OBJECTIVE SOLVED PROBLEMS

Problem 1: Two insulating cylinders *A* and *B* fitted with pistons contain equal amounts of an ideal diatomic gas at temperature 300K. The piston *A* is free to move, while that of *B* is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in *A* is 30K. Then the rise in temperature of the gas in *B* is,

(a) 30K (b) 18K
(c) 50K (d) 42K.

Solution : For cylinder A.

$dQ = nC_P dT$

$= n(C_V + R)dT = n(C_v + R)30$

For cylinder B

$dQ = nC_v dT_1$

From (i) and (ii)

$nC_V dT_1 = n(C_v + R)30$

$\therefore \quad dT_1 = \dfrac{(C_v + R)30}{C_v}$

For diatomic gas $C_v = \dfrac{5}{2}R$

$\therefore \quad dT_1 = 42K$.

Ans. (d)

Problem 2: Two identical containers *A* and *B* with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of gas contained in *A* is m_A and that in *B* is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The change in the pressure in *A* and *B* are found to be ΔP and $1.5\,\Delta P$ respectively. Then

(a) $4m_A = 9m_B$ (b) $2m_A = 3m_B$
(c) $3m_A = 2m_B$ (d) $9m_A = 4m_B$.

Solution : For gas in *A*, $P_1 = \left(\dfrac{m_A}{M}\right)\dfrac{RT}{V_1}$

$P_2 = \left(\dfrac{m_A}{M}\right)\dfrac{RT}{V_2}$

Putting $V_1 = V$ and $V_2 = 2V$ (Given $P_1 - P_2 = \Delta P$)

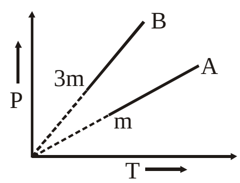
We get $\Delta P = \left(\dfrac{RT}{M}\right)\dfrac{m_A}{2V}$

Similarly for gas in *B*, $1.5\Delta P = \left(\dfrac{RT}{M}\right)\dfrac{m_B}{2V}$

From eq. (i) and (ii), we get $2m_B = 3m_A$.

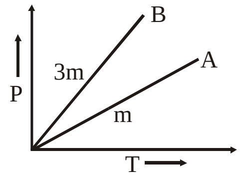
And. (c)

Problem 3. Two different masses m and 3m of an ideal gas are heated separately in a vessel of constant volume, the pressure *P* and absolute temperature *T*, graphs for these two cases and shown in the figure as *A* and *B*. The ratio of slopes of curves *B* to *A* is



(a) 3 : 1 (b) 1 : 9
(c) 9 : 1 (d) 1 : 8.

Solution : For a gas, $PV = \mu RT = \frac{m}{M}RT$



For graph A, $PV = \frac{m}{M}RT$

Slope of graph $A\left(\frac{dP}{dT}\right) = \frac{mR}{MV}$ …(i)

For graph B, $PV = \frac{3m}{M}RT$

Slope of graph B, $\left(\frac{dP}{dT}\right) = \frac{3mR}{MV}$ …(ii)

$$\frac{\text{Slope of curve B}}{\text{Slope of curve A}} = \frac{\frac{3mR}{MV}}{\frac{mR}{MV}} = \frac{3}{1}$$

Ans. (a)

Problem 4. n_1 and n_2 moles of two ideal gases of thermodynamics constant γ_1 and γ_2 respectively are mixed, $\frac{C_P}{C_V}$ for the mixture is

(a) $\frac{\gamma_1 + \gamma_2}{2}$ (b) $\frac{n_1\gamma_1 + n_2\gamma_2}{n_1 + n_2}$

(c) $\frac{n_1\gamma_2 + n_2\gamma_1}{n_1 + n_2}$ (d) $\frac{n_1\gamma_1(\gamma_2 - 1) + n_2\gamma_2(\gamma_1 - 1)}{n_1(\gamma_2 - 1) + n_2(\gamma_1 - 1)}$

