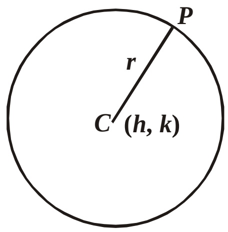
Lesson-3
CIRCLES

1. A circle is the locus of a point '*P*' which moves so that its distance from a fixed point, called the centre, is equal to a given distance, called the radius. Thus, if (h, k) denotes the centre and r = radius, then equation of the circle is



$$(x - h)^2 + (y - k)^2 = r^2$$

(a) The general equation of a circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Its centre is $(-g, -f)$ and radius = $\sqrt{g^2 + f^2 - c}$

(b) The general equation of the second degree

$$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a circle only if $a = b$ and $h = 0$

(c) The parametric form of the equation of a circle with centre (h, k) and radius r is

$$x = h + r\cos\theta\ ; \qquad y = k + r\sin\theta$$

where $0 \le \theta < 2\pi$. Here θ is the parameter

(d) Equation of a circle with diameter AB, where $A(x_1, y_1)$, $B(x_2, y_2)$, is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

2. Corresponding to a point $P(x_1, y_1)$, we define the following expressions

$$S_1 \equiv ax_1^2 + by_1^2 + 2gx_1 + 2fy_1 + c$$

$$T \equiv axx_1 + byy_1 + g(x + x_1) + f(y + y_1) + c$$

(a) **Position of a point w.r.t. the circle $S \equiv 0$.**

The point $P(x_1, y_1)$ lies within, on or outside the circle $S \equiv 0$ according as $S_1 < 0, = 0$, or > 0

(b) Equation of tangent at the point $P(x_1, y_1)$ lying on the circle $S \equiv 0$ is

$$T \equiv 0$$

(c) Equation of normal at the point $P(x_1, y_1)$ lying on the circle $S \equiv 0$ is

$$y - y_1 = \frac{y_1 + f}{x_1 + g}(x - x_1)$$. The normal of the circle will always pass through its centre.

(d) The line $y = mx + c$ intersects the circle $x^2 + y^2 = a^2$,

at real and distinct points if $c^2 < a^2(1 + m^2)$,

at real and coincident points if $c^2 = a^2(1 + m^2)$,

at imaginary points if $c^2 > a^2(1 + m^2)$

Thus, the two tangents of slope m to the circle $x^2 + y^2 = a^2$ are

$$y = mx \pm a\sqrt{1+m^2}$$

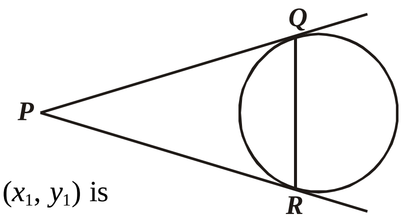
(e) The length of the tangent drawn from the point $P(x_1, y_1)$ to $S \equiv 0$ is $\sqrt{S_1}$

(f) The equation of the pair of tangents drawn from $P(x_1, y_1)$ to the circle $S \equiv 0$ is given by

$$SS_1 \equiv T^2$$

(g) The equation of the chord of contact (QR) of tangents drawn from $P(x_1, y_1)$ to $S \equiv 0$ is given by

$$T \equiv 0$$



(h) The equation of the chord (of $S \equiv 0$) with mid point (x_1, y_1) is

$$T \equiv S_1$$

Two or more circles:

Let $S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ be two circles.

1. Orthogonal circles

Circles $S_1 \equiv 0$ and $S_2 \equiv 0$ are said to intersect orthogonally when the tangents at their points of intersection are at right angles. The condition for orthogonality is

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

2. Radical axis

The radical axis of two circles is the locus of a point which moves so that the lengths of the tangents drawn from it to the two circles are equal.

The equation of the radical axis of $S_1 \equiv 0$ and $S_2 \equiv 0$ is

$$S_1 - S_2 \equiv 0$$

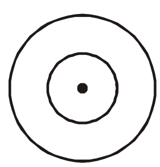
The radical axes of 3 circles taken in pairs, meet in a point called the radical centre of the 3 circles.

3. The position of two circles relative to each other.

Let $S_1 \equiv 0$ have center C_1 and radius r_1 and let $S_2 \equiv 0$ have center C_2 and radius r_2.

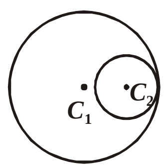
(i) One circle lies totally inside the other if

$$C_1C_2 < |r_2 - r_1|$$



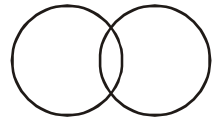
(ii) Circles touch each other (internally) if

$$C_1C_2 = |r_1 - r_2|$$



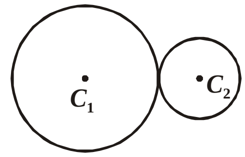
(iii) Circles intersect if

$$|r_1 - r_2| < C_1C_2 < r_1 + r_2$$



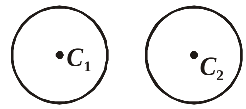
(iv) Circles touch each other (externally) if

$$C_1C_2 = r_1 + r_2$$



(v) Circles are non intersecting and lie outside each other if

$$C_1C_2 > r_1 + r_2$$



4. (i) For given circles $S_1 \equiv 0$ and $S_2 = 0$, the system $S_1 + \lambda S_2 = 0$, where λ is a parameter, represents a family of circles (which pass through the points of contact, if any, of $S_1 \equiv 0$ and $S_2 \equiv 0$)

(ii) If $u \equiv ax + by + c = 0$ is any straight line, then $S_1 + \lambda u \equiv 0$, where λ is a parameter, represents a family of circles (which pass through the points of contact, if any, of $S_1 \equiv 0$ and $u \equiv 0$)

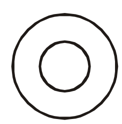
(iii) $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$, where λ is a parameter, represents a family of circles passing through the points $(x_i, y_i) : i = 1, 2$

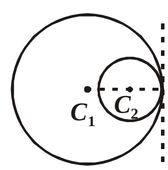
5. Common tangents of circles $S_1 \equiv 0$ and $S_2 \equiv 0$.

Let $S_1 \equiv 0$ have centre C_1 and radius r_1 and $S_2 \equiv 0$ have centre C_2 and radius r_2.

The number of common tangents of $S_1 \equiv 0$ and $S_2 \equiv 0$ is

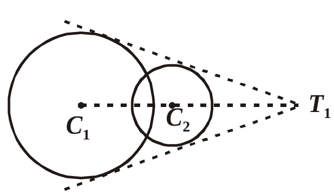
(i) zero if one circle lies totally inside the other.



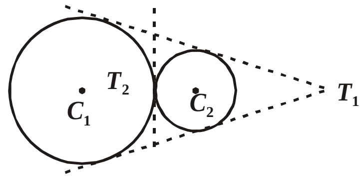


(ii) one, if the circles touch each other internally

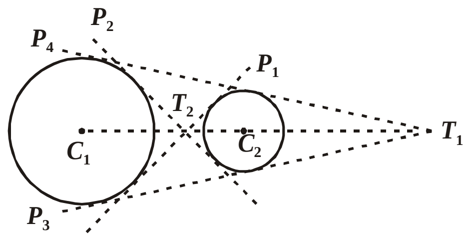
(iii) two, if the circles intersect each other (the tangents are called direct common tangents)



(iv) three, if the circles touch each other externally.



(v) four, if the circles lie outside each other.



P_4T_1 & P_3T_1 are called direct common tangents while P_2T_2 & P_1T_2 are called transverse common tangents. T_1 & T_2, the points of intersection of direct common tangents and transverse common tangents respectively, lie on the line joining C_1C_2.

