Lesson-2
SURDS

Introduction

We have studied that $\sqrt{2}, \sqrt{3}, \sqrt[3]{5}$,... etc. belong to the set of irrational numbers. Now we shall study irrational numbers of a particular type called surds.

Definition: A surd is defined as an irrational number of the type $\sqrt[n]{x}$, where it is not possible to extract exactly the n^{th} root of x where x is a positive number.

Alternatively: a number $\sqrt[n]{x}$, is called a *surd* if and only if

(*i*) it is an irrational number (*ii*) it is a root of a rational number.

Note 1: In the surd $\sqrt[n]{x}$, the symbol $\sqrt[n]{\ }$, is called the radical sign, the index n the order of the surd (or radical) and x the radicand.

Note 2: A surd which has unity as its rational factor, other factor being irrational is called pure surd and a surd having factor other than unity along with irrational factor is called mixed surd.

Note 3: Let n be a +ve integer and a be a real number.

(a) If a is irrational then $\sqrt[n]{a}$ is not a surd.

(b) If $\sqrt[n]{a}$ is rational, then also $\sqrt[n]{a}$ is not a surd.

Examples: $\sqrt{2}$ is a surd as $\sqrt{2}$ is an irrational number and is a root of rational number 2. Similarly $\sqrt[3]{5}, \sqrt[6]{8}, \sqrt[4]{32}$ and $\sqrt[4]{50}$ are all surds.

$\sqrt[3]{8}$ is not a surd as its root 2 is rational. Also $\sqrt{3+\sqrt{3}}$ although an irrational number is not a surd because it is the square root of an irrational number. Similarly $\sqrt{\sqrt{5}-\sqrt{2}}$ and $\sqrt{\pi}$ are not surds as the radicands are not rational numbers.

Order of Surds

The order of the surd is the number that indicates the root. Order of $\sqrt{12}$ is 2, order of $\sqrt[3]{5}$ is 3 and order of $\sqrt[n]{x}$ is n. Surds of order 2 are called *quadratic surds,* whereas surds of order 3 are called *cubic surds.* in the surd $5\sqrt[3]{2}$, 5 is called coefficient of the surd. When there is no coefficient in a surd, it is assumed that its coefficient is 1.

Example: $\sqrt{5}$ is a quadratic surd as its order is 2 and $\sqrt[3]{2}$ is a cubic surd as its order is 3.

The surds of the form $\sqrt[n]{a}$ can be split in the product form if a is not prime. For example, $\sqrt[4]{32} = 2 . \sqrt[4]{2}$ and

$\sqrt{\frac{2}{3}} = \frac{1}{3}.\sqrt{6} = \frac{1}{3}(\sqrt{3} \times \sqrt{2})$.

Algebraic Operations on Surds

As the surds can be expressed with fractional exponents, laws of indices studied in the earlier classes are applicable to them also. We recall the laws of indices here.

(i) $\sqrt[n]{x}\,\sqrt[n]{y} = \sqrt[n]{xy}$ or $x^{1/n} \times y^{1/n} = (xy)^{1/n}$

(ii) $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$ or $\frac{x^{1/n}}{y^{1/n}} = \left(\frac{x}{y}\right)^{1/n}$

(iii) $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x} = \sqrt[n]{\sqrt[m]{x}}$ or $(x^{1/n})^{1/m} = x^{1/mn} = (x^{1/m})^{1/n}$

(iv) $\sqrt[n]{x^m} = x^{m/n}$ or $(x^m)^{1/n} = x^{m/n}$

(v) $\sqrt[m]{x^a} = \sqrt[nm]{x^{an}}$ or $(x^a)^{1/m} = x^{a/m} = x^{an/mn} = (x^{an})^{1/mn}$

In the above results x, y and a are rational numbers and m and n are positive integers.

Examples:

(i) $\sqrt[3]{2}\,\sqrt[3]{5} = 2^{1/3} = (2.5)^{1/3} = 10^{1/3} = \sqrt[3]{10}$.