Solution : $n = n_1 + n_2$

$W = U_1 + U_2$

$nC_V = n_1C_{V1} + n_2C_{V2}$

$= n_1\frac{R}{\gamma_1 - 1} + n_2\frac{R}{\gamma_2 - 1}$

$nC_P = n_1\gamma_1\frac{R}{\gamma_1 - 1} + n_2\gamma_2\frac{R}{\gamma_2 - 1}$

$$\gamma = \frac{C_P}{C_V} = \frac{\frac{n_1\gamma R}{\gamma_1 - 1} + n_2\gamma_2\frac{R}{\gamma_2 - 1}}{\frac{R}{n_1(\gamma_2 - 1)} + \frac{R}{(\gamma_1 - 1)}}$$

$$= \frac{n_1\gamma_1(\gamma_2 - 1) + n_2\gamma_2(\gamma_1 - 1)}{n_1(\gamma_2 - 1) + n_2(\gamma_1 - 1)}.$$

Ans. (d)

Problem 5. A cylinder containing gas at 27º C is divided into two parts of equal volume each 100 cc and equal pressure bya piston of cross-sectional area 10.85 cm^2. The gas in one part is raised in temperatture to 100º C while the other volume is maintained at original temperatue. The piston and walls are perfect insulators. How far will be piston move during the change in temperature

(a) 1 cm
(b) 2 cm
(c) 0.5 cm
(d) 1.5 cm.

Solution : Let n be the number of moles in each compartment. If dV the change in volume, then

$$P_0(V - dV) = nRT\ ,$$

and $P_0(V + dV) = nRT'$

here T and T' are the temperature. Again

$$V(T' - T) = dV(T + T')$$

$$\therefore \quad \frac{dV}{V} = \frac{T' - T}{T + T'} = \frac{373 - 300}{373 + 300} \times 100$$

$$= 10.85\ \text{cm}$$

Distance moved by piston

= 10.85 / 10.85 = 1 cm.

Ans. (a)

Problem 6. A monatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the length of the gas column before and after expansion respectively, then $\frac{T_1}{T_2}$ is given by

(a) $\left(\frac{L_1}{L_2}\right)^{2/3}$
(b) $\frac{L_1}{L_2}$
(c) $\frac{L_2}{L_1}$
(d) $\left(\frac{L_2}{L_1}\right)^{2/3}$.

Solution: Here $TV^{\gamma-1}$ = constant

As $\gamma = 5/3$, hence $TV^{2/3}$ = constant

Now $T_1 L_1^{2/3} = T_2 L_2^{2/3}$ $\quad (\because V \propto L)$

or, $\frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{2/3}$.

Ans. (d)

Problem 7. *When a block of iron floats in mercury at $0°C$, a fraction k_1 of its volume is submerged, while at the temperature $60°C$, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is $\gamma_{Hg,}$ then the ratio k_1/k_2 can be expressed as*

(a) $\frac{1 + 60\gamma_{Fe}}{1 + 60\gamma_{Hg}}$
(b) $\frac{1 - 60\gamma_{Fe}}{1 + 60\gamma_{Hg}}$
(c) $\frac{1 + 60\gamma_{Fe}}{1 - 60\gamma_{Hg}}$
(d) $\frac{1 + 60\gamma_{Hg}}{1 + 60\gamma_{Fe}}$

Solution: Let ρ and σ be the densities of iron and mercury respectively :

Applying the law of floatation:

At 0°C, $W = V_0\rho_0 g = k_1 V_0 \sigma_0 g$

At 60°C, $W = V_{60}\rho_{60} g = k_2 V_{60}\sigma_{60} g$

Dividing , we get

$$\frac{\rho_0}{\rho_{60}} = \frac{k_1\sigma_0}{k_2\sigma_{60}}$$

or, $$\frac{k_1}{k_2} = \frac{\rho_0\sigma_{60}}{\sigma_0\rho_{60}} = \frac{\rho_{60}\left(1+60\gamma_{Fe}\right).\sigma_{60}}{\sigma_{60}\left(1+60\gamma_{Hg}\right)\rho_{60}}$$

$$\Rightarrow \quad \frac{k_1}{k_2} = \frac{1+60\gamma_{Fe}}{1+60\gamma_{Hg}}$$

Ans. (d)