T_1 divides C_1C_2 externally in the ratio $r_1 : r_2$.

T_2 divides C_1C_2 internally in the ratio $r_1 : r_2$.

SOLVED EXAMPLES

Ex.1: Find equation of the circle which passes through the centre of the circle $x^2 + y^2 + 8x + 10y - 7 = 0$ and is concentric with the circle $2x^2 + 2y^2 - 8x - 12y - 9 = 0$.

Sol.: Rewrite $2x^2 + 2y^2 - 8x - 12y - 9 = 0$, as $x^2 + y^2 - 4x - 6y - \frac{9}{2} = 0$

The centre of the required circle is C (2, 3) and it passes through the point A (–4, –5)

Its equation is

$$(x - 2)^2 + (y - 3)^2 \equiv AC^2 = 100$$

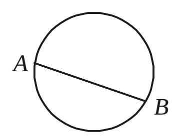
or $\quad x^2 + y^2 - 4x - 6y - 87 = 0.$

Ex.2: If $y = 2x$ is a chord AB of the circle $x^2 + y^2 - 10x = 0$, find the equation of the circle with AB as diameter.

Sol.: Solving $y = 2x$ and equation of circle, we get

$$x^2 + (2x)^2 - 10x = 0 \quad ; \quad x = 0, 2$$

Hence A is (0, 0) and B is (2, 4).



$\therefore$ Equation of the required circle on AB as diameter is

$$(x - 0)(x - 2) + (y - 0)(y - 4) = 0$$

or $\quad x^2 + y^2 - 2x - 4y = 0$

Ex.3: Find the equation of the tangent and the normal to the circle $x^2 + y^2 - 2ax = 0$ at the point $P\,[a\,(1 + \cos\alpha),\, a \sin\alpha] : 0 < \alpha < \frac{\pi}{2}$

Sol.: Equation of the tangent at P is

$$x.\,a\,(1 + \cos\alpha) + y\,.a\sin\alpha - a(x + a(1 + \cos\alpha)) = 0$$

or $\quad x\cos\alpha + y\sin\alpha = a(1 + \cos\alpha)$

Its slope is $-\cot\alpha$; $\therefore$ slope of normal at P is $\tan\alpha$.

$\therefore$ Equation of normal at P is

$$y - a\sin\alpha = \tan\alpha\,[x - a(1 + \cos\alpha)]$$

or $\quad x\sin\alpha - y\cos\alpha = a\sin\alpha.$

Ex.4: Find the equation of the circle through the point of intersection of the circles

$$x^2 + y^2 - 8x - 2y + 7 = 0 \text{ and } x^2 + y^2 - 4x + 10y + 8 = 0$$

and which passes through the point (–1, –2).

Sol.:

$$S_1 \equiv x^2 + y^2 - 8x - 2y + 7 = 0$$

$$S_2 \equiv x^2 + y^2 - 4x + 10y + 8 = 0$$

$S_1 + \lambda S_2 = 0$ passes through (–1, –2); hence, $\lambda = 8$

$\therefore$ Equation of required circle is $9x^2 + 9y^2 - 40x + 78y + 71 = 0$

Ex.5: Find the equation of a circle which cuts the circle $x^2 + y^2 - 6x + 4y - 3 = 0$ orthogonally and which passes through (3, 0) and touches the axis of y.

Sol.: Let centre of required circle be $C(h\,, k)$

$|h|$ = radius of circle = $\sqrt{(h-3)^2+k^2}$

$\therefore \quad (h-3)^2 + k^2 = h^2$

or $\quad k^2 - 6h + 9 = 0 \qquad$ ...(i)

Required circle is $(x-h)^2 + (y-k)^2 = h^2$

or $\quad x^2 + y^2 - 2hx - 2ky + k^2 = 0$

It is intersected by $x^2 + y^2 - 6x + 4y - 3 = 0$ orthogonally ;

$\therefore \quad 2(-3)(-h) + 2(2)(-k) = k^2 - 3$

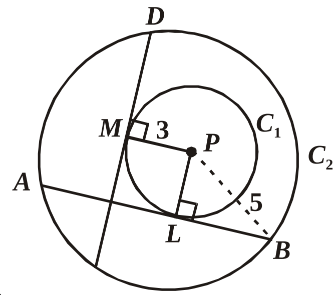
or $\qquad 6h - 4k + 3 = k^2 \qquad$ ...(ii)

Solve (i) and (ii) : $h = 3$, $k = 3$

Required circle is $x^2 + y^2 - 6x - 6y + 9 = 0$

Ex.6: Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C_1 (of diameter 6). If centre of C_1 lies in the first quadrant, find concentric circle C_2 which cuts intercepts of length 8 units on each given line.

Sol.: Let centre of circle C_1 be $P(h, k)$, where $h > 0$ and $k > 0$



$LB = 4$; $PL = 3$; $PB = 5$ = radius of C_2

$PM = PL = 3$

$$\Rightarrow \quad \frac{|5h+12k-10|}{13} = \frac{|5h-12k-40|}{13} = 3$$

Since $h > 0$, $k > 0$ we have : $5h + 12k - 10 > 5h - 12k - 40$.

Hence, $0 \le \dfrac{5h+12k-10}{13} = \dfrac{12k-5h+40}{13} = 3$ and $\therefore \quad h = 5,\ k = 2$

Hence, equation of required circle is $(x-5)^2 + (y-2)^2 = 25$

Ex.7: From a point P tangents are drawn to circles $x^2 + y^2 + x - 3 = 0$, $x^2 + y^2 - (5/3)x + y = 0$ and $4x^2 + 4y^2 + 8x + 7y + 9 = 0$, and they are of equal lengths. Find equation of a circle passing through P and touching the line $x + y = 5$ at $A(6, -1)$.

Sol.: Write third circle as $x^2 + y^2 + 2x + (7/4)y + (9/4) = 0$

By definition, ***P* is radical centre** of three circles. Equation of two of the radical axes are

$(8/3)x - y - 3 = 0$ and $x + (7/4)y + (21/4) = 0$ which intersect at $P(0, -3)$.

Let required circle be $x^2 + y^2 + 2gx + 2fy + c = 0$ with center $Q(-g, -f)$

$P(0, -3)$ lies on it $\Rightarrow -6f + c + 9 = 0 \qquad$ ...(i)

$A(6, -1)$ lies on it $\Rightarrow 12g - 2f + c + 37 = 0 \qquad$ ...(ii)

Since, QA is perpendicular to $x + y = 5$

$$\therefore \quad \left(\frac{-f+1}{-g-6}\right)(-1) = -1 \qquad \text{...(iii)}$$

$\Rightarrow \quad f - g = 7.$

Solving (i), (ii) and (iii) for f, g and c, we have :

$$f = \frac{7}{2},\ g = -\frac{7}{2} \text{ and } c = 12.$$

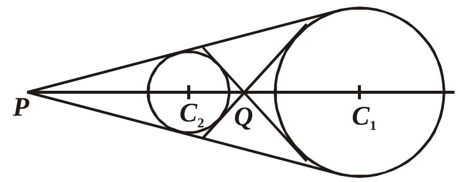
Hence equation of required circle is $x^2 + y^2 - 7x + 7y + 12 = 0$

Ex.8: Find all common tangents to

$S_1 \equiv x^2 + y^2 - 2x - 6y + 9 = 0$ and $S_2 \equiv x^2 + y^2 + 6x - 2y + 1 = 0$

Sol.: For S_1, centre C_1 is (1, 3), radius = 1 = r_1

For S_2, centre C_2 is (–3, 1), radius = 3 = r_2



Two direct common tangents intersect at P which divides C_1C_2 externally in ratio $r_1 : r_2$.