(ii) $\frac{(7)^{1/3}}{(11)^{1/3}} = \left(\frac{7}{11}\right)^{1/5} = \sqrt[5]{\frac{7}{11}}$

(iii) $\sqrt[5]{\sqrt[3]{2}} = \sqrt[5]{2^{1/3}} = (2^{1/3})^{1/5} = 2^{1/15} = \sqrt[15]{2} = \sqrt[5\times3]{2} = \sqrt[3]{\sqrt[5]{2}}$

(iv) $\sqrt[7]{3^4} = (3^4)^{1/7} = 3^{4/7}$

(v) $\sqrt[5]{2^3} = (2^3)^{1/5} = 2^{3/5} = 2^{9/15} = (2^9)^{1/15} = \sqrt[15]{2^9} = \sqrt[3\times5]{(2^{3\times3})}$

It is seen here that the order of the surd can be changed by multiplying the index of the surd and the index of the radicand by the same integer.

Similar Surds

Two surds are *similar* if they can be reduced to the same irrational factors. For example. $\sqrt{75}$ and $\sqrt{12}$ are similar surds as they can be rewritten as $5\sqrt{3}$ and $2\sqrt{3}$.

Simplest Form of a Surd

A surd is said to be in its simplest form if it has

(i) the smallest possible index of the radical

(ii) no fraction under the radical sign.

(iii) no factor of the form a^n where a is rational under the radical sign of index n.

Example: Simplest form of the surd $\sqrt[5]{2 \times 3^2}$ is $\sqrt[5]{18}$

Now we solve some examples.

Example: Express as a pure surd

(i) $3\sqrt{5}$ (ii) $\frac{5}{4}\sqrt{32}$

Solution: (i) $3\sqrt{5} = \sqrt{3^2}\sqrt{5} = \sqrt{9 \times 5} = \sqrt{45}$

(ii) $\frac{5}{4}\sqrt{32} = \sqrt{\left(\frac{5}{4}\right)^2} \times \sqrt{32} = \sqrt{\frac{25}{16} \times 32} = \sqrt{50}$

Example: Express as a mixed surd

(i) $\sqrt{80}$ (ii) $\sqrt[4]{405}$

Solution: (i) $\sqrt{80} = \sqrt{16 \times 5} = \sqrt{4^2 \times 5} = 4\sqrt{5}$

(ii) $\sqrt[4]{405} = \sqrt[4]{81 \times 5} = \sqrt[4]{3^4 \times 5} = 3\sqrt[4]{5}$.

Rationalisation of Surds

When the product of two surds is rational then each one of them is called the rationalising factor of the other. In other words we multiply a surd by another surd in such a manner that the product is a rational number. The process of converting the surds to rational numbers is called *rationalisation.* Thus to rationalise, we search a factor which when multiplied by the given surd gives us a rational number. We discuss below some examples of rationalisation:

(i) Rationalising factor of $\sqrt{x}$ is $\sqrt{x}$.

($\because$ $\sqrt{x} \times \sqrt{x} = x$, a rational number)

(ii) Rationalising factor of $3^{2/3}$ is $3^{1/3}$.

($\because$ $3^{2/3}.\ 3^{1/3} = 3$)

(iii) Rationalising factor of $\sqrt{3} + \sqrt{2}$ is $\sqrt{3} - \sqrt{2}$

($\because$ $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) = 3 - 2 = 1$)

(iv) Rationalising factor of $\sqrt{x} - \sqrt{y}$ is $\sqrt{x} + \sqrt{y}$.

Note: 1. The quantities $x - \sqrt{y}$ and $x + \sqrt{y}$ are called *conjugate binomial surds*. Their sum and products are always rational.

Thus $x - \sqrt{y} + x + \sqrt{y} = 2x$

and $(x - \sqrt{y})(x + \sqrt{y}) = x^2 - y^2$.

Note: 2. Rationalisation is usually used to rationalise denominators of a fraction. Now we take some examples.

Example 1: Find the simplest rationalising factor of $\sqrt{27}$.

Solution: $\sqrt{27} = \sqrt{3 \times 3 \times 3} = 3\sqrt{3}$.

$\therefore$ Rationalising factor is $\sqrt{3}$ . ... [$\because$ $3\sqrt{3} \times \sqrt{3} = 9$, a rational number]

Example 2: Find the rationalising factor of $\sqrt[3]{100}$.