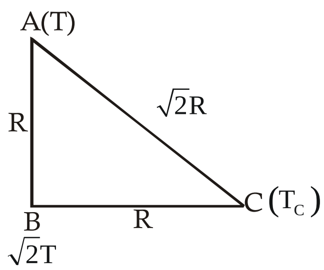
Problem 8. *Three rods of identical cross–sectional area and made from the same metal form the sides of an isosceles triangle ABC, right-angled at B. The points A and B are maintained at temperatures T and $\left(\sqrt{2}\right)T$ respectively. In the steady state, the temperature of the point C is T_C. Assuming that only heat conduction takes place, then T_C/T is*

(a) $\dfrac{1}{3}$ *(b)* $\dfrac{3}{\left(\sqrt{2}+1\right)}$

(c) $\dfrac{\sqrt{3}}{1}$ *(d)* $\left(\dfrac{1}{3}\right)^{1/3}$.

Solution: The thermal resistance of AB, BC and CA are R, R and $2R$ respectively.

$\therefore$ Thermal currents through BC and CA must be equal.



$$\therefore \quad \frac{\sqrt{2}T - T_C}{R} = \frac{T_C - T}{\sqrt{2}R}$$

or, $2T - \sqrt{2}T_C = T_C - T$

or, $$T_C/T = \frac{3}{\left(\sqrt{2}+1\right)}$$

Ans. (b)

Problem 9. *A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wien constant $b = 2.88\times10^6$ nm is K. Then* *(a)* $U_1 = 0$ *(b)* $U_3 = 0$

(c) $U_1 > U_2$ *(d)* $U_2 > U_1$.

Solution : From Wien's Law,

$$\lambda_m T = b$$

$$\therefore \quad \lambda_m = \frac{b}{T} = \frac{2.88\times10^6\,\text{nm.K}}{2880\text{K}}$$

$= 1000$ nm

Here, λ_m (between 999 nm & 1000 nm) is the wavelength corresponding to the maximum energy density $(= U_2)$

Hence U_1 and U_3 both will be less than U_2.

Ans. (d)

Problem 10: *Two rods of length ℓ_1 and ℓ_2 are made of materials whose coefficients of linear expansion are α_1 and α_2. If the difference between two lengths is independent of temperature then,*

(a) $\frac{\ell_1}{\ell_2} = \frac{\alpha_1}{\alpha_2}$ *(b)* $\frac{\ell_1}{\ell_2} = \frac{\alpha_2}{\alpha_1}$

(c) $\ell_2^2\alpha_1 = \ell_1^2\alpha_2$ *(d)* $\frac{\alpha_1^2}{\ell_1} = \frac{\alpha_2^2}{\ell_2}$

Solution : If change in length is Δl

Then $\Delta\ell$

$$\Delta\ell_1 = \ell_1\alpha_1\Delta T$$

$$\Delta\ell_2 = \ell_2\alpha_2\Delta T$$

difference in length is $\ell_2 - \ell_1 = (l_{02} + \Delta\ell_2) - (l_{01} + \Delta\ell_1)$

$= (l_{02} - l_{01}) + (\Delta\ell_2 - \Delta\ell_1)$

$l_{02} - l_{01}$ is independent of temperature so $\ell_2 - \ell_1$ to be independent of temperature.

i.e., $\Delta\ell_2 - \Delta\ell_1$ must be equal to zero

i.e. $l_1\alpha_1\Delta T = l_2\alpha_2\Delta T$

i.e. $l_1\alpha_1 = l_2\alpha_2$

i.e. $\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}$.