Two transverse common tangents intersect at Q which divides C_1C_2 internally in ratio $r_1 : r_2$.

Using these properties of common tangents and eqs. of circles we get P as (3, 4) and Q as (0, 5/2) by section formulae.

Direct Common tangents: Line through P viz. $y - 4 = M(x - 3)$, is a tangent to $S_1 = 0$, if perpendicular distance from centre C_1 to it is equal to radius of that circle. (Note: $x = 3$ does not satisfy this criterion)

$$\frac{|3 - 4 - M(1-3)|}{\sqrt{1+M^2}} = 1 \quad \Rightarrow \quad M = 0, \frac{4}{3}$$

Hence equations of **direct common tangents** are $y = 4$, $4x - 3y = 0$.

Transverse common tangents: Any line through Q is $y - \frac{5}{2} = m(x - 0)$ or $x = 0$

Proceeding as for direct common tangents we get, $m = -\frac{3}{4}$. We note that $x = 0$ is also a tangent.

$\therefore$ Transverse common tangents are $x = 0$ and $3x + 4y - 10 = 0$

Ex.9: If $P(2a, a + 1)$ is an interior point of the larger segment of the circle $x^2 + y^2 - 2x - 2y - 8 = 0$ made by line $x - y + 1 = 0$, find values of "a".

Sol.: Point $P(2a, a + 1)$ is an interior point of the circle

$\therefore$ $4a^2 + (a + 1)^2 - 2(2a) - 2(a + 1) - 8 < 0$

or $(5a - 9)(a + 1) < 0$

$\therefore$ $a \in (-1, 9/5)$ ...(i)

P lies in larger segment if, centre O (1, 1) and $P(2a, a + 1)$ lie on the same side of the line

$x - y + 1 = 0$.

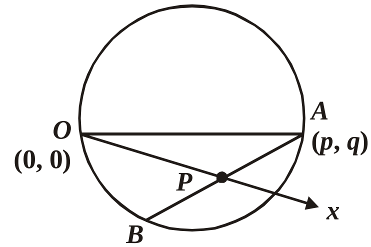
$\therefore$ $2a - (a + 1) + 1 > 0$ or $a > 0$ ...(ii)

Taking intersection of (i) and (ii);

$a \in (0, 9/5)$.

Ex.10: Two distinct chords drawn from the point $A(p, q)$ to the circle $x^2 + y^2 - px - qy = 0$, $(pq \neq 0)$, are bisected by the x-axis. Show that $p^2 > 8q^2$.

Sol.: Circle passes through (0, 0) and $A(p, q)$



Let the chord AB intersect x-axis at P; then, B is $(\alpha, -q)$

$\therefore \quad \alpha^2 + q^2 - p\alpha + q^2 = 0 \quad$ or $\quad \alpha^2 - p\alpha + 2q^2 = 0$

For two distinct real chords, roots of this equation are real and distinct

$\therefore \quad p^2 - 8q^2 > 0.$

Ex.11: Let T_1, T_2 be two tangents drawn from (–2, 0) to the circle $C : x^2 + y^2 = 1$. Determine circles touching C and having T_1, T_2 as their pair of tangents. Further find the equations of all posible common tangents to these circles, when taken two at a time.

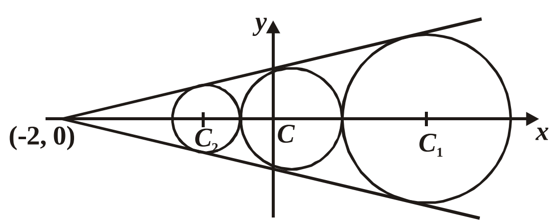
Sol.: Equation of any tangent to $x^2 + y^2 = 1$, is

$$y = mx \pm \sqrt{1+m^2}$$

As they are drawn from $A(-2,0)$, conditions are

$$0 = -2m \pm \sqrt{1+m^2}$$

$$\Rightarrow \quad m = \pm\frac{1}{\sqrt{3}}$$



Equations of tangents become

$$T_1 : \sqrt{3}y = x + 2$$

$$T_2 : \sqrt{3}y = -x - 2$$

Circles touching C and having T_1 and T_2 as tangents must have their center on x-axis (the angle bisector of T_1 and T_2).

Let C_1 and C_2 be the 2 circles and $M(h_1, 0)$ & $L(h_2, 0)$ be their respective centers where $h_1 > 0$ and $h_2 < 0$

By tangency of T_1, perpendicular distance from centre M is equal to radius r_1 of the circle C_1

$$\therefore \quad r_1 = \frac{h_1 + 2}{2}$$

As C_1 and C touch each other

$$r_1 = h_1 - 1$$

or $\quad \dfrac{h_1 + 2}{2} + 1 = h_1$ or $h_1 = 4$

$\therefore \quad$ For circle C_1 : centre is $M(4, 0)$ and radius = 3.

Similarly for circle C_2,

$$-h_2 - 1 = \left|\frac{h_2+2}{2}\right|$$

$\Rightarrow \quad -2h_2 - 2 = h_2 + 2$ ($\because \quad h_2 > -2$; see figure)

$\Rightarrow \quad -3h_2 = 4$

or $\quad h_2 = -\dfrac{4}{3}$

and radius $= \dfrac{1}{3}$.

Equations of two circles are $(x - 4)^2 + y^2 = 9$

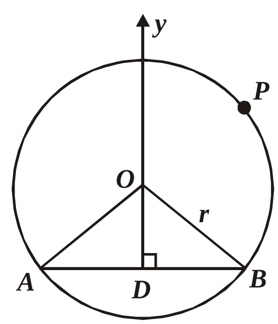
and $\left(x+\dfrac{4}{3}\right)^2 + y^2 = \dfrac{1}{9}$

C_1 & C have $x = 1$ as transverse common tangent and C_2 & C have $x = -1$ as transverse common tangent

Ex.12: Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre O. Show that the locus of the centroid of the triangle PAB as P moves on the circle is a circle .

Sol.: ΔODB is isosceles with $OD = DB = x$ (say)

We may assume AB is parallel to and below x-axis



$\therefore \quad x^2 + x^2 = r^2 \Rightarrow x = \dfrac{r}{\sqrt{2}}$

$\therefore \quad B$ is $\left(\dfrac{r}{\sqrt{2}}, -\dfrac{r}{\sqrt{2}}\right)$ and A is $\left(-\dfrac{r}{\sqrt{2}}, -\dfrac{r}{\sqrt{2}}\right)$

Let P be $(r\cos\theta, r\sin\theta)$ and centroid of ΔPAB be $G(x_1, y_1)$

$$\therefore \quad x_1 = \frac{r\cos\theta + \frac{r}{\sqrt{2}} - \frac{r}{\sqrt{2}}}{3}, \quad y_1 = \frac{r\sin\theta - \frac{r}{\sqrt{2}} - \frac{r}{\sqrt{2}}}{3}$$

$3x_1 = r\cos\theta$; $3y_1 = r(\sin\theta - \sqrt{2})$

Eliminating θ, we get

$$\therefore \quad \left(\frac{3x_1}{r}\right)^2 + \left(\frac{3y_1}{r}+\sqrt{2}\right)^2 = 1 \quad \text{or} \quad x_1^2 + \left(y_1 + \frac{\sqrt{2}r}{3}\right)^2 = \frac{r^2}{9}$$

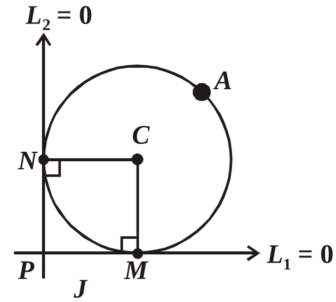
$\therefore$ Locus of (x_1, y_1) is a circle.

