|\cos x| + C$$

Illustration: $I = \int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$$= \int \frac{\sec^2 x\, dx}{a^2 + b^2\tan^2 x} \qquad (\text{Put } \tan x = t\, ;\ dt = \sec^2 x\, dx)$$

$$= \int \frac{dt}{a^2 + b^2 t^2}$$

$$= \frac{1}{b^2}\int \frac{dt}{\frac{a^2}{b^2} + t^2}$$

$$= \frac{1}{b^2}\frac{b}{a}\tan^{-1}\left(\frac{b}{a}t\right) + c$$

$$= \frac{1}{ab}\tan^{-1}\left(\frac{b}{a}\tan x\right) + c$$

Standard Results:

- $\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c; n+1 \neq 0$.
- $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$
- $\int \frac{f'(x)}{\sqrt{f(x)}}\,dx = 2\sqrt{f(x)} + c$
- $\int \tan x\,dx = \ln|\sec x| + c$
- $\int \cot x\,dx = \ln|\sin x| + c$
- $\int \sec x\;dx = \ln|\sec x + \tan x| + c$

  $\int \sec x\,dx = \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$
- $\int \text{cosec}\,x\,dx = \ln|\text{cosec}\,x - \cot x| + c$

  $\int \text{cosec}\,x\;dx = \ln\left|\tan\frac{x}{2}\right| + c$

Standard substitutions:

Expression substitution

$a^2 - x^2$, $x = a\sin\theta$ or $x = a\cos\theta$

$x^2 + a^2$, $x = a\tan\theta$ or $x = a\cot\theta$

$x^2 - a^2$, $x = a\sec\theta$ or $x = a\,\text{cosec}\,\theta$

Standard Results:

- $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c$

- $$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{|a|}\right) + c$$

- $$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left| x + \sqrt{x^2 - a^2} \right| + c$$

- $$\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln\left| x + \sqrt{x^2 + a^2} \right| + c$$

Integration of Rational Functions

Let $f(x) = \dfrac{P(x)}{Q(x)} = \dfrac{B_0 x^m + B_1 x^{m-1} + ...+ B_m}{A_0 x^n + A_1 x^{n-1} + ...+ A_n}$; $(m, n \in N)$

If $m \geq n$, divide $P(x)$ by $Q(x)$ to get

$$\frac{P(x)}{Q(x)} = M(x) + \frac{P_1(x)}{Q(x)}$$

where $P_1(x)$ is a polynomial of degree less than the degree of $Q(x)$. Break $Q(x)$ into factors. Then, evaluate $\int f(x)\,dx$ by using either partial fractions method or by using other standard methods.

Integration of Trigonometric Functions

- To evaluate $I = \int \sin^m x \cos^n x\,dx$,

  (a) substitute $\sin x = t$, if n is an odd positive integer

  (b) substitute $\cos x = t$, if m is an odd positive integer

  (c) substitute $\tan x = t$, if $(m + n)$ is a negative even integer, where m and n can be integers or fractions.

- To evaluate $I = \int R(\sin x, \cos x)\,dx$, where R is a rational function of $\sin x$ and $\cos x$, we substitute $\tan \frac{x}{2} = t$ to reduce integrand R into algebraic form.

Illustration: $I = \int \dfrac{dx}{\sin x(2 + \cos x - 2\sin x)}$ (put $\tan \frac{x}{2} = t$)

$$= \int \frac{1}{\frac{2t}{1+t^2}\left(2 + \frac{1-t^2}{1+t^2} - \frac{4t}{1+t^2}\right)} \frac{2\,dt}{(1+t^2)}$$

$$= \int \frac{1+t^2}{t(t^2 - 4t + 3)}\,dt$$

$$= \int \frac{1+t^2}{t(t-3)(t-1)}\,dt$$

$$= \int \left(\frac{A}{t} + \frac{B}{t-3} + \frac{C}{t-1} \right) dt$$

$$= \frac{1}{3} \int \frac{1}{t} dt + \frac{5}{3} \int \frac{dt}{t-3} - \int \frac{1}{t-1} dt$$

$$= \frac{1}{3} \log |t| + \frac{5}{3} \log |t-3| - \log |t-1| + C$$

$$= \frac{1}{3} \log \left| \tan \frac{x}{2} \right| + \frac{5}{3} \log \left| \tan \frac{x}{2} - 3 \right| - \log \left| \tan \frac{x}{2} - 1 \right| + C$$

Illustration: $I = \int (\sin^2 x (\cos x)^{-6} dx$

$$= \int \frac{\sin^2 x}{\cos^2 x} \frac{1}{\cos^2 x} \frac{1}{\cos^2 x} dx$$

$$= \int \tan^2 x (1 + \tan^2 x) \sec^2 x \, dx$$

Put $\tan x = t$; $\sec^2 x \, dx = dt$

$$= \int t^2 (1 + t^2) dt$$

$$= \frac{t^3}{3} + \frac{t^5}{5} + C$$

$$= \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$$

Integration of Irrational Forms

Illustration: $I = \int \frac{dx}{(1+x)^{1/3} + (1+x)^{1/2}}$

L.C.M of 3 and 2 is 6 ; put $x + 1 = t^6$: $t > 0$

$\therefore \quad I = \int \frac{6t^5 dt}{t^2 + t^3}$

$$= 6 \int \frac{t^3 dt}{1+t} = 6 \int \left(t^2 - t + 1 - \frac{1}{t+1} \right) dt$$

$$= 6 \left(\frac{t^3}{3} - \frac{t^2}{2} + t - \ln |t+1| \right) + C$$

$$= 2(x+1)^{1/2} - 3(x+1)^{1/3} + 6(x+1)^{1/6} - 6 \ln |1 + (x+1)^{1/6}| + C.$$

☸ **To evaluate $\int \frac{dx}{A\sqrt{B}}$, where A and B are linear or quadratic expressions in x, we use the following steps :**

1. **If B is linear, put $B = t^2$**
2. **If A is linear and B is quadratic, put $A = 1/t$**
3. **If both A and B are quadratic, put $x = 1/t$.**

We illustrate them by examples :

Illustration: $I = \int \frac{x}{(x-3)\sqrt{x+1}}\, dx$

$$= \int \left(1 + \frac{3}{x-3}\right) \frac{1}{\sqrt{x+1}}\, dx$$

$$= \int \frac{dx}{\sqrt{x+1}} + 3\int \frac{dx}{(x-3)\sqrt{x+1}}$$

$$= 2\sqrt{x+1} + I_1 \qquad \text{...(i)}$$

In I_1, both expressions are linear. Put $x + 1 = t^2$: $t > 0$

$$I_1 = 3\int \frac{2t}{(t^2-4)t}\, dt$$

$$= 6\int \frac{dt}{(t^2-4)} = 6 \cdot \frac{1}{4} \log\left|\frac{t-2}{t+2}\right|$$

$$= \frac{3}{2} \log\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + C \qquad \text{...(ii)}$$

From (i) and (ii), we get the required value of I

Illustration: $I = \int \frac{dx}{(x+1)\sqrt{x^2-1}}$: $x > 1$

In this case, A is linear and B is quadratic.

We put $\frac{1}{x+1} = t \Rightarrow dx = -\frac{1}{t^2}\, dt$

$$\therefore \quad I = \int \frac{t}{\sqrt{\frac{1}{t}\left(\frac{1}{t}-2\right)}} \left(-\frac{1}{t^2}\right) dt$$

$$= \int \frac{-dt}{\sqrt{1-2t}}$$

$$= \sqrt{1-2t} + C$$

$$= \sqrt{1-\frac{2}{x+1}} + C$$

$$= \sqrt{\frac{x-1}{x+1}} + C.$$

Illustration: $I = \int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$: $x > 0$

Here both A and B are quadratic expressions :

Put $x = 1/t$; $dx = -\frac{1}{t^2}dt$

$$I = \int \frac{-dt}{t^2\left(1+\frac{1}{t^2}\right)\sqrt{1-\frac{1}{t^2}}}$$

$$= \int \frac{-t\,dt}{(t^2+1)\sqrt{t^2-1}}$$ [put $t^2 - 1 = z^2$ where $z > 0$; $2t\,dt = 2z\,dz$]

$$= \int \frac{-z\,dz}{(z^2+2)z}$$

$$= -\int \frac{dz}{2+z^2}$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\frac{z}{\sqrt{2}} + C$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\,x}\right) + C \cdot$$

❖ **Integrals of the form :**

$$\int \frac{px+q}{ax^2+bx+c}dx,\ \int \frac{px+q}{\sqrt{ax^2+bx+c}}dx,\ \int (px+q)\sqrt{ax^2+bx+c}\,dx$$

Here we write $px + q = l$ (diff. coefficient of $ax^2 + bx + c$) $+ m$

Find l and m by comparing the coefficients of x and the constant term on both sides of the equation. In this way, the question reduces to the sum of two integrals which can be integrated easily.

❖ $\int \frac{dx}{a\cos^2 x + 2b\sin x\cos x + c\sin^2 x}$ or $\int \frac{dx}{a\cos^2 x + b}$ or $\int \frac{dx}{a + b\sin^2 x}$

or $\int \frac{dx}{a\sin 2x + b\cos 2x + c}$ or $\int \frac{dx}{a\sin^2 x + b\cos^2 x + c}$: put $\tan x = t$

- $\int \frac{dx}{(ax+b)^m (cx+d)^n}$, when $m + n \in I$. Put $\frac{ax+b}{cx+d} = t$.