SUBJECTIVE SOLVED PROBLEMS

Problem 1: *One mole of an ideal gas whose pressure changes with volume as $P = \alpha V$, where α is a constant, is expanded so that its volume increase η times. Find the change in internal energy and heat capacity of the gas.*

Solution : Let V be the initial volume of the gas. It is expanded to a volume η V. The work done in this process is given by

$$W = \int_V^{\eta V} PdV = \int_V^{\eta V} \alpha VdV = \alpha\left[\frac{V^2}{2}\right]_V^{\eta V}$$

$$= \frac{\alpha V^2}{2}\left[\eta^2 - 1\right]$$

The pressure of the gas varies with volume as $P = \alpha V$. So, the initial and final pressure will be α V and $\eta\alpha$ V. The change in internal energy is given by

$$dU = nC_V dT = \frac{R\left(T_f - T_i\right)}{\gamma - 1} = \frac{P_f V_f - P_i V_i}{\gamma - 1} = \frac{\eta^2 \alpha V^2 - \alpha V^2}{\gamma - 1} = \frac{\alpha V^2}{\gamma - 1}\left(\eta^2 - 1\right)$$

The heat exchange in this process is given by

$$Q = U + W$$

$$= \frac{\alpha V^2}{\gamma - 1}\left[\eta^2 - 1\right] + \frac{\alpha V^2}{2}\left[\eta^2 - 1\right] = \frac{\alpha V^2}{2}\left[\eta^2 - 1\right]\left[\frac{\gamma + 1}{\gamma - 1}\right]$$

Here $T_i = \frac{P_i V_i}{nR} = \frac{\alpha V^2}{nR}$ and $T_f = \frac{P_f V_f}{nR} = \frac{\eta^2 \alpha V^2}{nR}$

Now heat capacity $C = \frac{Q}{T_f - T_1}$

$$C = \frac{1}{T_f - T_i}\left[\frac{\alpha V^2}{2}\left(\eta^2 - 1\right)\left\{\frac{\gamma + 1}{\gamma - 1}\right\}\right]$$

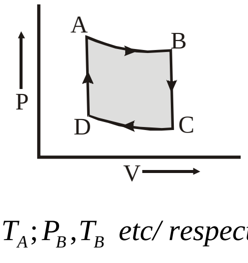
$$= \frac{nR}{\alpha V^2\left(\eta^2 - 1\right)}\left[\frac{\alpha V^2}{2}\left(\eta^2 - 1\right)\left\{\frac{\gamma + 1}{\gamma - 1}\right\}\right]$$

$$= \frac{nR}{2}\left[\frac{\gamma + 1}{\gamma - 1}\right]$$

Here n = 1

$$\therefore \quad C = \frac{R}{2}\left(\frac{\gamma + 1}{\gamma - 1}\right).$$

Problem 2: *One mole of monoatomic ideal gas is taken through the cycle shown in figure.*



$A \to B$ Adiabatic expansion

$B \to C$ Cooling at constant volume

$C \to D$ Adiabatic compression

$D \to A$ Heating at constant volume

The pressure and temperature at A, B etc., are denoted by $P_A, T_A; P_B, T_B$ etc/ respectively. Given $T_A = 1000K$, $P_B = (2/3)P_A$ and $P_C = (1/3)P_A$. Calculate

(a) The work done by the gas in the process $A \to B$

(b) *The heat lost by the gas in the process $B \to C$ and*

(c) *Temperature T_D given $(2/3)^{2/5} = 0.85$ and $R = 8.31$ J/mol K.*

Solution : (a) As for adiabatic change $PV^{\gamma} = \text{constant}$

i.e. $P\left(\frac{\mu RT}{P}\right)^{\gamma} = \text{constant}$ [as $PV = \mu RT$]

i.e. $\frac{T^{\gamma}}{P^{\gamma-1}} = \text{constant}$ so $\left(\frac{T_B}{T_A}\right)^{\gamma} = \left(\frac{P_B}{P_A}\right)^{\gamma-1}$ where $\gamma = \frac{5}{3}$

i.e. $T_B = T_A\left(\frac{2}{3}\right)^{1-\frac{1}{\gamma}} = 1000\left(\frac{2}{3}\right)^{2/5} = 850K$

so $W_{AB} = \frac{\mu R[T_i - T_f]}{[\gamma - 1]} = \frac{1 \times 8.31[1000 - 850]}{[(5/3) - 1]}$

i.e. $W_{AB} = (3/2) \times 8.31 \times 150 = 1869.75\text{ J}$

(b) For $B \to C, V = \text{constant}$ so $\Delta W = 0$

so from first law of thermodynamics

$\Delta Q = \Delta U + \Delta W = \mu C_V \Delta T + 0$

or $\Delta Q = 1 \times \left(\frac{3}{2}R\right)(T_C - 850)$ as $C_v = \frac{3}{2}R$