Ex.13: Find the equation of the circle passing through the point $A(2, 8)$ and touching the lines $4x - 3y - 24 = 0$ & $4x + 3y - 42 = 0$ and having both coordinates of the centre of the circle less than or equal to 8.

Sol.: Let C denote the center of required circle

Let $L_1 \equiv 4x - 3y - 24 = 0$

$L_2 \equiv 4x + 3y - 42 = 0$



& Let M and N denote the respective points of contact

Equations of bisectors of angles between the lines are:

$$\frac{4x-3y-24}{5} = \pm\frac{4x+3y-42}{5}$$

i.e., $y = 3$ & $x = \frac{33}{4}$

Since C lies on one of these bisectors and x-coordinate of C is less than or equal to 8,

$\therefore$ C lies on $y = 3$

Let C be $(a, 3)$.

Then, $CM = CA$

$$\Rightarrow \quad \frac{|4a-3(3)-24|}{5} = \sqrt{(a-2)^2+(3-8)^2}$$

or $\left(\frac{4a-33}{5}\right)^2 = (a-2)^2 + 25$

or $16a^2 - 264a + (33)^2 = 25\{a^2 - 4a + 29\}$

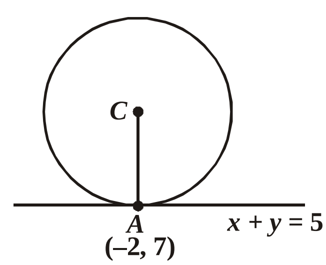
or $9a^2 + 164a - 364 = 0$

or $(a-2)(9a+182) = 0$

$\therefore$ $a = 2$ or $a = -\frac{182}{9}$ and radius $= CA$.

Ex.14: Find the equation of a circle which touches the line $x + y = 5$ at the point $A(-2, 7)$ and cuts the circle $x^2 + y^2 + 4x - 6y + 9 = 0$ orthogonally.

Sol.: Let C be the centre of the required circle; slope of $CA = 1$



Equation of CA is $\frac{x+2}{1/\sqrt{2}} = \frac{y-7}{1/\sqrt{2}} = \lambda (= AC)$

Centre C has coordinates $\left(-2+\frac{\lambda}{\sqrt{2}}, 7+\frac{\lambda}{\sqrt{2}}\right)$

Equation of circle is

$$\left(x+2-\frac{\lambda}{\sqrt{2}}\right)^2 + \left(y-7-\frac{\lambda}{\sqrt{2}}\right)^2 = \lambda^2$$

or $x^2 + y^2 + 2\left(2-\frac{\lambda}{\sqrt{2}}\right)x - 2\left(7+\frac{\lambda}{\sqrt{2}}\right)y + \left(2-\frac{\lambda}{\sqrt{2}}\right)^2 + \left(7+\frac{\lambda}{\sqrt{2}}\right)^2 - \lambda^2 = 0$

or $x^2 + y^2 + 2\left(2 - \frac{\lambda}{\sqrt{2}}\right)x - 2\left(7 + \frac{\lambda}{\sqrt{2}}\right)y + 53 + \frac{10}{\sqrt{2}}\lambda = 0$ ...(i)

It intersects $x^2 + y^2 + 4x - 6y + 9 = 0$ orthogonally.

$$\therefore \quad 2\left(2 - \frac{\lambda}{\sqrt{2}}\right)2 + 2\left(7 + \frac{\lambda}{\sqrt{2}}\right)3 = 53 + \frac{10}{\sqrt{2}}\lambda + 9$$

$$= 62 + \frac{10}{\sqrt{2}}\lambda$$

$$\therefore \quad \lambda = -\frac{3}{\sqrt{2}}$$

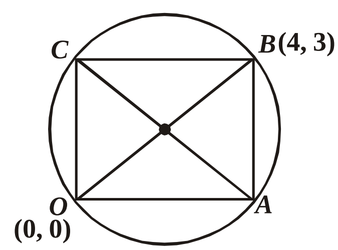
∴ Required circle is

$$x^2 + y^2 + 7x - 11y + 38 = 0$$

Ex.15: Extremities of a diagonal of a rectangle are (0, 0) and (4, 3). Find the equations of the tangents to the circumcircle of the rectangle which are parallel to this diagonal.

Sol.: Two extremities are $O(0, 0)$ and $B(4, 3)$.

Middle point of the diagonal OB is $D\left(2, \frac{3}{2}\right)$



which is the centre of the circumscribed circle

and radius is $OD = \sqrt{4 + \frac{9}{4}} = \frac{5}{2}$

A line parallel to OB is

$$y = \frac{3}{4}x + C$$

It is a tangent to the circumscribed circle.

Therefore length of perpendicular from $D\left(2, \frac{3}{2}\right)$ to it $= \frac{5}{2}$

$$\Rightarrow \quad \frac{\left|\frac{3}{4}(2) - \frac{3}{2} + C\right|}{\sqrt{1 + \frac{9}{16}}} = \frac{5}{2}$$

or $C = \pm\frac{5}{2}.\frac{5}{4} = \pm\frac{25}{8}$

Hence tangents are $y = \frac{3}{4}x \pm \frac{25}{8}$

or $3x - 4y \pm \frac{25}{2} = 0.$

OBJECTIVE ASSIGNMENT

Choose the correct option in the following :

1. Equation of the circle through the points of intersectionof circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ and passing through (1, 1), is $x^2 + y^2 + \lambda x + a = 0$, where λ and a are

(a) –6, 1 (b) –3, 1 (c) –3, –2 (d) none

2. The locus of the centre of that circle of radius 2, which rolls on the outside of the circle $x^2 + y^2 + 3x - 6y - 9 = 0$, is a circle with centre at

(a) $\left(-\frac{3}{2}, 3\right)$ (b) $\left(-\frac{3}{2}, 1\right)$ (c) $\left(-\frac{3}{2}, 2\right)$ (d) none of these

3. The area of the circle with centre (1, 2) and passing through (4, 6), is

(a) 5π (b) 10π (c) 25π (d) none of these

4. The circles $x^2 + y^2 - 2x - 3 = 0$ and $x^2 + y^2 - 4x - 6y - 8 = 0$

(a) touch each other (b) intersect each other

(c) one lies inside the other (d) none of these

5. The locus of the middle points of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin, is

(a) $x + y = 2$ (b) $x^2 + y^2 = 2$ (c) $x^2 + y^2 = 1$ (d) none of these

6. Angle between two tangents drawn from origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\pi/2$ (d) none of these

7. The number of common tangents to the circles : $x^2 + y^2 + 2x + 8y - 23 = 0$ and $x^2 + y^2 - 4x - 10y + 19 = 0$ is

(a) 1 (b) 2 (c) 3 (d) none of these

8. If the line $3x + 4y = K$ touches the circle $x^2 + y^2 = 10x$, then the value of K is equal to

(a) +30 (b) +10 (c) 40 (d) none of these

9. The equation of the circle whose centre is (3, –1) and which cuts off a chord of length 6 units on the line $2x - 5y + 18 = 0$, is

(a) $x^2 + y^2 - 7x + y = 2$ (b) $x^2 + y^2 - 3x + y + 7 = 0$

(c) $x^2 + y^2 - 6x + 2y - 28 = 0$ (d) none of these

10. Equation of a circle through (–1, –2) and concentric with the circle $x^2 + y^2 - 3x + 4y - c = 0$ is

(a) $x^2 + y^2 - 3x + 4y - 1 = 0$ (b) $x^2 + y^2 - 3x + 4y = 0$

(c) $x^2 + y^2 - 3x + 4y + 2 = 0$ (d) none of these

11. Points (2, 0), (0, 1), (4, 5) and (0, a) are concyclic for a =

(a) 14/3 or 1 (b) 14 or 1/3 (c) –14/3 or –1 (d) none of these

12. Equation of the normal to the circle $x^2 + y^2 - 2ax = 0$ at the point $[a(1 + \cos\alpha), a \sin\alpha]$ is given by

(a) $y = (x - a)\tan\alpha$ (b) $y = (x + a)\cot\alpha$ (c) $y = x\tan\alpha + a\cot\alpha$ (d) none of these

13. The tangents to the circles $x^2 + y^2 - 169 = 0$ at the points (5, 12) and (12, –5) respectively, are

(a) parallel (b) coincident (c) perpendicular (d) none of these

14. The number of points of intersection of the circle $x^2 + y^2 + 6x - 4y + 8 = 0$ with the line $4x + 3y - 12 = 0$

(a) at two distinct real points (b) at two imaginary points

(c) at two coincident real points (d) none of these

15. The angle between a pair of tangents drawn from a point P to the circle

$x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α.