Solution: $\sqrt[3]{100} = \sqrt[3]{2 \times 2 \times 5 \times 5} = \sqrt[3]{2^2 . 5^2} = 2^{2/3} \cdot 5^{2/3}$.

$\therefore$ Rationalising factor is $2^{1/3} \cdot 5^{1/3}$ or $10^{1/3}$.

Example 3: Find the rationalising factor of $\sqrt[5]{160}$.

Solution: $\sqrt[5]{160} = \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2 \times 5}$

$= \sqrt[5]{2^5 \cdot 5} = 2 \cdot 5^{1/5}$

$\therefore$ Rationalising factor is $5^{4/5}$ or $\sqrt[5]{625}$

SOLVED EXAMPLES

Ex.1: Express as a mixed surd in its simplest form

(*i*) $\sqrt[6]{320}$ (*ii*) $\sqrt[3]{189}$ (*iii*) $\sqrt[4]{1250}$

Sol.: (*i*) $\sqrt[6]{320} = \sqrt[6]{64 \times 5}$

$= \sqrt[6]{2^6 \times 5} = 2\sqrt[6]{5}$

(*ii*) $\sqrt[3]{189} = \sqrt[3]{27 \times 7}$

$= \sqrt[3]{3^3 \times 7} = 3\sqrt[3]{7}$

(*iii*) $\sqrt[4]{1250} = \sqrt[4]{625 \times 2} = \sqrt[4]{625} \times \sqrt[4]{2}$

$= \sqrt[4]{5^4} \times \sqrt[4]{2} = 5\sqrt[4]{2}$

Ex.2: Simplify each of the following

(*i*) $7\sqrt{2} + 5\sqrt{8}$ (*ii*) $4\sqrt{3} + \sqrt{27}$ (*iii*) $3\sqrt{5} - \sqrt{5}$ (*iv*) $15\sqrt{6} - \sqrt{216}$

Sol.: (*i*) $7\sqrt{2} + 5\sqrt{8} = 7\sqrt{2} + 5\sqrt{4 \times 2}$

$= 7\sqrt{2} + 10\sqrt{2} = (7 + 10)\sqrt{2}$

$= 17\sqrt{2}$

(*ii*) $4\sqrt{3} + \sqrt{27} = 4\sqrt{3} + \sqrt{9 \times 3} = 4\sqrt{3} + 3\sqrt{3}$

$= (4 + 3)\sqrt{3} = 7\sqrt{3}$

(*iii*) $3\sqrt{5} - \sqrt{5} = (3 - 1)\sqrt{5}$

$= 2\sqrt{5}$

(*iv*) $15\sqrt{6} - \sqrt{216} = 15\sqrt{6} - \sqrt{36 \times 6}$

$= 15\sqrt{6} - 6\sqrt{6}$

$= (15 - 6)\sqrt{6}$

$= 9\sqrt{6}.$

Ex.3: Multiply $5\sqrt[3]{2}$ by $7\sqrt[3]{4}$.

Sol.: $5\sqrt[3]{2} \times 7\sqrt[3]{4} = 5 \times 7 \times \sqrt[3]{2 \times 4} = 35 \times \sqrt[3]{8}$

$= 35 \times \sqrt[3]{2^3} = 35 \times 2 = 70.$

Ex.4: Multiply $17\sqrt[3]{6}$ by $3\sqrt{2}$.

Sol.: $17\sqrt[3]{6} \times 3\sqrt{2} = 17 \times 3 \times \sqrt[3]{6} \times \sqrt{2}$

$= 51(6^2)^{1/(3\times2)} \times (2^3)^{1/(3 \times 2)}$

$= 51(36)^{1/6} \times (8)^{1/6}$

$= 51\ (36 \times 8)^{1/6} = 51\ (288)^{1/6}$

Ex.5: Divide $11\sqrt[3]{7}$ by $8\sqrt[6]{2}$.