- $\int \frac{1+x^2}{1+x^4} dx$ and $\int \frac{1-x^2}{1+x^4} dx$

  Put $x - \frac{1}{x} = t$ and $x + \frac{1}{x} = t$ respectively.

- $\int \frac{1}{a\sin x + b\cos x} dx = \frac{1}{\sqrt{a^2+b^2}} \int \frac{1}{\sin(x+\alpha)} dx$ (where $\alpha = \tan^{-1} b/a$)

$$= \frac{1}{\sqrt{a^2+b^2}} \int \operatorname{cosec}(x+\alpha)\, dx$$

$$= \frac{1}{\sqrt{a^2+b^2}} \log\left|\tan\left(\frac{x+\alpha}{2}\right)\right| + c$$

Method 2: Integration by parts

If $u(x)$ and $v(x)$ are functions of x, which are differentiable, then

$$\int uv\, dx = uv_1 - \int v_1 \frac{du}{dx} dx \quad \text{where } v_1 = \int v dx$$

The choice of function $u(x)$ is usually done by following the sequence *ILATE*, where *I* stands for inverse circular function, *L* stands for logarithmic function, *A* stands for Algebraic function, *T* stands for Trigonometric function, and *E* stands for Exponential function.

Illustration: $I = \int \frac{x\tan^{-1} x}{\sqrt{1+x^2}} dx$

Integrating by parts,

$$I = (\tan^{-1} x)\sqrt{1+x^2} - \int \frac{1}{1+x^2}.\sqrt{1+x^2}\, dx$$

$$= (\tan^{-1} x)\sqrt{1+x^2} - \int \frac{dx}{\sqrt{1+x^2}}.$$

$$= \sqrt{1+x^2}\tan^{-1} x - \log|x + \sqrt{x^2+1}| + c.$$

Illustration: $I = \int \frac{dx}{\cos(x-\alpha)\cos(x-\beta)}$

$$= \frac{1}{\sin(\alpha-\beta)} \int \frac{\sin[(x-\beta)-(x-\alpha)]}{\cos(x-\alpha)\cos(x-\beta))} dx$$

$$= \frac{1}{\sin(\alpha-\beta)}\int \frac{\sin(x-\beta)\cos(x-\alpha)-\cos(x-\beta)\sin(x-\alpha)}{\cos(x-\alpha)\cos(x-\beta)}\,dx$$

$$= \frac{1}{\sin(\alpha-\beta)}\int [\tan(x-\beta)-\tan(x-\alpha)]dx$$

$$= \frac{1}{\sin(\alpha-\beta)}\left[-\log|\cos(x-\beta)|+\log|\cos(x-\alpha)|\right]+c$$

$$= \frac{1}{\sin(\alpha-\beta)}\log\left|\frac{\cos(x-\alpha)}{\cos(x-\beta)}\right|+c$$

Illustration: $I = \int \cos x \cos 2x \cos 3x \, dx$

$$= \int \frac{1}{2}(\cos 3x+\cos x)\cos 3x \, dx$$

$$= \frac{1}{4}\int 2\cos^2(3x)dx+\frac{1}{4}\int(2\cos x\cos 3x)dx$$

$$= \frac{1}{4}\int(1+\cos 6x)dx+\frac{1}{4}\int(\cos 4x+\cos 2x)dx$$

$$= \frac{1}{4}\left(x+\frac{\sin 6x}{6}\right)+\frac{1}{4}\left(\frac{\sin 4x}{4}+\frac{\sin 2x}{2}\right)+c$$

$$= \frac{1}{4}x+\frac{1}{24}\sin 6x+\frac{1}{16}\sin 4x+\frac{1}{8}\sin 2x+c$$

Standard Results:

- $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{|a|}\right)+c$
- $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right|+c$
- $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|+c$
- $\int e^x(f(x)+f'(x))\,dx = e^x f(x)+c$

SOLVED EXAMPLES

Ex.1: Evaluate $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}\, dx.$

Sol.: Method : To evaluate

$$I = \int \frac{P(x)}{Q(x)}\, dx,$$

If $P(x) = AQ(x) + BQ'(x)$ ...(i)

where A and B are constants, then

$$I = \int \frac{AQ(x) + BQ'(x)}{Q(x)}\, dx$$

$$= A\int dx + B\int \frac{Q'(x)}{Q(x)}\, dx$$

$$= Ax + B \log |Q(x)| + C$$

From (i), by comparing coefficients of same type of terms, one gets constants A and B.

In the present problem

$$I = \int \frac{(4e^x + 6e^{-x})}{9e^x - 4e^{-x}}\, dx$$

Denominator $Q(x) = 9e^x - 4e^{-x}$

Numerator $P(x) = 4e^x + 6e^{-x}$

As $Q'(x) = 9e^x + 4e^{-x}$, we take

$$4e^x + 6e^{-x} = A(9e^x - 4e^{-x}) + B(9e^x + 4e^{-x})$$

By comparing the coefficients of e^x and e^{-x}, we get

$$4 = 9A + 9B$$

$$6 = -4A + 4B$$

$$\therefore \quad A = -\frac{19}{36}, \quad B = \frac{35}{36}$$

$$\therefore \quad I = \int \frac{A(9e^x - 4e^{-x}) + B(9e^x + 4e^{-x})}{9e^x - 4e^{-x}}\, dx$$

$$= A\int dx + B\int \frac{9e^x + 4e^{-x}}{9e^x - 4e^{-x}}\, dx$$

$$= Ax + B \log |9e^x - 4e^{-x}| + C$$

$$= -\frac{19}{36}x + \frac{35}{36}\log |9e^x - 4e^{-x}| + C$$

Ex.2: Evaluate $I = \int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x}\, dx.$

Sol.: $I = \int \frac{\cos^3 x(1+\cos^2 x)}{\sin^2 x(1+\sin^2 x)}\,dx$

Since power of cos *x* is odd, put sin *x* = *t* ; then cos *x dx* = *dt*

$$I = \int \frac{(1-t^2)(1+1-t^2)}{t^2(1+t^2)}\,dt$$

$$= \int \frac{(1-t^2)(2-t^2)}{t^2(1+t^2)}\,dt$$

For partial fractions, integrand is a function of even powers of *t*.

$$\therefore \quad \frac{(1-t^2)(2-t^2)}{t^2(1+t^2)} = \frac{(1-z)(2-z)}{z(1+z)}$$

$$= 1 + \frac{2-4z}{z(1+z)}$$

$$= 1 + \frac{2}{z} - \frac{6}{1+z}$$

$$= 1 + \frac{2}{t^2} - \frac{6}{1+t^2}$$

$$\therefore \quad I = \int \left(1 + \frac{2}{t^2} - \frac{6}{1+t^2}\right)dt$$

$$= t - \frac{2}{t} - 6\tan^{-1} t + C = \sin x - 2\operatorname{cosec} x - 6\tan^{-1}(\sin x) + C.$$

Ex.3: Evaluate $\int \frac{1+x^2}{1-x^2}\frac{dx}{\sqrt{1+3x^2+x^4}}$. : $x > 0$

Sol.: $I = \int \frac{1+x^2}{1-x^2}\frac{dx}{\sqrt{1+3x^2+x^4}}$

$$= \int \frac{(1+x^2)dx}{x\left(\frac{1}{x}-x\right)x\sqrt{\frac{1}{x^2}+3+x^2}}$$

$$= \int \frac{(\frac{1}{x^2}+1)dx}{\left(\frac{1}{x}-x\right)\sqrt{(x-\frac{1}{x})^2+5}}$$

Put $x - 1/x = t$; $\left(1+\frac{1}{x^2}\right)dx = dt$

$$I = -\int \frac{dt}{t\sqrt{t^2+5}}$$

Put $t^2 + 5 = z^2$: $z > 0$; $2t\,dt = 2z\,dz$

$$I = -\int \frac{dz}{z^2 - 5}$$

$$= -\frac{1}{2\sqrt{5}} \ln\left|\frac{z-\sqrt{5}}{z+\sqrt{5}}\right| + C$$

$$= -\frac{1}{2\sqrt{5}} \ln\left|\frac{\sqrt{\left(x^2+\frac{1}{x^2}+3\right)}-\sqrt{5}}{\sqrt{\left(x^2+\frac{1}{x^2}+3\right)}+\sqrt{5}}\right| + C.$$

Ex.4: Evaluate $\int \operatorname{cosec}^2 x.\ln(\cos x + \sqrt{\cos 2x})dx \ : \sin x > 0$