Now along path BC, V = constant; $P \propto T$

i.e. $\frac{P_C}{P_B} = \frac{T_C}{T_B}$, $T_C = \frac{(1/3)P_A}{(2/3)P_A} \times T_B = \frac{T_B}{2} = \frac{850}{2} = 425\text{ K}$ …(ii)

So $\Delta Q = 1 \times \frac{3}{2} \times 8.31(425 - 850) = -5297.625\text{ J}$

[Negative heat means, heat is lost by the system]

(c) $\text{D} \to \text{A}$ process is isochoric

$\frac{P_D}{P_A} = \frac{T_D}{T_A}$, i.e. $P_D = P_A\frac{T_D}{T_A}$

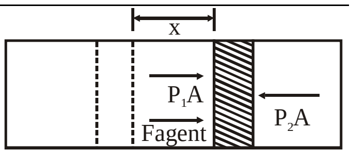
But C and D are on the same adiabatic

$$\left(\frac{T_D}{T_C}\right)^{\gamma} = \left(\frac{P_D}{P_C}\right)^{\gamma-1} = \left(\frac{P_A T_D}{P_C T_A}\right)^{\gamma-1}$$

or $(T_D)^{1/\gamma} = T_C\left[\frac{P_A}{P_C T_A}\right]^{1-\frac{1}{\gamma}}$, i.e. $T_C^{3/5} = \left(\frac{T_B}{2}\right)\left[\frac{P_A}{(1/3)P_A 1000}\right]^{2/5}$

i.e. $T_D^{3/5} = \left[\frac{1}{2}\left(\frac{2}{3}\right)^{2/3} \times 1000\right]\left[\frac{3}{1000}\right]^{2/5}$ i.e. $T_D = 500\text{ K}$

Problem 3: *A piston can freely move inside a horizontal cylinder closed from both ends. Initially, the piston separates the inside space of the cylinder into two equal parts each of volume V_0, in which an ideal gas is contained under the same pressure P_0 and at the same temperature. What work has to be performed in order to increase isothermally the volume of one part of gas η times compared to that of the other by slowly moving the piston ?*



Solution : Let the agent move as shown.
In equilibrium position,

$$P_1A + F_{\text{agent}} = P_2A$$

$$F_{\text{agent}} = (P_2 - P_1)A$$

Elementary work done by the agent

$$F_{\text{agent}}\,dx = (P_2 - P_1)A \times dx = (P_2 - P_1)dV \quad \ldots \text{(i)}$$

Applying PV = constant for two parts, we have

$P_1(V_0 + Ax) = P_0V_0$ and $P_2(V_0 - Ax) = P_0V_0$

$$P_1 = \frac{P_0V_0}{(V_0 + Ax)} \quad \text{and} \quad P_2 = \frac{P_0V_0}{(V_0 - Ax)}$$

$$\therefore \quad P_2 - P_1 = \frac{P_0V_0(2Ax)}{V_0^2 - A^2x^2} = \frac{2P_0V_0V}{V_0^2 - V^2}$$

When the volume of the left end is η times the volume of right end, we have

$$(V_0 + V) = \eta(V_0 - V)$$

$$V = \left(\frac{\eta - 1}{\eta + 1}\right)V_0 \quad \ldots \text{(ii)}$$

The work done by the agent is given by

$$W = \int_0^V (P_2 - P_1)dV = \int_0^V \frac{2P_0V_0V}{(V_0^2 - V^2)}dV$$

$$= -P_0V_0[\ln\left(V_0^2 - V^2\right)]_0^V = -P_0V_0[\ln\left(V_0^2 - V^2\right) - \ln V_0^2]$$

$$= -P_0V_0\left[\ln\left\{V_0^2 - \left(\frac{\eta - 1}{\eta + 1}\right)^2 V_0^2\right\} - \ln V_0^2\right]$$

$$= -P_0V_0\left[\ln\{4\eta/(\eta + 1)^2\}\right] = P_0V_0\ln\left[\frac{(\eta + 1)^2}{4\eta}\right].$$