Sol.: $11\sqrt[3]{7} \div 8\sqrt[6]{2}$

$= \frac{11}{8}[(7)^{1/3} \div (2)^{1/6}]$

$= \frac{11}{8}[(7^2)^{1/(3 \times 2)} \div (2)^{1/6}]$

$= \frac{11}{8}[(49)^{1/6} \div (2)^{1/6}]$

$= \frac{11}{8}\sqrt[6]{\frac{49}{2}}$

Ex.6: Simplify : $3\sqrt{48} - \frac{5}{2}\sqrt{\frac{1}{3}} + 4\sqrt{3}$.

Sol.: $3\sqrt{48} - \frac{5}{2}\sqrt{\frac{1}{3}} + 4\sqrt{3}$

$= 3\sqrt{16 \times 3} - \frac{5}{2}\sqrt{\frac{3}{3 \times 3}} + 4\sqrt{3}$

$= 3.4\sqrt{3} - \frac{5}{2}\cdot\frac{\sqrt{3}}{3} + 4\sqrt{3}$

$= 12\sqrt{3} - \frac{5}{6}\sqrt{3} + 4\sqrt{3}$

$= \left(12 - \frac{5}{6} + 4\right)\sqrt{3} = \frac{91}{6}\sqrt{3}.$

Ex.7: Simplify : $8\sqrt{45} - 8\sqrt{20} + \sqrt{245} - 3\sqrt{125}$

Sol.: $8\sqrt{45} - 8\sqrt{20} + \sqrt{245} - 3\sqrt{125}$

$= 8\sqrt{9 \times 5} - 8\sqrt{4 \times 5} + \sqrt{49 \times 5} - 3\sqrt{25 \times 5}$

$= 8\sqrt{3 \times 3 \times 5} - 8\sqrt{2 \times 2 \times 5} + \sqrt{7 \times 7 \times 5} - 3\sqrt{5 \times 5 \times 5}$

$= 8 \times 3\sqrt{5} - 8 \times 2\sqrt{5} + 7\sqrt{5} - 3 \times 5\sqrt{5}$

$= 24\sqrt{5} - 16\sqrt{5} + 7\sqrt{5} - 15\sqrt{5}$

$= 31\sqrt{5} - 31\sqrt{5} = 0$

Ex.8: Simplify and express the result in the simplest form: $2\sqrt{50} \times 3\sqrt{32} \times 4\sqrt{18}$

Sol.: $2\sqrt{50} \times 3\sqrt{32} \times 4\sqrt{18}$

$= 2\sqrt{5 \times 5 \times 2} \times 3\sqrt{4 \times 4 \times 2} \times 4\sqrt{3 \times 3 \times 2}$

$= 2 \times 5\sqrt{2} \times 3 \times 4\sqrt{2} \times 4 \times 3\sqrt{2}$

$= 10\sqrt{2} \times 12\sqrt{2} \times 12\sqrt{2}$

$= 1440\sqrt{2} \times \sqrt{2} \times \sqrt{2} = 1440 \times 2\sqrt{2}$

$= 2880\sqrt{2}.$

Ex.9: Which is greater $\left(\frac{1}{2}\right)^{1/2}$ or $\left(\frac{1}{3}\right)^{1/3}$?

Sol.: L.C.M. of 2 and 3 is 6

$$\therefore \quad \left(\frac{1}{2}\right)^{1/2} = \left[\left(\frac{1}{2}\right)^3\right]^{\frac{1}{2} \times \frac{1}{3}} = \left(\frac{1}{8}\right)^{1/6}$$

$$\text{and} \quad \left(\frac{1}{3}\right)^{1/3} = \left[\left(\frac{1}{3}\right)^2\right]^{\frac{1}{3} \times \frac{1}{2}} = \left(\frac{1}{9}\right)^{1/6}$$

As $\frac{1}{8} > \frac{1}{9}$, we have $\left(\frac{1}{8}\right)^{1/6} > \left(\frac{1}{9}\right)^{1/6}$

i.e., $\left(\frac{1}{2}\right)^{1/2} > \left(\frac{1}{3}\right)^{1/3}$

Ex.10: Rationalise: $\frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}}$.