Sol.: $$I = \int \operatorname{cosec}^2 x.\ln\left[\sin x\left(\cot x + \frac{\sqrt{\cos^2 x - \sin^2 x}}{\sin x}\right)\right]dx$$

$$= \int \operatorname{cosec}^2 x.\ln[\sin x.(\cot x + \sqrt{\cot^2 x - 1})]\,dx$$

$$= \int \operatorname{cosec}^2 x.\ln\sin x\,dx + \int \operatorname{cosec}^2 x.\ln[\cot x + \sqrt{\cot^2 x - 1}\,]dx$$

$$= I_1 + I_2$$

$$I_1 = \int \operatorname{cosec}^2 x.\ln\sin x\,dx$$ (integrate by parts)

$$= (-\cot x).\ln\sin x - \int(-\cot x).\cot x\,dx$$

$$= -\cot x.\ln\sin x + \int(\operatorname{cosec}^2 x - 1)\,dx$$

$$= -\cot x.\ln\sin x - \cot x - x$$

$$I_2 = \int \operatorname{cosec}^2 x.\ln[\cot x + \sqrt{\cot^2 x - 1}\,]dx$$

Put $\cot x = t; \quad -\operatorname{cosec}^2 x\ dx = dt$

$$I_2 = -\int \ln[t + \sqrt{t^2 - 1}\,]dt$$ (integrate by parts)

$$= -t.\ln(t + \sqrt{t^2-1}) + \int t.\frac{1+\frac{t}{\sqrt{t^2-1}}}{t+\sqrt{t^2-1}}\,dt$$

$$= -t.\ln(t + \sqrt{t^2-1}) + \int \frac{t}{\sqrt{t^2-1}}\,dt$$

$$= -t.\ln(t + \sqrt{t^2-1}) + \sqrt{t^2-1} + C$$

$$= -\cot x.\ln(\cot x + \sqrt{\cot^2 x - 1}) + \sqrt{\cot^2 x - 1} + C$$

Ex.5: Evaluate : $I = \int \frac{\sin x}{\sin^3 x + \cos^3 x}dx$.

Sol.: If the integrand contains odd powers in sinx and cos x, put tan $x = t$ to evaluate I.

$$I = \int \frac{1}{\cos^3 x} \frac{\sin x}{(1+\tan^3 x)} dx$$

$$= \int \frac{\tan x.\sec^2 x}{1+\tan^3 x} dx \qquad \text{(put } \tan x = t\text{)}$$

$$= \int \frac{t}{1+t^3} dt$$

$$= -\frac{1}{3}\int \frac{dt}{1+t} + \frac{1}{3}\int \frac{t+1}{t^2-t+1} dt \qquad \text{(by partial fractions)}$$

$$= -\frac{1}{3}\log|t+1| + \frac{1}{6}\int \frac{(2t-1)+3}{t^2-t+1} dt$$

$$= -\frac{1}{3}\log|t+1| + \frac{1}{6}\log|t^2-t+1| + \frac{1}{2}\int \frac{dt}{(t-\frac{1}{2})^2+\frac{3}{4}}$$

$$= -\frac{1}{3}\log|t+1| + \frac{1}{6}\log|t^2-t+1| + \frac{1}{2}\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t-\frac{1}{2}}{\sqrt{3}/2}\right) + C$$

$$= -\frac{1}{3}\log|1+\tan x| + \frac{1}{6}\log|1-\tan x+\tan^2 x| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right) + C$$

Ex.6: If $I_{m,n} = \int \cos^m x.\cos nx\, dx$, show that $(m+n) I_{m,n} = \cos^m x \sin nx + m\, I_{m-1,\, n-1}$

Sol.: Integrating by parts,

$$I_{m,n} = \cos^m x \frac{\sin nx}{n} + \frac{m}{n}\int \cos^{m-1} x \sin x \sin nx\, dx \qquad \text{...(i)}$$

But $\cos (n-1)x = \cos (nx - x)$

$= \cos nx \cos x + \sin nx \sin x$

$\sin x \sin nx = \cos(n-1)x - \cos nx \cos x$ ...(ii)

From (i) and (ii):

$$I_{m,n} = \frac{1}{n}\cos^m x.\sin nx + \frac{m}{n}\int \cos^{m-1} x[\cos(n-1)x - \cos nx \cos x]dx$$

$$I_{m,n} = \frac{1}{n}\cos^m x \sin nx + \frac{m}{n} I_{m-1,n-1} - \frac{m}{n} I_{m,n}$$

$$I_{m,n} = \frac{1}{m+n}\cos^m x \sin nx + \frac{m}{m+n} I_{m-1,\, n-1}.$$

Ex.7: Evaluate : $I = \int (x+\sqrt{1+x^2})^n dx$.

Sol.: Let $x + \sqrt{1+x^2} = t$; then,

$$\left(1+\frac{x}{\sqrt{1+x^2}}\right)dx = dt \qquad \text{or} \qquad \frac{t}{\sqrt{1+x^2}}dx = dt$$

$\therefore \quad dx = \sqrt{1+x^2}\,\frac{dt}{t}$

As $\sqrt{1+x^2} + x = t$

$\Rightarrow \quad \frac{1}{t} = \frac{1}{\sqrt{1+x^2}+x} = \frac{\sqrt{1+x^2}-x}{1}$

$\therefore \quad 2\sqrt{1+x^2} = t + \frac{1}{t} = \frac{t^2+1}{t}$

Thus $\quad I = \int t^n . \frac{t^2+1}{2t}\frac{dt}{t}$

$= \frac{1}{2}\int t^{n-2}(t^2+1)\,dt$

$= \frac{1}{2}\int (t^n + t^{n-2})\,dt$

$= \frac{1}{2}\left(\frac{t^{n+1}}{n+1} + \frac{t^{n-1}}{n-1}\right) + C$

where $\quad t = x + \sqrt{1+x^2}$.

Ex.8: Evaluate: $I = \int \frac{2\sin^3(x/2)\,dx}{(\cos(x/2))\sqrt{\cos^3 x + 3\cos^2 x + \cos x}}$: $\cos x > 0$

Sol.: $\quad I = \int \frac{(2\sin(x/2)\cos(x/2))\,(2\sin^2(x/2))\,dx}{(2\cos^2(x/2))\sqrt{\cos^3 x + 3\cos^2 x + \cos x}}$

$\therefore \quad I = \int \frac{(1-\cos x)\sin x\,dx}{(1+\cos x)\sqrt{\cos^3 x + 3\cos^2 x + \cos x}}$ [put $\cos x = t$]

$= \int \frac{(t-1)dt}{(1+t)\sqrt{t^3+3t^2+t}}$

$= \int \frac{(t^2-1)dt}{(t+1)^2 t\sqrt{t+3+\frac{1}{t}}}$

$= \int \frac{t^2(1-\frac{1}{t^2})dt}{t(t^2+2t+1)\sqrt{t+\frac{1}{t}+3}}$

$= \int \frac{(1-\frac{1}{t^2})\,dt}{(t+\frac{1}{t}+2)\sqrt{(t+\frac{1}{t})+3}}$

Put $\quad t + \frac{1}{t} + 3 = z^2$: $z > 0$; then $\left(1 - \frac{1}{t^2}\right)dt = 2zdz$

$I = \int \frac{2zdz}{(z^2-1).z}$

$$= 2\int \frac{dz}{z^2-1} = \log\left|\frac{z-1}{z+1}\right| + C$$

$$\therefore \quad I = \log\left|\frac{\sqrt{\cos x+\sec x+3}-1}{\sqrt{\cos x+\sec x+3}+1}\right| + C.$$

Ex.9: Evaluate $\int \dfrac{e^x(2-x^2)dx}{(1-x)\sqrt{1-x^2}}$.

Sol.: $$I = \int e^x \frac{(1-x^2)+1}{(1-x)\sqrt{1-x^2}}\,dx$$

$$= \int e^x \left\{\frac{1+x}{\sqrt{1-x^2}} + \frac{1}{(1-x)\sqrt{1-x^2}}\right\}dx$$

But $$\frac{d}{dx}\left(\frac{1+x}{\sqrt{1-x^2}}\right) = \frac{1}{\sqrt{1-x^2}} + (1+x)\frac{x}{(1-x^2)^{3/2}}$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{(1+x)x}{(1-x)(1+x)\sqrt{1-x^2}}$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{x}{(1-x)\sqrt{1-x^2}}$$

$$= \frac{1-x+x}{(1-x)\sqrt{1-x^2}} = \frac{1}{(1-x)\sqrt{1-x^2}}$$

Hence, integrand is of type $e^x\,(f(x) + f'(x))$

$$\therefore \quad I = e^x \frac{1+x}{\sqrt{1-x^2}} + C.$$

Ex.10: Evaluate : $I = \int \dfrac{(\sin x - \cos x)dx}{(\sin x+\cos x)\sqrt{\sin x\cos x+\sin^2 x\cos^2 x}}$.

Sol.: Let $\sin x + \cos x = t$; then, $(\cos x - \sin x)dx = dt$

Also, $t^2 = (\sin x + \cos x)^2 = 1 + 2\sin x\cos x$

$$\therefore \quad \sin x\cos x = \frac{t^2-1}{2}$$

$$I = -\int \frac{dt}{t\sqrt{\frac{t^2-1}{2}\left(1+\frac{t^2-1}{2}\right)}}$$

$$= -\int \frac{dt}{t\sqrt{\frac{(t^2-1)(t^2+1)}{4}}}$$

$$= -2\int \frac{t^3 dt}{t^4\sqrt{t^4-1}} \qquad [\text{Put } t^4 - 1 = z^2 : z > 0]$$

$$= -2\int \frac{1}{4}\frac{2z\,dz}{(z^2+1)z}$$

$$= -\int \frac{dz}{1+z^2}$$

$$= -\tan^{-1} z + C$$

$$= -\tan^{-1}\sqrt{t^4-1} + C$$

$$= -\tan^{-1}\sqrt{(1+\sin 2x)^2 - 1} + C$$

$$= -\tan^{-1}\sqrt{\sin^2 2x + 2\sin 2x} + C.$$

Ex.11: Evaluate : $I = \int \frac{dx}{1+\sqrt{x^2+x+2}}$.