The equation of the locus of the point P is

(a) $x^2 + y^2 + 4x - 6y + 4 = 0$ (b) $x^2 + y^2 + 4x - 6y - 9 = 0$

(c) $x^2 + y^2 + 4x - 6y - 4 = 0$ (d) $x^2 + y^2 + 4x - 6y + 9 = 0$

16. The area of triangle formed by the tangent and the normal at the point (4, 3) to the circle $x^2 + y^2 = 25$ and x-axis is

(a) 75/4 (b) 75/8 (c) 25/4 (d) none of these

17. The equation of circle which touches $x = 0$, $x = a$ and $3x + 4y + 5a = 0$ is

(a) $x^2 + y^2 - ax + 2ay + a^2 = 0$ (b) $x^2 + y^2 = a^2$

(c) $x^2 + y^2 - ax = 0$ (d) none of these

18. The circle passing through the points (1, 0), (0, 1) and (0, 0) passes through the point $(2k + 1, 3k + 1)$ if

(a) k is any integer (b) $k = -5/13$ (c) $k = 0$ (d) none of these

19. The equation of the circle passing through the points of intersection of the circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$, and the point (1, 1) is

(a) $x^2 + y^2 - 3x + 1 = 0$ (b) $x^2 + y^2 - 7x + 6 = 0$

(c) $x^2 + y^2 - x + 7 = 0$ (d) none of these

20. A circle passing through the origin and cutting off the equal chords of length $\sqrt{2}$ from the rays $y = |x|$, is

(a) $x^2 + y^2 - 2y = 0$ (b) $x^2 + y^2 + 2x = 0$ (c) $x^2 + y^2 - 2x - 2y = 0$ (d) none of these

21. The equation of the circle passing through (2, 0) and (0, 4) and having the minimum radius is

(a) $x^2 + y^2 + x + y = 3$ (b) $x^2 + y^2 - 2x - 4y = 0$

(c) $x^2 + y^2 - 7x - y + 8 = 0$ (d) none of these

22. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle with AB as diameter is

(a) $x^2 + y^2 = 25$ (b) $x^2 + y^2 - x - y = 0$

(c) $x^2 + y^2 \pm 2x \pm 3y = 0$ (d) none of these

23. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ bisects the circumference of the circle

$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0,$

then the condition is

(a) $2g_1(g - g_1) + 2f_1(f - f_1) = c_1 - c$ (b) $2g_1(g - g_1) + 2f_1(f - f_1) = c - c_1$

(c) $2g_1(g - g_1) + 2f_1(f - f_1) = c + c_1$ (d) none of these

24. The locus of the mid-points of the chords of the circle $x^2 + y^2 = 4$, which subtends a right angle at the centre is

(a) $3x + 4y = 1$ (b) $4(x^2 + y^2) + 2x = 0$ (c) $x^2 + y^2 = 2$ (d) none of these

25. The equation of the circle cutting orthogonally the three circles :

$x^2 + y^2 - 2x + 3y - 7 = 0,$

$x^2 + y^2 + 5x - 5y + 9 = 0,$

$x^2 + y^2 + 7x - 9y + 29 = 0$ is

(a) $x^2 + y^2 - 16x - 18y - 4 = 0$ (b) $x^2 + y^2 - 7x + 11y + 6 = 0$

(c) $x^2 + y^2 + 2x - 8y + 9 = 0$ (d) none of these

26. α, β and γ are parametric angles of three points P, Q and R respectively, on the circle $x^2 + y^2 = 1$ and A is the point (–1, 0). If the length of the chords AP, AQ and AR are in GP, then cos α/2, cos β/2 and cos γ/2 are in

(a) AP (b) GP (c) HP (d) none of these

27. The area bounded by the circles $x^2 + y^2 = r^2$, $r = 1, 2$ and the rays given by $2x^2 - 3xy - 2y^2 = 0$, $y > 0$ is

(a) $\frac{\pi}{4}$ sq unit (b) $\frac{\pi}{2}$ sq unit (c) $\frac{3\pi}{4}$ sq unit (d) π sq unit

28. The circle $x^2 + y^2 = 4$ cuts the line joining the points A(1, 0) and B(3, 4) in two points P and Q. Let $\frac{BP}{PA} = \alpha$ and $\frac{BQ}{QA} = \beta$, then α and β are roots of the quadratic equation

(a) $x^2 + 2x + 7 = 0$ (b) $3x^2 + 2x - 21 = 0$ (c) $2x^2 + 3x - 27 = 0$ (d) none of these

29. If α, β, are the roots of $ax^2 + bx + c = 0$ and α', β' those of $a'x^2 + b'x + c' = 0$, the equation of the circle having A(α, α') and B(β, β') as diameter is

(a) $cc'(x^2 + y^2) + ac'x + a'cy + a'b + ab' = 0$

(b) $cc'(x^2 + y^2) + a'cx + ac'y + a'b + ab' = 0$

(c) $bb'(x^2 + y^2) + a'bx + ab'y + a'c + ac' = 0$

(d) $aa'(x^2 + y^2) + a'bx + ab'y + a'c + ac' = 0$

30. The equation of the circle touching the lines $|y| = x$ at a distance $\sqrt{2}$ unit from the origin is

(a) $x^2 + y^2 - 4x + 2 = 0$ (b) $x^2 + y^2 + 4x - 2 = 0$

(c) $x^2 + y^2 + 4x + 2 = 0$ (d) none of these

MORE THAN ONE CORRECT ANSWERS

31. The tangents drawn from the origin to the circle $x^2 + y^2 - 2px - 2qy + q^2 = 0$ are perpendicular, if

(a) $p = q$ (b) $p^2 = q^2$ (c) $q = -p$ (d) $p^2 + q^2 = 1$

32. An equation of a circle touching the axes of coordinates and the line $x \cos \alpha + y \sin \alpha = 2$ can be

(a) $x^2 + y^2 - 2gx - 2gy + g^2 = 0$; where $g = 2/(\cos \alpha + \sin \alpha + 1)$

(b) $x^2 + y^2 - 2gx - 2gy + g^2 = 0$; where $g = 2/(\cos \alpha + \sin \alpha - 1)$

(c) $x^2 + y^2 - 2gx + 2gy + g^2 = 0$; where $g = 2/(\cos \alpha - \sin \alpha + 1)$

(d) $x^2 + y^2 - 2gx + 2gy + g^2 = 0$; where $g = 2/(\cos \alpha - \sin \alpha - 1)$