Problem 4: *Three moles of an ideal gas being initially at a temperature $T_0 = 273\,K$ were isothermally expanded $\eta = 5.0$ time its initial volume and then isochorically heated so that the pressure in the final state became equal to that in the initial state. The total amount of heat transferred to the gas during the process equals $Q = 80$ KJ. Find the ratio $\gamma = C_P / C_V$ for this gas.*

Solution : In isothermal process, the heat transferred to the gas is given by

$$Q_1 = nRT_0\ln(V_2 / V_1) = nRT_0\ln\eta \quad \ldots \text{(i)}$$

$$[\therefore \quad \eta = (V_2 / V_1) = (P_1 / P_2)]$$

In isochroric process, $Q_2 = \Delta U$ (W = 0)

$\therefore \quad Q_2 = nC_V \Delta T = n\{R/(\gamma - 1)\}\Delta T$ …(ii)

Now $\quad \frac{P_2}{P_1} = \frac{T_0}{T} \quad$ or $\quad T = T_0\left(\frac{P_1}{P_2}\right) = \eta T_0$

$\therefore \quad \Delta T = \eta T_0 - T_0 = (\eta - 1)T_0$ …(iii)

substituting the value of ΔT from equation (iii) in equation (ii), we get

$$Q_2 = n\left(\frac{R}{\gamma - 1}\right)(\eta - 1)T_0$$

$$\therefore \quad Q = nRT_0 \ln\eta + n\left(\frac{R}{\gamma - 1}\right)(\eta - 1)T_0$$

or $$\frac{Q}{nRT_0} - \ln\eta = \left(\frac{\eta - 1}{\gamma - 1}\right)$$

or $$\gamma - 1 = \frac{\eta - 1}{\frac{Q}{nRT_0} - \ln\eta}$$

$$\therefore \quad \gamma = 1 + \frac{\eta - 1}{\frac{Q}{nRT_0} - \ln\eta}$$

Substituting given values, we get

$$\gamma = 1 + \frac{(5 - 1)}{\frac{80 \times 10^3}{3 \times 8.3 \times 273} - \ln 5}$$

Solving we get $\gamma = 1.4$

Problem 5: *One end of a rod of length 20 cm is inserted in a furnace at 800 K. the sides of the rod are covered with an insulating material and the other end emits radiation like a blackbody. The temperature of this end is 750 K in the steady state. The temperature of the surrounding air is 300 K. Assuming radiation to be the only important mode of energy transfer between the surrounding and the open end of the rod, find the thermal conductivity of the rod. Stefan constant* $\sigma = 6.0 \times 10^{-8} W/m^2 - K^4$

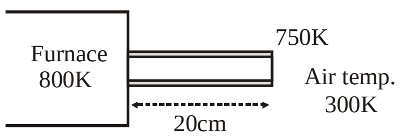
Solution : Quantity of heat flowing through the rod in steady state

$$\frac{dQ}{dt} = \frac{K.A.d\theta}{x} \quad \text{…(i)}$$

Quantity of heat radiated from the end of the rod in steady state

$$\frac{dQ}{dt} = A\sigma\left(T^4 - T_0^4\right) \quad \text{…(ii)}$$

From (i) and (ii)



$$\frac{K.d\theta}{x} = \sigma\left(T^4 - T_0^4\right)$$

$$\frac{K \times 50}{0.2} = 6.0 \times 10^{-8}[(7.5)^4 - (3)^4] \times 10^8$$

or K = 74 W/m – K.