Sol.: $I = \int \frac{dx}{1+\sqrt{(x+\frac{1}{2})^2+\frac{7}{4}}}$

Put $x + \frac{1}{2} = \frac{\sqrt{7}}{2}\tan\theta : -\frac{\pi}{2} < \theta < \frac{\pi}{2}$; then $dx = \frac{\sqrt{7}}{2}\sec^2\theta\, d\theta$

$$I = \int \frac{\sqrt{7}}{2}\frac{\sec^2\theta d\theta}{1+\frac{\sqrt{7}}{2}\sec\theta}$$

$$= \frac{\sqrt{7}}{2}\int \frac{d\theta}{\cos\theta(\cos\theta+\frac{\sqrt{7}}{2})}$$

$$= \int \left(\frac{1}{\cos\theta} - \frac{1}{\cos\theta+\frac{\sqrt{7}}{2}}\right)d\theta$$

$$= \log|\sec\theta+\tan\theta| - \int \frac{d\theta}{a+\cos\theta};\ a = \frac{\sqrt{7}}{2}$$

$$I = \log|\sec\theta + \tan\theta| - I_1 \qquad ...(i)$$

where $I_1 = \int \frac{d\theta}{a+\cos\theta}$

Put $\tan\frac{\theta}{2} = t$; $\cos\theta = \frac{1-t^2}{1+t^2}$

$$I_1 = \int \frac{2\,dt}{1+t^2}\frac{1}{a+\frac{1-t^2}{1+t^2}}$$

$$= 2\int \frac{dt}{a(1+t^2)+1-t^2}$$

$$= \frac{2}{a-1}\int \frac{dt}{\frac{a+1}{a-1}+t^2}$$

$$= \frac{2}{a-1}\sqrt{\frac{a-1}{a+1}}\tan^{-1}\left(\sqrt{\frac{a-1}{a+1}}\,t\right)+C$$

$$= \frac{2}{\sqrt{a^2-1}}\tan^{-1}\left(\sqrt{\frac{a-1}{a+1}}\tan\frac{\theta}{2}\right)+C \qquad \text{...(ii)}$$

From (i) and (ii), we get I.

Ex.12: Evaluate: $I = \int \frac{2+3\cos\theta}{\sin\theta+2\cos\theta+3}\,d\theta$.

Sol.: Write Numerator = l (denominator) + m (derivative of denominator) + n

$$2 + 3\cos\theta = l(\sin\theta + 2\cos\theta + 3) + m(\cos\theta - 2\sin\theta) + n$$

Comparing constant term and the coefficients of $\cos\theta$ and $\sin\theta$,

$3l + n = 2 \qquad 2l + m = 3 \qquad l - 2m = 0$

$\therefore \quad l = 6/5 \qquad m = 3/5 \qquad$ and $\qquad n = -8/5$

$$\therefore \quad I = \int \frac{6}{5}\,d\theta + \frac{3}{5}\int \frac{\cos\theta - 2\sin\theta}{\sin\theta+2\cos\theta+3}\,d\theta - \frac{8}{5}\int \frac{d\theta}{\sin\theta+2\cos\theta+3}$$

$$= \frac{6}{5}\theta + \frac{3}{5}\ln|\sin\theta+2\cos\theta+3| - \frac{8}{5}I_3$$

where $I_3 = \int \frac{d\theta}{\sin\theta+2\cos\theta+3}$

$$= \int \frac{d\theta}{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)+4\cos^2\left(\frac{\theta}{2}\right)-4\sin^2\left(\frac{\theta}{2}\right)+3\sin^2\left(\frac{\theta}{2}\right)+3\cos^2\left(\frac{\theta}{2}\right)}$$

$$= \int \frac{\sec^2(\theta/2)\,d\theta}{-\tan^2\frac{\theta}{2}+2\tan\frac{\theta}{2}+7} \; : \; \text{put } \tan\left(\frac{\theta}{2}\right) = t\,;$$

$$= -2\int \frac{dt}{t^2-2t-7} = -2\int \frac{dt}{(t-1)^2-8} = 2\int \frac{dt}{(2\sqrt{2})^2-(t-1)^2}$$

$$= \frac{1}{2\sqrt{2}}\log_e\left|\frac{(2\sqrt{2}-1)+\tan(\theta/2)}{(2\sqrt{2}+1)+\tan(\theta/2)}\right|$$

BASIC LEVEL ASSIGNMENT

Evaluate the following integrals :

1. $\int \frac{\sec x}{\sec x + \tan x} dx$

2. $\int \frac{x^4 - 1}{x^2 (x^4 + x^2 + 1)^{1/2}} . dx$

3. $\int \tan^{-1}\left(\frac{\sin 2x}{1 + \cos 2x}\right) dx \ : x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

4. $\int \frac{1 + \tan x}{x + \log \sec x} dx$

5. $\int \frac{2\cos x - 3\sin x}{3\cos x + 2\sin x} dx$

6. $\int \frac{(x-1)dx}{x\sqrt{x^2 - 1}}$

7. $\int \frac{dx}{\sqrt{1 - 3x} - \sqrt{5 - 3x}}$

8. $\int x^2 e^{x^3} \cos (e^{x^3}) \, dx$

9. $\int \frac{\sec^2 (2\tan^{-1} x)}{1 + x^2} dx$

10. $\int \frac{dx}{(2\sin x + 3\cos x)^2}$

11. $\int \cos^{3/5} x \sin^3 x \, dx$

12. $\int \frac{\log x}{x^2} dx$

13. $\int \sin^{-1} \sqrt{\frac{x}{a + x}} \, dx \ : a > 0$

14. $\int e^x \, \frac{2 + \sin 2x}{1 + \cos 2x} dx$

15. $\int \frac{dx}{x \, [6(\log x)^2 + 7\log x + 2]}$

16. $\int \frac{x^2 + 1}{(x+3)(x-1)^2} dx$

17. $\int \frac{1}{1 - \tan x} dx$

18. If $f'(x) = x - \frac{1}{x^2}$ and $f(1) = \frac{1}{2}$, find $f(x)$.

19. For any natural number m, evaluate $\int (x^{3m} + x^{2m} + x^m)(2x^{2m} + 3x^m + 6)^{1/m} \, dx, \ x > 0$

20. $\int \frac{\sin(x + \alpha)}{\cos(x + \alpha) + \cos(x - \alpha)} dx$

21. $\int \frac{dx}{\sin(x - \alpha)\cos(x - \beta)}$

22. $\int \frac{dx}{a\cos x + b\sin x}$

23. $\int \frac{dx}{(x + a)^{8/7} (x - b)^{6/7}}$

24. $\int \frac{\sqrt{x}}{\sqrt{x} + \sqrt[3]{x}} dx$

25. $\int \frac{e^{m\tan^{-1} x}}{(1 + x^2)^{3/2}} dx$

26. $\int \frac{dx}{(x - \beta)\sqrt{(x - \alpha)(\beta - x)}}$

27. $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$

28. $\int \left(x + \frac{1}{x}\right)^{n+5} \left(\frac{x^2 - 1}{x^2}\right) dx$

29. $\int \frac{dx}{(a\cos x - b\sin x)^2}$

30. $\int x^3 . \sqrt{1 + x^2} \, dx$

ADVANCED LEVEL ASSIGNMENT

Evaluate the following integrals :

1. $\int \frac{dx}{\sin x + \sec x}$

2. $\int \frac{\sqrt{\cos 2x}}{\sin x} dx \; : \cos x > 0$

3. $\int \frac{\sqrt{x^2+1}\,(\log(x^2+1) - 2\log x)}{x^4} dx$

4. $\int \frac{\sin x}{\sin x - \cos x} dx$

5. $\int x \sin^{-1}\left(\frac{1}{2}\sqrt{\frac{2a-x}{a}}\right) dx$

6. $\int \sec^4 x \operatorname{cosec}^2 x \, dx$

7. $\int \frac{d\theta}{(a+b\cos\theta)^2} \; : a > b > 0$

8. $\int \frac{2dx}{(1-x^3)^{1/3}}$

9. $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+\alpha)}}$

10. $\int \frac{\tan^{-1} x}{x^4} dx$

11. $\int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$

12. $\int \frac{\cos 8x - \cos 7x}{1 + 2\cos 5x} dx$

13. $\int \frac{x^3+1}{x(x-1)^3} dx$

14. $\int \frac{1+x^{2/3}}{1+x} dx$

15. $\int \frac{dx}{x + \sqrt{x^2 - x + 1}}$

16. $\int \frac{dx}{(x^2+a^2)^3}$

17. $\int \frac{dx}{\sqrt[3]{\sin^{11} x . \cos x}}$

18. $\int \frac{(x-\sin x)^{3/2}}{\sqrt{x}} . \left\{ \frac{6x^2 \sin^2 \frac{x}{2}}{x - \sin x} + 3x \right\} dx$

19. Evaluate $\int \frac{f(x)}{x^3-1} dx$, where $f(x)$ is polynomial of the second degree in x such that $f(0) = f(1) = 3f(2) = -3$.