Sol.: $\frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}} = \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}} \times \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}}$

...[Multiplying and dividing by the rationalising factor of the denominator $\sqrt{2} - \sqrt{3}$ *i.e.,* by $\sqrt{2} + \sqrt{3}$]

$$= \frac{(\sqrt{2}+\sqrt{3})^2}{(\sqrt{2})^2-(\sqrt{3})^2}$$

$$= \frac{2+3+2\sqrt{2}\sqrt{3}}{2-3}$$

$$= \frac{5+2\sqrt{6}}{-1}$$

$$= -5-2\sqrt{6}.$$

Ex.11: Rationalise: $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$.

Sol.: $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} \times \frac{\sqrt{48}-\sqrt{18}}{\sqrt{48}-\sqrt{18}}$

...[Multiplying and dividing by the rationalising factor of $\sqrt{48}+\sqrt{18}$]

$$= \frac{(4\sqrt{3}+5\sqrt{2})(4\sqrt{3}-3\sqrt{2})}{48-18}$$

...[$\because \sqrt{48} = 4\sqrt{3}$ and $\sqrt{18} = 3\sqrt{2}$]

$$= \frac{4\sqrt{3}(4\sqrt{3}-3\sqrt{2})+5\sqrt{2}(4\sqrt{3}-3\sqrt{2})}{30}$$

$$= \frac{48-12\sqrt{6}+20\sqrt{6}-30}{30}$$

$$= \frac{18+8\sqrt{6}}{30} = \frac{9+4\sqrt{6}}{15}.$$

Ex.12: If $\sqrt{2} = 1.414$ find the value of $\frac{1}{\sqrt{2}+1}$.

Sol.: $\frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$$= \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

$$= 1.414 - 1 = 0.414.$$

Note: Before substituting the values of irrational numbers, we rationalise the denominator.

Ex.13: If $x = \frac{1}{2-\sqrt{3}}$, find the value of $x^3 - 2x^2 - 7x + 5$.

Sol.: We are given $x = \frac{1}{2-\sqrt{3}}$

$$\therefore \quad x = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{2+\sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$= \frac{2+\sqrt{3}}{4-3}$$

$$\therefore \quad x = 2+\sqrt{3}$$

or $x - 2 = \sqrt{3}$

or $(x-2)^2 = (\sqrt{3})^2$

or $x^2 - 4x + 4 = 3$

or $x^2 - 4x + 1 = 0$

Now $x^3 - 2x^2 - 7x + 5 = (x^3 - 4x^2 + x) + 2x^2 - 8x + 2 + 3$

$= x(x^2 - 4x + 1) + 2(x^2 - 4x + 1) + 3$

$= x(0) + 2(0) + 3$

$= 3.$

Ex.14: If $x = 9 - 4\sqrt{5}$, find (*i*) $\sqrt{x} - \frac{1}{\sqrt{x}}$ (*ii*) $x^2 + \frac{1}{x^2}$

Sol.: (*i*) We have $x = 9 - 4\sqrt{5}$

$$\therefore \quad \frac{1}{x} = \frac{1}{9-4\sqrt{5}}$$

$$= \frac{1}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}}$$

$$= \frac{9+4\sqrt{5}}{81-80}$$

$$= 9 + 4\sqrt{5}$$

$$\therefore \quad x + \frac{1}{x} = (9 - 4\sqrt{5}) + (9 + 4\sqrt{5}) = 18$$

$$\text{Now}\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 = x+\frac{1}{x}-2$$