33. If α is the angle subtended at P(x_1, y_1) by the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then

(a) $\cos \alpha = \dfrac{\sqrt{S_1}}{\sqrt{(g^2 + f^2 - c)}}$ (b) $\cot \alpha/2 = \dfrac{\sqrt{S_1}}{\sqrt{(g^2 + f^2 - c)}}$

(c) $\tan \alpha = \dfrac{2\sqrt{(g^2 + f^2 - c)}}{\sqrt{S_1}}$ (d) $\alpha = 2 \tan^{-1}\left(\dfrac{\sqrt{(g^2 + f^2 - c)}}{\sqrt{S_1}}\right)$

34. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts each of the circles $x^2 + y^2 - 4 = 0$, $x^2 + y^2 - 6x - 8y = 0$ and $x^2 + y^2 + 2x - 4y - 2 = 0$ at the extremities of a diameter, then

(a) $c = -4$ (b) $g + f = c - 1$ (c) $g^2 + f^2 - c = 17$ (d) $gf = 6$

35. From the point A(0, 3) on the $x^2 + 4x + (y - 3)^2 = 0$, a chord AB is drawn and extended to a point M, such that AM = 2AB. An equation of the locus of M is

(a) $x^2 + 6x + (y - 2)^2 = 0$ (b) $x^2 + 8x + (y - 3)^2 = 0$

(c) $x^2 + y^2 + 8x - 6y + 9 = 0$ (d) $x^2 + y^2 + 6x - 4y + 4 = 0$

36. If a chord of the circle $x^2 + y^2 - 4x - 2y - c = 0$ is trisected at the points (1/3, 1/3) and (8/3, 8/3), then

(a) $c = 10$ (b) $c = 20$ (c) $c = 15$ (d) $c^2 - 40c + 400 = 0$

37. Consider the circles $C_1 \equiv x^2 + y^2 - 2x - 4y - 4 = 0$ and $C_2 \equiv x^2 + y^2 + 2x + 4y + 4 = 0$ and the line $L \equiv x + 2y + 2 = 0$, then

(a) L is the radical axis of C_1 and C_2

(b) L is the common chord of C_1 and C_2

(c) L is the common chord of C_1 and C_2

(d) L is perpendicular to the line joining centres of C_1 and C_2

38. The equation of a circle is $S_1 \equiv x^2 + y^2 = 1$. The orthogonal tangents to S_1 meet at another circle S_2 and the orthogonal tangents to S_2 meet at the third circle S_3. Then

(a) radius of S_2 and S_3 are in the ratio $1 : \sqrt{2}$ (b) radius of S_2 and S_3 are in the ratio 1 : 2

(c) the circles S_1, S_2 and S_3 are concentric (d) none of these

39. If the area of the quadrilateral formed by the tangents from the origin to the circle $x^2 + y^2 + 6x - 10y + c = 0$ and the pair of radii at the points of contact of these tangents to the circle is 8 sq unit, then the value of c is

(a) 2 (b) 4 (c) 16 (d) 32

40. The equation of a circle in which the chord joining the points (1, 2) and (2, –1) subtends an angle of $\pi/4$ at any point on the circumference is

(a) $x^2 + y^2 - 5 = 0$ (b) $x^2 + y^2 - 6x - 2y + 5 = 0$

(c) $x^2 + y^2 + 6x + 2y - 15 = 0$ (d) $x^2 + y^2 - 2x - 4y + 4 = 0$

MISCELLANEOUS ASSIGNMENT

Comprehension-I

Through two given points infinite circles can pass. All such circles are together called family of circles as they have a common relation among them (i.e. passing through the same two given points). There are number of ways in which two points are obtained, hence number of ways of getting equation of family of circles are

(i) Two circles $S_1 = 0$ and $S_2 = 0$ intersecting at two points. The equation of family of circles is $S_1 + \lambda S_2 = 0$, where $\lambda \in R - \{-1\}$

(ii) A circle, $S_1 = 0$ and a line $L_1 = 0$ intersecting at two points, then equation of family of circles is $S_1 + \lambda L_1 = 0$, where $\lambda \in R$

(iii) Two points $A(x_1, y_1)$ and $B(x_2, y_2)$ are given, then equation of family of circles is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

If one more condition is given then we can find the specific member of the family by getting the value of λ.

1. Equation of circle passing through the point of intersection of line $x + y = 1$ and $x^2 + y^2 = 16$ and passing through $(-1, 0)$ is

(a) $x^2 + y^2 + 2x + 2y + 8 = 0$ (b) $x^2 + y^2 + x = 0$

(c) $2x^2 + 2y^2 - 15x - 15y - 17 = 0$ (d) none of these

2. In $S_1 + \lambda S_2 = 0$, if $\lambda = 1$, then equation comes out to be

(a) circle (b) straight line (c) point (d) none of these

3. Equation of the circle passing through (1, 1) and (3, 1) and touching the x-axis at (2, 0) is

(a) $x^2 + y^2 - 4x - 2y - 4 = 0$ (b) $x^2 + y^2 - 2x - 4y + 4 = 0$

(c) $x^2 + y^2 - 4x - 2y + 4 = 0$ (d) none of these

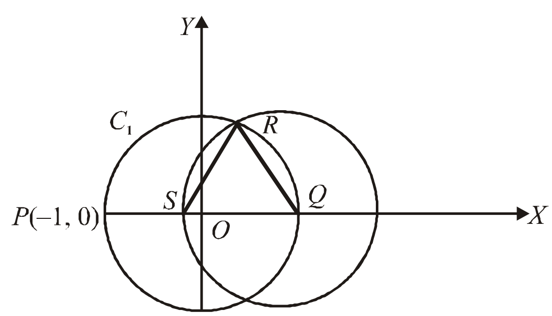
4. Equation of the circle passing through the point of intersection of $x^2 + y^2 = 4$ and $x^2 + y^2 - 4x - 4y = 0$ and whose diameter is the common chord is

(a) $x^2 + y^2 - x - y + 4 = 0$ (b) $2x^2 + 2y^2 - 4x - 2y + 6 = 0$

(c) $x^2 + y^2 - 4x = 0$ (d) $x^2 + y^2 - x - y - 3 = 0$

Comprehension-II

The circle $x^2 + y^2 = 1$ cuts the x-axis at P and Q. Another circle with center at Q and variable radius intersects the first circle at R above the x-axis and the line segment PQ at S.



5. For maximum or minimum area ΔQSR, radius of the circle is

(a) $\sqrt{\frac{2}{3}}$ (b) $2\sqrt{\frac{2}{3}}$ (c) $\sqrt{\frac{3}{2}}$ (d) $2\sqrt{\frac{3}{2}}$

6. The maximum area of ΔQSR is

(a) $\sqrt{\frac{2}{3}}$ sq. units (b) $\frac{4}{\sqrt{3}}$ sq. units (c) $\frac{2}{3\sqrt{3}}$ sq. units (d) $\frac{4}{3\sqrt{3}}$ sq. units

MATRIX MATCH TYPE QUESTIONS

7.

A.	The minimum radius of circle belonging to family of circles passing through point of intersection of $x^2 + y^2 - 6x - 8y = 0$ and $y = 4$ is	(p)	5
B.	The maximum value of minimum radius in above case is	(q)	4
C.	A circle is drawn cutting parabola $y^2 = 16x$ at A and B and touching its directrix then minimum length of AB is	(r)	8
D.	A point P moves such that it subtends a obtuse angle at line segment AB where $A \equiv (0, 0)$, $B \equiv (4, 0)$, minimum area of the region is A_0 then A_0/π is	(s)	10

8.