20. If $I_{n,m} = \int x^{n-1}.(1-x)^m \, dx$, then prove that

$$I_{n,m} = \frac{m}{n} I_{n+1,m-1} + \frac{x^n (1-x)^m}{n}.$$

21. $\int \frac{\sin^{-1}\sqrt{x}}{\sqrt{1-x}}\,dx$

22. $\int \frac{dx}{x^4 + x^6}$

23. $\int \cos^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+5}}\right)dx$

24. $\int \sqrt{4+\sqrt{1+\frac{x}{3}}}\;dx$

25. $\int \frac{x\log x}{(x^2-1)^{3/2}}\,dx$

26. $\int \frac{dx}{\sqrt{1+x}-\sqrt[3]{1+x}}$

27. $\int \sec 2x.\sqrt{\tan x}\;dx$

28. $\int \frac{\sec^2 x}{(\sec x+\tan x)^n}dx \quad (n > 1)$

29. $\int \frac{dx}{\sin^4 x+\cos^4 x}$

30. $\int \frac{x^3+3x+2}{(x^2+1)^2(x+1)}\;dx$

OBJECTIVE ASSIGNMENT

Choose the correct option in the following :

1. If the derivative of $f(x)$ w.r.t. x is $\dfrac{\frac{1}{2}-\sin^2 x}{f(x)}$, then $f(x)$ is a periodic function with period

(a) π (b) 2π (c) $\pi/2$ (d) none of these

2. If $\int \dfrac{x^4}{1+x^2}\,dx = A\dfrac{x^3}{3}+Bx+\tan^{-1}x+C$, then A, B are

(a) 1, –1 (b) –1, 1 (c) 1, 1 (d) none of these

3. $\int (\tan x+\cot x)^2\,dx =$

(a) $\tan x + \cot x + A$ (b) $\tan x - \cot x + A$ (c) $\tan x - 2\cot x + A$ (d) none of these

4. If $\int \dfrac{6x-7}{(3x^2-7x+5)^2}\,dx = \dfrac{K}{3x^2-7x+5}+A$, then $K =$

(a) –1 (b) 1 (c) 2 (d) none of these

5. $\int \dfrac{1}{x\log x}\,dx =$

(a) $\log|\log x| + A$ (b) $(\log x)^2 + A$ (c) $(\log x)^{-1} + A$ (d) none of these

6. If $\int \dfrac{1}{\cos x-\sin x}\,dx = a\log\left|\tan\left(\dfrac{\pi}{8}-\dfrac{x}{2}\right)\right|+A$, then $a =$

(a) $\dfrac{1}{\sqrt{2}}$ (b) $-\dfrac{1}{\sqrt{2}}$ (c) $-\sqrt{2}$ (d) none of these

7. $\int e^{2x}\left(\dfrac{2x-1}{4x^2}\right)dx =$

(a) $\dfrac{e^{2x}}{4x}+A$ (b) $\dfrac{e^{2x}}{2x}+A$ (c) $\dfrac{e^{2x}}{x}+A$ (d) none of these

8. $\int \dfrac{x\tan^{-1}x}{(1+x^2)^{3/2}}\,dx = a\dfrac{\tan^{-1}x}{\sqrt{1+x^2}}+\dfrac{x}{\sqrt{1+x^2}}+A$, then $a =$

(a) 1 (b) 2 (c) –1 (d) none of these

9. $\int \frac{(2x^{12}+5x^9)}{(x^5+x^3+1)^3}dx$ is equal to

(a) $\frac{x^{10}}{2(x^5+x^3+1)^2}+c$

(b) $\frac{x^2+2x}{(x^5+x^3+1)}+c$

(c) $\ln(x^5+x^3+1+\sqrt{2x^7+5x^4})+c$

(d) none of these

10. If $\int \frac{2\sin 2x-\cos x}{4-\cos^2 x-4\sin x}dx = a\log|\sin x-1|+b\log|\sin x-3|+A$, then a and b are

(a) $-\frac{3}{2},\frac{11}{2}$

(b) $\frac{3}{2},\frac{11}{2}$

(c) $\frac{3}{2},-\frac{11}{2}$

(d) none of these

11. Let $A=\int e^{ax}\cos bx\,dx$ and $B=\int e^{ax}\sin bx\,dx$, then the value of $(A^2+B^2)(a^2+b^2)^2$ is

(a) e^{2x}

(b) e^{2ax}

(c) $e^{a\sqrt{x}}$

(d) none of these

12. The value of integral $\int \frac{\sqrt{\tan x}}{\sin x\cos x}dx$ is equal to

(a) $2\sqrt{\tan x}+C$

(b) $\sqrt{x}+C$

(c) $2^{x^{3/2}}$

(d) $2\sin^{3/2}x+C$

13. $\int \frac{x}{x^4+x^2+1}dx =$

(a) $\frac{1}{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+C$

(b) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+C$

(c) $\frac{1}{\sqrt{3}}\cot^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+C$

(d) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2}{3}\right)+C$

14. The value of the integral $\int \frac{dx}{\sqrt{(x-1)(2-x)}}$ is

(a) $\sin^{-1}(2x-3)+C$

(b) $\sin^{-1}\frac{x}{2}$

(c) $\sin^{-1}\frac{3x}{2}$

(d) none of these

15. The values of a and b whch satisfy $f'(1)=2$, $\int_0^3 f(x)\,dx=7$, $f(x)=a2^x+b$, are

(a) 1, 2

(b) $\frac{1}{\log_e 2}$, 0

(c) $\frac{1}{\log_e 2},\frac{3[(\log_e 2)^2-1]}{7(\log 2)^2}$

(d) $\frac{1}{\log 2},\frac{7(\log 2)^2-7}{3(\log 2)^2}$

16. $\int 7^{7^{7^x}}.7^{7^x}.7^x\,dx$

(a) $7^{7^{7^x}}(\log 7)^3 + C$ (b) $\dfrac{7^{7^{7^x}}}{(\log 7)^3} + C$ (c) $\dfrac{7^{7^{7^x}}}{(\log 7)}$ (d) none of these

17. $\int \dfrac{dx}{(1+x^2)\sqrt{1-x^2}} =$

(a) $-\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{\sqrt{2}\,x}\right)+c$ (b) $\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{\sqrt{2}\,x}\right)+c$

(c) $\dfrac{3}{2}\tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{\sqrt{2}\,x}\right)+c$ (d) none of these

18. $\int \dfrac{\sin x + \cos x}{\sqrt{1+\sin 2x}}\,dx =$

(a) $\log|\sin x + \cos x| + c$ (b) $-x + c$ for all x

(c) $\begin{cases} x+c, & \text{if } \sin x + \cos x > 0 \\ -x+c, & \text{if } \sin x + \cos x < 0 \end{cases}$ (d) $x + c$ for all x.

19. If $\dfrac{3\pi}{4} < x < \dfrac{7\pi}{4}$, then $\int \dfrac{\sin x + \cos x}{\sqrt{1+\sin 2x}}\,dx =$

(a) $-x + c$ (b) $x + c$

(c) $\log|\sin x + \cos x| + c$ (d) none of these

20. $\int xe^x \cos x\,dx = f(x) + c,$ then $f(x)$ equal to

(a) $-\dfrac{e^x}{2}\{(1-x)\sin x - x\cos x\} + c$ (b) $\dfrac{e^x}{2}\{(1-x)\sin x + x\cos x\} + c$

(c) $\dfrac{e^x}{2}\{(1+x)\sin x - x\cos x\} + c$ (d) none of these

21. If $\int \dfrac{\cos 4x + 1}{\cot x - \tan x}\,dx = A\cos 4x + B$ where A & B are constants, then :

(a) A = – 1/4 & B may have any value (b) A = – 1/8 & B may have any value

(c) A = – 1/2 & B = – 1/4 (d) none of

these

22. $\int \sqrt{\cot x}\ \sec^4 x\ d\,x$ =

(a) $\sqrt{2\tan x} + \frac{2}{5}\sqrt{\tan^5 x} + c$ (b) $2\sqrt{\tan x} + \frac{2}{5}\sqrt{\tan^5 x} + c$

(c) $\sqrt{\tan x} + \frac{2}{5}\sqrt{\tan^5 x} + c$ (d) $\sqrt{\tan x} + \frac{1}{5}\sqrt{\tan^5 x} + c$

23. $\int \frac{\tan^{-1} x - \cot^{-1} x}{\tan^{-1} x + \cot^{-1} x}\, dx$ is equal to :

(a) $\frac{4}{\pi}\, x \tan^{-1} x + \frac{2}{\pi} \ln (1 + x^2) - x + c$ (b) $\frac{4}{\pi}\, x \tan^{-1} x - \frac{2}{\pi} \ln (1 + x^2) + x + c$

(c) $\frac{4}{\pi}\, x \tan^{-1} x + \frac{2}{\pi} \ln (1 + x^2) + x + c$ (d) $\frac{4}{\pi}\, x \tan^{-1} x - \frac{2}{\pi} \ln (1 + x^2) - x + c$