$$= 18 - 2 = 16$$

$$\therefore \quad \sqrt{x}-\frac{1}{\sqrt{x}} = 4.$$

(*ii*) Again $\left(x+\dfrac{1}{x}\right)^2 = 18^2$

$$\therefore \quad x^2+\frac{1}{x^2}+2 = 324$$

$$\therefore \quad x^2+\frac{1}{x^2} = 324 - 2 = 322.$$

BASIC LEVEL ASSIGNMENT

1. Simplify each of the following

(*i*) $\sqrt{50} - 4\sqrt{8} + 7\sqrt{18}$

(*ii*) $6\sqrt[3]{54} - 2\sqrt[3]{16} + 10\sqrt[3]{128}$

2. Multiply

(*i*) $\sqrt[4]{15}$ by $\sqrt[4]{250}$

(*ii*) $\sqrt[3]{2}$ by $\sqrt[4]{3}$

3. Divide

(*i*) $4\sqrt{28}$ by $3\sqrt{7}$

(*ii*) $\sqrt{24}$ by $\sqrt[3]{320}$

4. Simplify each of the following

(*i*) $12\sqrt{18} + 6\sqrt{20} - 6\sqrt{147} + 3\sqrt{50} + 8\sqrt{45}$

(*ii*) $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

5. Which is greater

(*i*) $\sqrt{2}$ or $\sqrt[3]{3}$

(*ii*) $\sqrt[4]{5}$ or $\sqrt[3]{4}$

(*iii*) $\sqrt[5]{10}$ or $\sqrt[4]{9}$

(*iv*) $\sqrt[3]{9}$ or $\sqrt[4]{12}$

(*v*) $\sqrt[3]{6}$ or $\sqrt[4]{8}$

6. Arrange in ascending order, $\sqrt{2}, \sqrt{3}, \sqrt[3]{4}$.

7. Arrange in descending order : $\sqrt[3]{2}$, $\sqrt[4]{3}$, $\sqrt[3]{4}$.

8. Simplify: $\dfrac{7 + 3\sqrt{5}}{3 + \sqrt{5}} - \dfrac{7 - 3\sqrt{5}}{3 - \sqrt{5}}$

9. Find the continued product of $\sqrt{2} + \sqrt{3} + \sqrt{5}, \sqrt{2} - \sqrt{3} + \sqrt{5}, \sqrt{2} + \sqrt{3} - \sqrt{5}$ and $-\sqrt{2} + \sqrt{3} + \sqrt{5}$.

10. Given $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$, find the value of each of the following:

$\dfrac{2 + \sqrt{3}}{2 - \sqrt{3}} + \dfrac{2 - \sqrt{3}}{2 + \sqrt{3}} + \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$.

11. If $x = 7 + 4\sqrt{3}$, find the value of

(*i*) $\sqrt{x} + \dfrac{1}{\sqrt{x}}$

(*ii*) $x^2 + \dfrac{1}{x^2}$

(*iii*) $x^3 + \dfrac{1}{x^3}$

12. If $\dfrac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} = p + q\sqrt{15}$, find p and q where p and q are rational numbers.

13. If $x = \dfrac{1}{7 + 4\sqrt{3}}$ and $y = \dfrac{1}{7 - 4\sqrt{3}}$, then find the value of

(*i*) $x^2 + y^2$

(*ii*) $x^3 + y^3$

14. Prove that $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots + \frac{1}{\sqrt{8}+\sqrt{9}} = 2$

15. Similify each of the its following

(*i*) $\frac{3}{5-\sqrt{3}} + \frac{2}{5+\sqrt{3}}$ (*ii*) $\frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}}$

16. Prove that $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} = 0$

17. Show that $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} = 5$

ANSWERS

Basic Level Assignment

1. (i) $18\sqrt{2}$, (ii) $54\sqrt[3]{2}$

2. (i) $5\sqrt[4]{6}$, (ii) $\sqrt[12]{432}$

3. (i) $\frac{8}{3}$, (ii) $\sqrt[6]{\frac{27}{200}}$

4. (i) $36\sqrt{5} - 42\sqrt{3} + 51\sqrt{2}$, (ii) 0

5. (i) $\sqrt[3]{3}$, (ii) $\sqrt[3]{4}$, (iii) $\sqrt[4]{9}$, (iv) $\sqrt[3]{9}$, (v) $\sqrt[3]{6}$

6. $\sqrt{2}$, $\sqrt[3]{4}$, $\sqrt{3}$

7. $\sqrt[3]{4}$, $\sqrt[4]{3}$, $\sqrt[3]{2}$

8. $\sqrt{2}$

9. 24

10. 14.268

11. (i) 4, (ii) 144, (iii) 2702

12. $p = 4$, $q = 1$

13. (i) 194, (ii) 2707

15. (i) $\frac{25+\sqrt{3}}{22}$, (ii) $\frac{42}{11}$