A.	The point $(\lambda, 2 + \lambda)$ lies inside the circle $x^2 + y^2 = 4$, then the value of λ can be	(p)	–1
B.	The point $(\lambda, \lambda + 2)$ will lies outside the circle $x^2 + y^2 - 2x + 4y = 0$, then the value of λ can be	(q)	–1/2
C.	Both the equations $x^2 + y^2 + 2\lambda x + 4 = 0$ and $x^2 + y^2 - 4\lambda y + 8 = 0$ represent real circles, then the value of λ can be	(r)	1
		(s)	3
		(t)	5

INTEGER TYPE QUESTIONS

9. If a circle passes through the points of intersection of the co-ordinate axes with the lines $\lambda x - y + 1 = 0$ and $x - 2y + 3 = 0$, then the value of 3λ is.

10. Length of chord on the line $4x - 3y - 10 = 0$ cut off by the circle $x^2 + y^2 - 2x + 4y - 4 = 0$ is.

11. The length of y-intercept of the circle $x^2 + y^2 + 4x - 7y + 12 = 0$ is.

12. The maximum number of points with rational coordinates on a circle whose centre is $(\sqrt{5}, 0)$ is.

13. If number of points with integral coordinates that are interior to the circle $x^2 + y^2 = 17$ is λ then the value of $\lambda/7$ is.

14. Tangents are drawn from P(6, 8) to the circle $x^2 + y^2 = r^2$, then the radius of the circle such that the area of the Δ formed by tangents and chord of contact is maximum must be

15. If $\left(m_i, \frac{1}{m_i}\right), m_i > 0,$ $i = 1, 2, 3, 4$ are four distinct points on a circle, then the value of $\prod_{i=1}^{4} m_i$ must be

16. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the coordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + k(x^2 + y^2)^{1/2} = 0$, then the value of k must be .

17. The area of a quadrilateral formed by a pair of tangents from the point (4, 5) to the circle $(x-2)^2 + (y-1)^2 = 16$ with a pair of radii where tangents touch the circle is λ sq unit, then λ must be

18. If the line $y + x = 0$ bisects two chords drawn from a point $\left(\frac{1+a\sqrt{2}}{2}, \frac{1-a\sqrt{2}}{2}\right)$ to the circle $2x^2 + 2y^2 - (1 + a\sqrt{2})x - (1 - a\sqrt{2})y = 0$, then lies in the interval $(-\infty, -\lambda) \cup (\lambda, \infty)$, the numerical quantity λ should be equal to

PREVIOUS YEAR QUESTIONS

IIT-JEE/JEE-ADVANCE QUESTIONS

1. A square is inscribed in the circle $x^2 + y^2 - 2x + 4y + 3 = 0$. Its sides are parallel to the coordinate axes. Then one vertex of the square is

(a) $(1 + \sqrt{2}, -2)$ (b) $(1 - 2\sqrt{2}, -2)$ (c) $(1, -2 + \sqrt{2})$ (d) none of these

2. If from any point on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha = 0$, then the angle between the tangents is

(a) α (b) 2α (c) 4α (d) none of these

3. If $3x + y = 0$ is a tangent to the circle which has its centre at $(2, -1)$ then the equation of the other tangent to the circle from origin is

(a) $x - 3y = 0$ (b) $x + 3y = 0$ (c) $3x - y = 0$ (d) $x - y = 0$

4. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is touched by the line $y = x$ at P where $OP = 6\sqrt{2}$ (O being origin) then

(a) $c = 36$ (b) $c = 72$ (c) $c = 144$ (d) $c = 27$

5. The equation of circle which touches the line $2x - y = 1$ t $(1, 1)$ and also touches the line $2x + y = 4$ is

(a) $x^2 + y^2 - 3y + 1 = 0$ (b) $x^2 + y^2 + 3y + 1 = 0$

(c) $4(x^2 + y^2) + 10x + 7y + 9 = 0$ (d) none of these

6. The number of points with integral coordinates which are interior to the circle $x^2 + y^2 = 16$ is

(a) 43 (b) 45 (c) 49 (d) 51

7. For each $k \in N$, let C_k denote the circle whose equation is $x^2 + y^2 = k^2$. On the circle C_k, a particle moves k units in the anticlockwise direction. After completing its motion on C_k, the particle moves to C_{k+1} in the radial direction. The motion of particle continues in this manner. The particle starts at $(1, 0)$. If the particle crosses the positive x-axis for the first time on the circle C_n, then n is equal to

(a) 2 (b) 6 (c) 7 (d) 9

8. Two distinct chords drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ $(pq \neq 0)$ are bisected by x-axis then

(a) $p^2 = q^2$ (b) $p^2 = 8q^2$ (c) $p^2 < 8q^2$ (d) $p^2 > 8q^2$

9. The centre of the smallest circle passing through origin lies on the straight line $y = x + 1$ The centre of the circle is

(a) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{4}, \frac{5}{4}\right)$ (c) $\left(-\frac{3}{2}, \frac{1}{2}\right)$ (d) $(-1, 0)$

10. The locus of the points of intersection of perpendicular tangents to the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ is

(a) $x^2 + y^2 = a^2 + b^2$ (b) $x^2 + y^2 = (a^2 + b^2)^2$

(c) $x^2 + y^2 = \sqrt{a^4 + b^4}$ (d) $x^2 + y^2 = |a^2 - b^2|$

11. Two circles each of radius r having centres at points (2, 3) and (5, 6) respectively cut each other orthogonally then r is equal to

(a) 1 (b) 2 (c) 3 (d) 4

12. If the tangent at a point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis then the length of PQ is

(a) 4 (b) $2\sqrt{5}$ (c) $3\sqrt{5}$ (d) 5

13. The number of common tangents to the circles $x^2 + y^2 - x = 0$ and $x^2 + y^2 + x = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4

14. The length of common chord of the circles $x^2 + y^2 + 5x + 7y + 9 = 0$ and $x^2 + y^2 = 7x + 5y + 9 = 0$ is

(a) 6 (b) 7 (c) 8 (d) 9

15. If $a > 2b > 0$ then the positive value of m for which the line $y = mx - b\sqrt{1 + m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is

(a) $\dfrac{2b}{\sqrt{a^2 - 4b^2}}$ (b) $\dfrac{\sqrt{a^2 - 4b^2}}{2b}$ (c) $\dfrac{2b}{\sqrt{a^2 - 2b^2}}$ (d) $\dfrac{2a}{\sqrt{a^2 - 2b^2}}$

16. Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$.

Statement-1: The tangents are mutually perpendicular.

because

Statement-2: The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

17. Let $ABCD$ be a quadrilateral with area 18, with side AB parallel to the side CD and $AB = 2CD$. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral $ABCD$ touching all the sides, then its radius is

(a) 3 (b) 2 (c) 3/2 (d) 1

Comprehension (Q.18 to Q.20)

A square $ABCD$ of side length 2 units. C_1 is the circle touching all sides and C_2 is the circle touching the vertices of the square $ABCD$. L is the line through A.