Problem 6. *The intensity of solar radiation, just outside the earth's atmosphere, is measured to be 1.4* kW/m^2. *If the radius of the sun* $7\times10^8\text{m}$, *while the earth-sun distance is* $150\times10^6\,\text{km}$, *then find*

(i) the intensity of solar radiation at the surface of the sun,

(ii) the temperature at the surface of the sun assuming it to be a black body,

(iii) the most probable wavelength in solar radiation,

Solution: Assuming the sun to be a "blackbody" at a temperature T_0,

we can write,

W = intensity of solar radiation on the sun's surface $=\sigma T_0^{\,4}$,

Where σ is the Stefan-Boltzmann constant

(i) The radiation emitted from the solar surface per unit time is spread over the surface of a sphere having a radius equal to earth-sun distance where it is received on the earth (just outside the atmosphere)

$$\therefore\ W\times4\pi R_S^2 = I_0\times4\pi D_{Se}^2$$

where D_{Se} is the distance between the sun and the earth, and I_0 is the intensity outside the earth's atmosphere.

$$I_0 = W\times\left(\frac{R_S}{D_{Se}}\right)^2$$

Now, $R_S = 7\times10^8\,\text{m}, D_{Se} = 150\times10^9\,\text{m}$

and $I_0 = 1.4\times10^3\ \text{W/m}^2$.

$$\therefore\ 1.4\times10^3 = W\times\left(\frac{7\times10^8}{150\times10^9}\right)^2 = W\times\frac{49}{225}\times10^{-4}$$

or $W = 6.4\times10^7\,\text{W/m}^2$

(ii) Assuming the sun to be a blackbody,

$$6.4\times10^7 = \sigma T_0^4 = \left(5.67\times10^{-8}\right)T_0^4$$

$$\therefore\ T_0^4 = \frac{6.4}{5.67}\times10^{15}$$

or $T_0 \approx 0.58\times10^4\,\text{K} = 5800\text{K}$

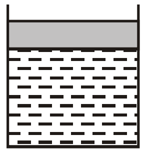
(iii) Using Wien's displacement law,

$$\lambda_{\text{mp}}T_0 = 0.29\text{cm-K} = 2.9\times10^{-3}\,\text{m-K}$$

or $\lambda_{mp} = \dfrac{2.0\times10^{-3}}{5800} = 5\times10^{-7}\,\text{m} = 5000A^\circ$

[Note : λ_{mp} is also referred to as λ_{max}]

Problem 7. *Consider a lake that is getting frozen at an atmospheric temperature of -10 °* C. *Assuming that most of the heat that is lost comes from the latent heat of fusion released when the water freezes. Find the rate at which the thickness of ice increases as a function of time. Take the conductivity of ice as K and the density of ice $\approx$ density of water $= \rho$*



Solution: The water just beneath the ice is almost at 0°C. Assume that the thickness of ice at time t is $x(t)$, that the area of the lake is A_0 and that the density of ice is ρ.

If the latent heat of ice is L, then

$$\frac{dQ}{dt} = \frac{LA_0 dx\rho}{dt} = \frac{KA_0}{x}\left[0-(-10)\right] = \frac{10KA_0}{x}$$

or, $\dfrac{dx}{dt} = \dfrac{10K}{xL\rho}$

or, $\int xdx = \dfrac{10K}{L\rho}\int dt$

or $\dfrac{x^2}{2} = \dfrac{10K}{L\rho}t + \text{constant}$

At $t = 0$, we assume that $x = 0$: i.e. initially the lake is not frozen.

Therefore, $x^2 = \dfrac{20K}{L\rho}t$

or $x(t) = \sqrt{\dfrac{20K}{L\rho}}\sqrt{t} = C\sqrt{t},$

where $C = \sqrt{\dfrac{20K}{L\rho}}$ is a constant.

Problem 8. *A solid copper sphere of density ρ, specific heat c and radius r is at temperature T_1. It is suspended inside a chamber whose walls are at temperature 0 K. What is the time required for the temperature of sphere to drop to T_2? Take the emmissivity of the sphere to be equal to e.*

Solution: The rate of loss of energy due to radiation,

$$P = eA\sigma T^4$$

This rate must be equal to $mc\dfrac{dT}{dt}$.