24. $\int \frac{\ln |x|}{x\sqrt{1 + \ln |x|}}\, dx$ equals :

(a) $\frac{2}{3}\sqrt{1 + \ln |x|}\ (\ln|x| - 2) + c$ (b) $\frac{2}{3}\sqrt{1 + \ln |x|}\ (\ln|x| + 2) + c$

(c) $\frac{1}{3}\sqrt{1 + \ln |x|}\ (\ln|x| - 2) + c$ (d) $2\sqrt{1 + \ln |x|}\ (3\ln|x| - 2) + c$

25. $\int \sin x \,.\, \cos x \,.\, \cos 2x \,.\, \cos 4x \,.\, \cos 8x \,.\, \cos 16x\ dx$ equals :

(a) $\frac{\sin 16x}{1024} + c$ (b) $-\frac{\cos 32x}{1024} + c$

(c) $\frac{\cos 32x}{1096} + c$ (d) $-\frac{\cos 32x}{1096} + c$

26. $\int \frac{(x+1)}{x\left(1 + xe^x\right)^2}\, dx$ =

(a) $\ln\left(\frac{x\,e^x}{1+x\,e^x}\right) + \frac{1}{1+e^x} + c$

(b) $\ln\left(\frac{x\,e^x}{1+e^x}\right) + \frac{1}{1+x\,e^x} + c$

(c) $\ln\left(\frac{x\,e^x}{1+x\,e^x}\right) + \frac{x}{1+x\,e^x} + c$

(d) $\ln\left(\frac{x\,e^x}{1+x\,e^x}\right) + \frac{1}{1+x\,e^x} + c$

27. $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x}\, dx$:

(a) $\sin x - 6\tan^{-1}(\sin x) + c$

(b) $\sin x - 2\sin^{-1} x + c$

(c) $\sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + c$

(d) $\sin x - 2(\sin x)^{-1} + 5\tan^{-1}(\sin x) + c$

28. If $\int \frac{e^{4x}-1}{e^{2x}} \log\left(\frac{e^{2x}+1}{e^{2x}-1}\right) dx = \frac{t^2}{2}\log t - \frac{t^2}{4} - \frac{u^2}{2}\log u + \frac{u^2}{4} + C$, then

(a) $t = u = e^x + e^{-x}$

(b) $t = e^x - e^{-x}$, $u = e^x + e^{-x}$

(c) $t = e^x + e^{-x}$, $u = e^x - e^{-x}$

(d) none of these

29. The primitive of $\frac{1}{(x-a)^{3/2}(b-x)^{1/2}}$ is

(a) $\frac{1}{b-a}\left[\frac{b-x}{x-a}\right]^{1/2} + C$

(b) $\frac{3}{4(b-a)}\left[\frac{b-x}{x-a}\right]^{1/2} + C$

(c) $\frac{1}{b-a}\left[\frac{x-a}{b-a}\right]^{1/2} + C$

(d) $\frac{2}{a-b}\sqrt{\frac{b-x}{x-a}} + C$

30. The value of $\int \frac{dx}{x^{13}(x^6-1)}$ is

(a) $\log\left(\left|\frac{x^6-1}{x^6}\right|\right) + x^{-6} + x^{-12} + C$

(b) $\frac{1}{6}\log\left|\frac{x^6-1}{x^6}\right| + \frac{1}{6}x^{-6} + \frac{1}{4}x^{-12} + C$

(c) $\frac{1}{6}\left(\log\left|\frac{x^6-1}{x^6}\right| + x^{-6} + \frac{1}{2}x^{-12}\right) + C$

(d) none of these

MORE THAN ONE CORRECT ANSWERS

31. $\int \frac{x^2+\cos^2 x}{1+x^2}$ cosec$^2 x$ dx is equal to :

(a) $\cot x - \cot^{-1} x + c$ (b) $c - \cot x + \cot^{-1} x$

(c) $-\tan^{-1} x - \frac{\text{cosec } x}{\sec x} + c$ (d) $-e^{\ln \tan^{-1} x} - \cot x + c$

where 'c' is constant of integration .

32. $\int \frac{\ln(\tan x)}{\sin x \cos x}\, dx$ equal :

(a) $\frac{1}{2}\ln^2(\cot x) + c$ (b) $\frac{1}{2}\ln^2(\sec x) + c$

(c) $\frac{1}{2}\ln^2(\sin x \sec x) + c$ (d) $\frac{1}{2}\ln^2(\cos x \text{ cosec } x) + c$

33. If $\frac{dx}{5+4\cos x} = K\tan^{-1}\left(M\tan\frac{x}{2}\right) + C$, then

(a) K = 1 (b) K = 2/3 (c) M = 1/3 (d) M = 2/3

34. If $\int \frac{xe^x}{\sqrt{1+e^x}}\, dx = f(x)\sqrt{1+e^x} - 2\log g(x) + C$, then

(a) $f(x) = x - 1$ (b) $g(x) = \frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}$

(c) $g(x) = \frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}$ (d) $f(x) = 2(x-2)$

35. If $\int xe^{-5x^2}\sin 4x^2\, dx = Ke^{-5x^2}(A\sin 4x^2 + B\cos 4x^2) + C$. The

(a) $K = -\frac{1}{82}$ (b) $K = \frac{1}{82}$ (c) A = 5 (d) none of these

36. If the antiderivative of $\sin^{-1}\sqrt{\frac{x}{x+1}}$ is $x\sin^{-1}\sqrt{\frac{x}{x+1}} - \sqrt{x} + fog(x) + C$ then

(a) $f(x) = \sin^{-1} x$ (b) $g(x) = \sqrt{x+1}$ (c) $f(x) = \tan^{-1} x$ (d) $g(x) = \sqrt{x}$

37. $\int \frac{\left(x+\sqrt{1+x^2}\right)^{15}}{\sqrt{1+x^2}} dx =$

(a) $\frac{\left(x+\sqrt{1+x^2}\right)^{15}}{15} + c$ (b) $\frac{\left(x+\sqrt{1+x^2}\right)^{15}}{16} + c$

MISCELLANEOUS ASSIGNMENT

Comprehension-1

$y = f(x)$ is a polynomial function passing through point (0, 1) and which increases in the intervals (1, 2) and (3, ∞) and decreases in the intervals (–∞, 1) and (2, 3)

1. If $f(1) = -8$, then the value of $f(2)$ is

(a) – 3 (b) –6 (c) –20 (d) –7

2. If $f(1) = -8$, then the range of $f(x)$ is

(a) $[3, \infty)$ (b) $[-8, \infty)$ (c) $[-7, \infty)$ (d) $(-\infty, 6]$

3. If $f(x) = 0$ has four real roots, then the range of values of leading co-efficient of polynomial is

(a) [4/9, 1/2] (b) [4/9, 1] (c) [1/3, 1/2] (d) none of these

Comprehension-2

Let $f(x)$ be a polynomial of degree 3 such that $f(0) = 1$, $f(1) = 2$, $x = 0$ is a cirtical point but $f(x)$ does not have local extremum at $x = 0$.

4. $f(x)$ will be

(a) $x^3 + x^2 + 1$ (b) $x^3 + 1$ (c) $x^3 - x + 1$ (d) x^3

5. $\int \frac{f(x)}{\sqrt{x^2+7}}\,dx =$

(a) $\frac{\sqrt{x^2+7}}{3}(x^2-14)+\ln|x+\sqrt{x^2+7}|+C$ (b) $\frac{\sqrt{x^2+7}}{3}(x^2-14)+\ln(x+\sqrt{x^2+7})+C$

(c) both (a) & (b) (d) none of these

6. If origin is shifted at (0, 1), then w.r.t. new system, function will be

(a) even (b) odd

(c) symmetric about origin (d) no symmetry

7. Match the following

A. The value of $\int \sqrt{3x+2}\,dx$ is (p) $2\log(1+\sqrt{x}) + C$

B. The value of $\int \frac{1}{\sqrt{x}+\sqrt{x+1}}\,dx$ is (q) $\sin^{-1}x + \sqrt{1-x^2} + C$

C. The value of $\int \sqrt{\frac{1-x}{1+x}}\,dx$ is (r) $-\frac{2}{3}x^{3/2} + \frac{2}{3}(x+1)^{3/2} + C$

D. The value of $\int \frac{1}{x+\sqrt{x}}\,dx$ is (s) $\frac{2}{9}(3x+2)^{3/2} + C$

8. $\int f(x)\,dx$ if

A. $f(x) = \dfrac{1}{(x^2+1)\sqrt{x^2+2}}$

B. $f(x) = \dfrac{1}{(x+2)\sqrt{x^2+6x+7}}$

C. $f(x) = \dfrac{x^4+x^8}{(1-x^4)^{7/2}}$

D. $f(x) = \sqrt{\dfrac{1-\sqrt{x}}{1+\sqrt{x}}}$

(p) $\dfrac{x^5}{5(1-x^4)^{5/2}} + C$

(q) $\sin^{-1}\left(\dfrac{x+1}{(x+2)\sqrt{2}}\right) + C$

(r) $-2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{x}\sqrt{1-x} + C$

(s) $-\tan^{-1}\sqrt{1+2/x^2} + C$

INTEGER TYPE QUESTIONS

9. If $\int \dfrac{(x-x^3)^{1/3}}{x^4}dx = a\left(\dfrac{1}{x^2}-1\right)^b + c,$ then the value of $1/|ab|$ is

10. Let $f(x) = \int\left(\dfrac{\cos x}{x} - \log x^{\sin x}\right)dx$ and $f(1) = 0$, then the value of $f(\pi/2)$ is

11. If $f(x) = \int \dfrac{2+\sqrt{x}}{(x+1+\sqrt{x})^2}dx$ and $f(0) = 0$, then the value of $[f(4)]$ is ...... (where [.] represents the greatest integer function.

12. If $\int \dfrac{\cos^2 x + \sin 2x}{(2\cos x - \sin x)^2}dx = \dfrac{\cos x}{2\cos x - \sin x} + ax + b \ln|2\cos x - \sin x| + c$, then $|a + 2b|$ is

13. $\dfrac{\sin x}{\sin x - \cos x}dx = \dfrac{1}{m}x + \dfrac{1}{2}\log|\sin x - \cos x| + C$, then m is

14. If $\int \sin^{-7/5} x \cos^{-3/5} x\,dx = -\dfrac{\lambda}{2}(\cot x)^{2/5} + C,$ then λ is

15. If $\int(\sqrt{\tan x}+\sqrt{\cot x})dx = A\tan^{-1}\left[\dfrac{\sqrt{\tan x}-\sqrt{\cot x}}{B}\right] + C$, then the value of A/B is

16. If $\int(\sin 4x)e^{\tan^2 x}dx = a\cos^b x\, e^{\tan^2 x} + k$

17. If $\int \dfrac{\cos^4 x\,dx}{\sin^3 x[\sin^5 x + \cos^5 x]^{3/5}} = -\dfrac{1}{A}\left(\dfrac{1+\tan^5 x}{\tan^5 x}\right)^B + k$, then the value of $A + 5B$.

18. If $\int \dfrac{x+(\cos^{-1}3x)^2}{\sqrt{1-9x^2}}dx = -\dfrac{1}{A}\sqrt{1-9x^2} - \dfrac{1}{B^2}(\cos^{-1}3x)^3 + C$, then A is

PREVIOUS YEAR QUESTIONS

IIT-JEE QUESTIONS

1. $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx$

(a) $\frac{\sqrt{2x^4-2x^2+1}}{x}+c$

(b) $\frac{\sqrt{2x^4-2x^2+1}}{x^2}+c$

(c) $\frac{\sqrt{2x^4-2x^2+1}}{x^3}+c$

(d) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2}+c$

2. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for $n \geq 2$ and $g(x) = \underbrace{(f \circ f \circ \cdots \circ f)}_{f \text{ occurs } n \text{ times}}(x)$. Then $\int x^{n-2} g(x)\,dx$ equals

(a) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$

(b) $\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}}+K$

(c) $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}}+K$

(d) $\frac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}}+K$

3. Let $I = \int \frac{e^x}{e^{4x}+e^{2x}+1}dx$, $J = \int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$

Then, for an arbitrary constant C, the value of $J - I$ equals

(a) $\frac{1}{2}\log\left(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)+C$

(b) $\frac{1}{2}\log\left(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right)+C$

(c) $\frac{1}{2}\log\left(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)+C$

(d) $\frac{1}{2}\log\left(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+C$

4. The integral $\int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$ equals (for some arbitrary constant K)

(a) $-\frac{1}{(\sec x-\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(b) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(c) $-\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(d) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

DCE QUESTIONS

1. $\int \frac{\cos 2x}{\cos x} dx$ is equal to

(a) $2 \sin x + \log (\sec x - \tan x) + c$ (b) $2 \sin x - \log (\sec x - \tan x) + c$

(c) $2 \sin x + \log (\sec x + \tan x) + c$ (d) $2 \sin x - \log (\sec x + \tan x) + c$

2. $\int \frac{2x^2 + 3}{(x^2 - 1)(x^2 + 4)} dx = a \log \left(\frac{x+1}{x-1}\right) + b \tan^{-1} \frac{x}{2}$, then (a, b) is

(a) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (c) $(-1, 1)$ (d) $(1, -1)$

3. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx =$

(a) $e^{\sqrt{x}}$ (b) $\frac{e^{\sqrt{x}}}{2}$ (c) $2.e^{\sqrt{x}}$ (d) $\sqrt{x}.e^{\sqrt{x}}$

4. What is the value of the integral

$$I = \int \frac{dx}{(1+e^x)(1+e^{-x})}$$

(a) $\frac{-1}{1+e^x}$ (b) $\frac{e^x}{1+e^x}$ (c) $\frac{1}{1+e^x}$ (d) none of these

5. $\int 5^{5^{5^x}}.5^{5^x}.5^x \, dx$ is equal to

(a) $\frac{5^{5^x}}{(\log 5)^3} + C$ (b) $5^{5^{5^x}} (\log 5)^3 + C$ (c) $\frac{5^{5^{5^x}}}{(\log 5)^3} + C$ (d) none of these

6. The value of the $\int \frac{\sin x + \cos x}{3 + \sin 2x} dx$ is

(a) $\frac{1}{4} \ln\left(\frac{2 + \sin x - \cos x}{2 - \sin x + \cos x}\right) + c$ (b) $\frac{1}{2} \ln\left(\frac{2 + \sin x}{2 - \sin x}\right) + c$

(c) $\frac{1}{4} \ln\left(\frac{1 + \sin x}{1 - \sin x}\right) + c$ (d) none of these

7. The value of $\int \frac{dx}{x+\sqrt{x-1}}$ is

(a) $\log\left(x+\sqrt{x-1}\right)+\sin^{-1}\left(\sqrt{\frac{x-1}{x}}\right)+c$

(b) $\log\left(x+\sqrt{x-1}\right)+c$

(c) $\ln\left(x+\sqrt{x-1}\right)-\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)+c$

(d) none of these

8. The value of $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}}\,dx$ is

(a) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+\text{C}$ (b) $\frac{4}{3}\left(\frac{x+1}{x+2}\right)^{1/4}+\text{C}$ (c) $\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{1/4}+\text{C}$ (d) $\frac{4}{3}\left(\frac{x-1}{x-2}\right)^{1/4}+\text{C}$

AIEEE/JEE-MAINS QUESTIONS

1. If $\int \frac{\sin x}{\sin(x-\alpha)}\,dx = Ax + B\log\sin(x-\alpha) + C$, then value of (A, B) is

(a) $(\sin\alpha, \cos\alpha)$ (b) $(-\cos\alpha, \sin\alpha)$ (c) $(-\sin\alpha, \cos\alpha)$ (d) $(\cos\alpha, \sin\alpha)$

2. $\int \frac{1}{\cos x - \sin x}\,dx$ is equal to

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C$

(b) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$

(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+C$

(d) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+C$

3. $\int \left\{\frac{(\log x-1)}{1+(\log x)^2}\right\}^2 dx$ is equal to

(a) $\frac{xe^x}{1+x^2}+C$ (b) $\frac{x}{(\log x)^2+1}+C$ (c) $\frac{\log x}{(\log x)^2+1}+C$ (d) $\frac{x}{x^2+1}+C$

4. The value of integral $\int \sqrt{2x-x^2}\,dx$ is

(a) $\frac{(x-1)\sqrt{2x-x^2}}{2}+\frac{\sin^{-1}(x-1)}{2}+c$

(b) $\frac{x\sqrt{2x-x^2}}{2}+\sin^{-1}(x-1)+c$

(c) $\frac{\sqrt{2x-x^2}}{2}+\sin^{-1}(x-1)+c$

(d) none of these

5. $\int \frac{1}{\cos x + \sqrt{3}\sin x} dx$ equals

(a) $\log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$

(b) $\log \tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$

(c) $\frac{1}{2}\log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$

(d) $\frac{1}{2}\log \tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$

6. If the integral $\int \frac{5\tan x}{\tan x - 2} dx = x + a\ln|\sin x - 2\cos x| + k$, then a is equal to

(a) -1 (b) -2 (c) 1 (d) 2

7. If $\int f(x)\,dx = \psi(x)$, then $\int x^5 f(x^3)\,dx$ is equal to

(a) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$

(b) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)dx\right] + C$

(c) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)dx\right] + C$

(d) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)dx + C$

8. The integral $\int\left(1 + x - \frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to

(a) $(x-1)e^{x+\frac{1}{x}} + c$ (b) $xe^{x+\frac{1}{x}} + c$ (c) $(x+1)e^{x+\frac{1}{x}} + c$ (d) $-xe^{x+\frac{1}{x}} + c$