Hence, $-mc\dfrac{dT}{dt} = eA\sigma T^4$

Negative sign is used as temperature decreases with time. In this equation,

$$m = \left(\frac{4}{3}\pi r^3\right)\rho \text{ and } A = 4\pi r^2$$

$\therefore \qquad -\frac{dT}{dt} = \frac{3e\sigma}{\rho cr} T^4$

or, $\qquad -\int_0^t dt = \frac{r\rho c}{3e\sigma} \int_{T_1}^{T_2} \frac{dT}{T^4}$

Solving this, we get $t = \frac{r\rho c}{9e\sigma}\left(\frac{1}{T_2^3} - \frac{1}{T_1^3}\right)$.

Problem 9. *As insulated container is divided into two equal portions. One portion contains an ideal gas at pressure P and temperature T, while the other portion is a perfect vacuum. If a hole is opened between the two portions, find the change in internal energy and temperature of the gas.*

Solution : As the system is thermally insulated,

$\Delta Q = 0$

Further as here the gas is expanding against vacuum (surroundings), the process is called free expansion and for it,

$\Delta W = \int PdV = 0$ [as for vacuum P = 0]

So in accordance with first law of thermodynamics, i.e. $\Delta Q = \Delta U + \Delta W$, we have

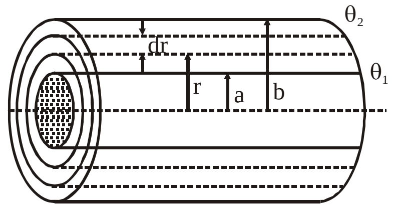
$0 = \Delta U + 0$, i.e. $\Delta U = 0$ or $U =$ constant

So in this problem internal energy of the gas remains constant, i.e. $\Delta U = 0$. Now as for an ideal gas

$U = \frac{3}{2}\mu RT$, i.e. $U \propto T$

So temperature of the gas will also remain constant, i.e. $\Delta T = 0$.

Problem 10: *A 2m long wire of resistance 4 ohm and diameter 0.64 mm is coated with plastic insulation of thickness 0.06 mm. When a current of 5 ampere flows through the wire, find the temperature difference across the insulation in steady state if*
$[K = 0.16 \times 10^{-2}\, cal/cm - {}^\circ C\, s]$



Solution : Considering a concentric cylindrical shell of radius *r* and thickness *dr* as shown in figure. The radial rate of flow of heat through this shell in steady state will be

$H = \frac{dQ}{dt} = -KA\frac{d\theta}{dr}$ Negative sign is used as with increase in *r*, θ decreases

Now as for cylindrical shell A = $2\pi r$ L

$H = -2\pi rLK\frac{d\theta}{dr}$

or $\qquad \int_a^b \frac{dr}{r} = -\frac{-2\pi LK}{H}\int_{\theta_1}^{\theta_2} d\theta$

which on integration and simplification gives

$$H = \frac{dQ}{dt} = -\frac{2\pi LK(\theta_1 - \theta_2)}{\ln\left(b/a\right)} \qquad \ldots(i)$$

Here, $H = \frac{I^2 R}{4.2} = \frac{(5)^2 \times 4}{4.2} = 24 \frac{\text{cal}}{\text{s}}$

$L = 2m = 200\,\text{cm}$

$r_1 = (0.64/2)\,\text{mm} = 0.032$ cm and $R_2 = r_1 + d = 0.032 + 0.006 = 0.038\,\text{cm}$

So $(\theta_1 - \theta_2) = \frac{24 \times \ln\left(38/32\right)}{2 \times 2.3026[\log_{10} 38 - \log_{10} 32]}$

$= \frac{24 \times 2.3026[\log_{10} 38 - \log_{10} 32]}{3.14 \times 0.64}$

or $(\theta_1 - \theta_2) = \frac{55 \times [1.57 - 1.50]}{2} = 2\,^{\circ}\text{C}.$