ANSWERS

Basic Level Assignment

1. $\tan x - \sec x + C$

2. $\dfrac{\sqrt{x^4+x^2+1}}{x} + c$

3. $\dfrac{x^2}{2} + C$

4. $\log|x + \log(\sec x)| + C$

5. $\log|3\cos x + 2\sin x| + C$

6. $\log|x+\sqrt{x^2-1}| - \sec^{-1}(x) + c$

7. $\dfrac{1}{18}\left[(1-3x)^{3/2} + (5-3x)^{3/2}\right] + C$

8. $\dfrac{1}{3}\sin(e^{x^3}) + C$

9. $\dfrac{1}{2}\tan(2\tan^{-1}x) + C$

10. $-\dfrac{1}{2.(2\tan x + 3)} + C$

11. $-\dfrac{5}{8}\cos^{8/5}x + \dfrac{5}{18}\cos^{18/5}x + C$

12. $\dfrac{-\log x}{x} - \dfrac{1}{x} + C$

13. $(x+a)\tan^{-1}\sqrt{\dfrac{x}{a}} - \sqrt{ax} + C$

14. $e^x \tan x + C$

15. $\log|2\log x + 1| - \log|3\log x + 2| + C$

16. $\dfrac{5}{8}\log|x+3| + \dfrac{3}{8}\log|x-1| - \dfrac{1}{2(x-1)} + C$

17. $\dfrac{1}{2}x - \dfrac{1}{2}\log|\cos x - \sin x| + C$

18. $f(x) = \dfrac{x^2}{2} + \dfrac{1}{x} - 1$

19. $\dfrac{1}{6(m+1)}(2x^{3m} + 3x^{2m} + 6x^m)^{m+1/m} + C$

20. $\dfrac{1}{2}\log(\sec x) + \dfrac{1}{2}x\tan\alpha + c$

21. $\sec(\alpha - \beta).\log\left(\dfrac{\sin(x-\alpha)}{\cos(x-\beta)}\right) + c$

22. $\dfrac{1}{\sqrt{a^2+b^2}}\log\tan\left(\dfrac{\pi}{4} + \dfrac{x-\alpha}{2}\right) + c$

23. $\dfrac{7}{(a+b)}\left(\dfrac{x-b}{x+a}\right)^{1/7} + c$

24. $x - \dfrac{6}{5}x^{5/6} + \dfrac{3}{2}x^{2/3} - 2x^{1/2} + 3x^{1/3} - 6x^{1/6} + 6\log(1 + x^{1/6}) + c$

25. $\frac{(x+m)e^{m\tan^{-1}x}}{(1+m^2)\sqrt{1+x^2}}+c$

26. $\frac{-2}{(\beta-\alpha)}\sqrt{\frac{x-\alpha}{(\beta-x)}}+c$

27. $x\log(\log x)-\frac{x}{\log x}+c$

28. $\frac{\left(x+\frac{1}{x}\right)^{n+6}}{n+6}+c$

29. $\frac{1}{b(a-b\tan x)}+c$

30. $\frac{3x^2-2}{15}(1+x^2)^{3/2}+c$

Advanced Level Assignment

1. $I=\tan^{-1}(\sin x+\cos x)-\frac{1}{2\sqrt{3}}\log\left|\frac{\sin x-\cos x-\sqrt{3}}{\sin x-\cos x+\sqrt{3}}\right|+C$

2. $\sqrt{2}\log\left(\sqrt{2}\cos x+\sqrt{\cos 2x}\right)-\frac{1}{2}\log\left|\frac{\cos x+\sqrt{\cos 2x}}{\cos x-\sqrt{\cos 2x}}\right|+C$

3. $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+C$

4. $\frac{1}{2}x+\frac{1}{2}\log|\sin x-\cos x|+C$

5. $\frac{x^2}{2}\sin^{-1}\left(\frac{1}{2}\sqrt{\frac{2a-x}{a}}\right)+\frac{a^2}{2}\left(\sin^{1}\frac{x}{2a}-\frac{x}{2a}\sqrt{1-\frac{x^2}{4a^2}}\right)+C$

6. $\frac{1}{3}\tan^3 x+2\tan x-\cot x+C$

7. $\frac{1}{b\sin\theta}\times\frac{1}{a+b\cos\theta}+\frac{1}{b}(aI_2-bI_3)+C$ where

$$I_2=-\frac{1}{(a^2-b^2)\sin\theta}-\frac{b}{(a^2-b^2)^{3/2}}\tan^{-1}\frac{b\sin\theta}{\sqrt{a^2-b^2}}$$

$$I_3=\frac{-1}{a^2-b^2}\left(\cot\theta+\frac{a}{\sqrt{a^2-b^2}}\tan^{-1}\frac{a\tan\theta}{\sqrt{a^2-b^2}}\right)$$

8. $\frac{1}{3}\ln\frac{(z+1)^2}{z^2-z+1}-\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2z-1}{\sqrt{3}}\right)+C$; where $z=\left(\frac{1}{x^3}-1\right)^{1/3}$.

9. $-\frac{2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}}+C$

10. $-\frac{\tan^{-1}x}{3x^3}+\frac{1}{6}\log\left|\frac{x^2+1}{x^2}\right|-\frac{1}{6x^2}+c$

11. $\frac{2(\sqrt{x}-1)}{\sqrt{1-x}}+C$

12. $\frac{1}{3}\sin 3x-\frac{1}{2}\sin 2x+C$

13. $-\frac{1}{(x-1)^2}-\frac{1}{(x-1)}+2\log|(x-1)|-\log|x|+C$

14. $\frac{3}{2}x^{2/3}+\log\left|1+x^{1/3}\right|-\frac{1}{2}\log|x^{2/3}-x^{1/3}+1|-\sqrt{3}\tan^{-1}\frac{2x^{1/3}-1}{\sqrt{3}}+C$

15. $2\log|x+\sqrt{x^2-x+1}|-\frac{3}{2}\log|2x-1+2\sqrt{x^2-x+1}|-\frac{3}{2}\times\frac{1}{(2x-1)+2\sqrt{x^2-x+1}}+c$

16. $I=\frac{x}{4a^2(x^2+a^2)^2}+\frac{3}{4a^2}\left\{\frac{x}{2a^2(x^2+a^2}+\frac{1}{2a^3}\tan^{-1}\left(\frac{x}{a}\right)\right\}+c$

17. $\frac{-3[4\tan^2 x+1]}{8\tan^2 x\tan^{2/3}x}+c$

18. $2x^{3/2}(x-\sin x)^{3/2}+c$

19. $\log\frac{x^2+x+1}{|x-1|}+\frac{2}{\sqrt{3}}\tan^{-1}\frac{2x+1}{\sqrt{3}}+c$

21. $2(\sqrt{x}-\sqrt{1-x}.\sin^{-1}\sqrt{x})+c$

22. $-\frac{1}{3x^3}+\frac{1}{x}+\tan^{-1}x+c$

23. $(x+1)\cot^{-1}\left(\frac{x+1}{2}\right)+\log(x^2+2x+5)+c$

24. $\frac{4}{5.\sqrt[4]{27}}(4\sqrt{3}+\sqrt{3+x})^{3/2}(\sqrt{3x+9}-8)+c$

25. $\sec^{-1}x-\frac{\log x}{\sqrt{x^2-1}}+c$

26. $2\sqrt{1+x}+3\sqrt[3]{1+x}+6\sqrt[6]{1+x}+6\log(\sqrt[6]{1+x}-1)+c$

27. $\frac{1}{2}\log\left|\frac{1+\sqrt{\tan x}}{1-\sqrt{\tan x}}\right|-\tan^{-1}\sqrt{\tan x}+c$

28. $-\frac{1}{2}\frac{1}{(\sec x+\tan x)^{n-1}}\left\{\frac{1}{n-1}+\frac{1}{n+1}\frac{1}{(\sec x+\tan x)^2}\right\}+c$

29. $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\cot 2x\right)+c$

30. $\frac{1}{4}\log(x^2+1)-\frac{1}{2}\log(x+1)+\frac{3}{2}\tan^{-1}x+\frac{x}{x^2+1}+C$

Objective Assignment

1.	(a)	2.	(a)	3.	(b)	4.	(a)	5.	(a)
6.	(b)	7.	(a)	8.	(c)	9.	(a)	10.	(a)
11.	(b)	12.	(a)	13.	(b)	14.	(a)	15.	(d)
16.	(b)	17.	(a)	18.	(c)	19.	(a)	20.	(a)
21.	(b)	22.	(b)	23.	(d)	24.	(a)	25.	(b)
26.	(d)	27.	(c)	28.	(c)	29.	(d)	30.	(c)
31.	(b,c,d)	32.	(a,c,d)	33.	(b,c)	34.	(b,d)	35.	(a,c)
36.	(c,d)	37.	(a,d)	38.	(a,b)	39.	(a,b,c,d)	40.	(b,c,d)

Miscellaneous Assignment

1. (d) 2. (b) 3. (a) 4. (b) 5. (a)

6. (b) 7. A-(s); B-(r); C-(q); D-(p) 8. A-(s); B-(q); C-(p); D-(r)

9.	(2)	10.	(0)	11.	(1)	12.	(1)	13.	(2)
14.	(5)	15.	(1)	16.	(2)	17.	(4)	18.	(9)

Previous Year Questions

IIT-JEE/JEE-Advance

1.	(d)	2.	(a)	3.	(c)	4.	(c)

DCE

1.	(d)	2.	(a)	3.	(c)	4.	(a)	5.	(c)
6.	(a)	7.	(c)	8.	(a)				

Mains Questions

1.	(d)	2.	(b)	3.	(b)	4.	(a)	5.	(c)
6.	(d)	7.	(a)	8.	(b)				