18. P is any point on C_1 and Q is any point on C_2. The value of $\frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$ is

(a) 0.25 (b) 0.75 (c) 1 (d) 1.25

19. A circle touch the line L and the circle C_1 externally such that both the circles are on the same side of the line, then the locus of centre of the circle is

(a) ellipse (b) hyperbola

(c) parabola (d) parts of straight line

20. A line M through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts M at T_2 and T_3 and AC at T_1, then area of $\Delta T_1T_2T_3$ is

(a) $\frac{1}{2}$ sq. unit (b) $\frac{2}{3}$ sq. unit (c) 1 sq. unit (d) 2 sq. unit

Comprehension (Q.21 to Q.23)

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

21. The ratio of the areas of the triangles PQS and PQR is

(a) $1 : \sqrt{2}$ (b) $1 : 2$ (c) $1 : 4$ (d) $1 : 8$

22. The radius of the circumcircle of the triangle PRS is

(a) 5 (b) $3\sqrt{3}$ (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

23. The radius of the incircle of the triangle PQR is

(a) 4 (b) 3 (c) 8/3 (d) 2

Paragraph for Question Nos. 24 to 26

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}\,x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

24. The equation of circle C is

(a) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$

(b) $(x - 2\sqrt{3})^2 + (y + 1/2)^2 = 1$

(c) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$

(d) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

25. Points E and F are given by

(a) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$

(b) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$

(c) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(d) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

26. Equations of the sides QR, RP are

(a) $y = \frac{2}{\sqrt{3}}x + 1, y = -\frac{2}{\sqrt{3}}x - 1$

(b) $y = \frac{1}{\sqrt{3}}x, y = 0$

(c) $y = \frac{\sqrt{3}}{2}x + 1, y = -\frac{\sqrt{3}}{2}x - 1$

(d) $y = \sqrt{3}x, y = 0$

27. Match the statements in **Column I** with the properties in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

	Column-I		**Column-II**
A.	Two intersecting circles	(p)	have a common tangent
B.	Two mutually external circles	(q)	have a common normal
C.	Two circles, one strictly inside the other	(r)	do not have a common tangent
D.	Two branches of a hyperbola	(s)	do not have a common normal

28. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is

29. The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point

(a) $\left(-\frac{3}{2}, 0\right)$ (b) $\left(-\frac{5}{2}, 2\right)$ (c) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (d) (–4, 0)

30. The locus of the mid-point of the chord of contact of tangents drawn from point lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is

(a) $20(x^2 + y^2) - 36x + 45y = 0$

(b) $20(x^2 + y^2) + 36x - 45y = 0$

(c) $36(x^2 + y^2) - 20x + 45y = 0$

(d) $36(x^2 + y^2) + 20x - 45y = 0$

Paragraph for Question 31 to 32

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P\left(\sqrt{3},1\right)$. A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$.

31. A common tangent of the two circles is

(a) $x = 4$ (b) $y = 2$ (c) $x+\sqrt{3}y=4$ (d) $x+2\sqrt{2}y=6$

32. A possible equation of L is

(a) $x-\sqrt{3}y=1$ (b) $x+\sqrt{3}y=1$ (c) $x-\sqrt{3}y=-1$ (d) $x+\sqrt{3}y=5$

DCE QUESTIONS

1. There are two circles with equations $x^2 + y^2 = 9$ and $x^2 + y^2 - 8x - 6y + n^2 = 0$ ($n \in I$). If the two circles have exactly two common tangents then the number of possible values of n is

(a) 2 (b) 7 (c) 9 (d) 8

2. A is the set of all circles with radii 3 whose centres lie on the circle $x^2 + y^2 = 25$. The locus of any point on any of the member of set A is given by

(a) $x^2 + y^2 \le 25$ (b) $4 \le x^2 + y^2 \le 8$

(c) $(x - 3)^2 + (y - 5)^2 \le 25$ (d) $4 \le x^2 + y^2 \le 64$

3. The equation of the locus of a point, the tangents from which to the circle

$$x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$$

contain angle 2α between then is

(a) $x^2 + y^2 + 4x - 6y - 9 = 0$ (b) $x^2 + y^2 + 4x - y + 9 = 0$

(c) $x^2 + y^2 + 4x - 6y - 4 = 0$ (d) $x^2 + y^2 + 4x - 6y + 4 = 0$

4. A line meets the coordinate axes in A and B. A circle is described about the triangle OAB. If m and n are distances of the tangent to the circle at the origin from points A and B respectively then the diameter of the circle is

(a) $m(m + n)$ (b) $m + n$ (c) $n(m + n)$ (d) $\frac{1}{2}(m + n)$

5. If the two circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g_1x + 2f_1y = 0$ touch each other then

(a) $f_1g = fg_1$ (b) $ff_1 = gg_1$ (c) $f^2 + g^2 = f_1^2 + g_1^2$ (d) none of these

6. Circles are drawn from the point (2, 0) to cut intercepts of length 5 units on the x-axis. If their centre lie in the first quadrant then their equation is

(a) $x^2 + y^2 - 9x + 2ky + 14 = 0$ (b) $3x^2 + 3y^2 + 27x - 2ky + 43 = 0$

(c) $x^2 + y^2 - 9x - 2ky + 14 = 0$ (d) $x^2 + y^2 - 2kx - 9y + 14 = 0$

7. If the line $ax + by = 2$ is a normal to the circle $x^2 + y^2 - 4x - 4y = 0$ and tangent to the circle $x^2 + y^2 = 1$ then

(a) $a = \frac{1+\sqrt{7}}{2}, b = \frac{1-\sqrt{7}}{2}$

(b) $a = 1, b = \sqrt{3}$

(c) $a = \frac{1}{2}, b = \frac{1}{2}$

(d) $a = \frac{\sqrt{7}}{2}, b = -\frac{\sqrt{7}}{2}$

8. The radical centre of three circles described on the three sides of a triangle as diameters is

(a) the orthocentre of Δ

(b) the circumcentre of Δ

(c) the incentre of Δ

(d) the centroid of Δ

9. The centre of a circle passing through the points (0, 0) & (1, 0) and touching the circle $x^2 + y^2 = 9$ is

(a) $\left(\frac{1}{2}, \frac{3}{2}\right)$

(b) $\left(\frac{3}{2}, \frac{1}{2}\right)$

(c) $\left(\frac{1}{2}, -\frac{3}{2}\right)$

(d) $\left(\frac{1}{2}, -2^{1/2}\right)$

10. The distance from the centre of the circle $x^2 + y^2 = 2x$ to the straight line passing through the intersection of the two circles $x^2 + y^2 + 5x - 8y + 1 = 0$ and $x^2 + y^2 - 3x + 7y - 25 = 0$

(a) 1

(b) 3

(c) 2

(d) 1/3

11. Four vertices of a square are (±3, ±3). If (a, a) is a point inside the square but falls outside the circle $x^2 + y^2 = 1$, then the set of possible values of a is

(a) $\left(\frac{1}{\sqrt{2}}, 3\right)$

(b) $\left(-3, -\frac{1}{\sqrt{2}}\right)$

(c) $\left(-3, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 3\right)$

(d) (–3, 3)

12. Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre, then the locus of the centroid of ΔPAB where P moves on the circle

(a) a parabola

(b) a circle

(c) an ellipse

(d) a pair of lines

13. The equation of the circle concentric to the circle $2x^2 + 2y^2 - 3x + 6y + 2 = 0$ and having area double the area of this circle is

(a) $8x^2 + 8y^2 - 24x + 48y - 13 = 0$

(b) $16x^2 + 16y^2 + 24x - 48y - 13 = 0$

(c) $16x^2 + 16y^2 - 24x + 48y - 13 = 0$

(d) $8x^2 + 8y^2 + 24x - 48y - 13 = 0$

14. Let $x^2 + y^2 - 2x - 6y + 6 = 0$ and $x^2 + y^2 - 6x - 4y + 12 = 0$ are two circles, then equation of the circle having diamter as their common chord is

(a) $5x^2 + 5y^2 + 26x - 22y + 54 = 0$

(b) $5x^2 + 5y^2 + 26x + 22y + 54 = 0$

(c) $5x^2 + 5y^2 - 26x - 22y + 54 = 0$

(d) $5x^2 + 5y^2 - 26x - 22y - 54 = 0$

AIEEE/JEE-MAINS QUESTIONS

1. The circle $x^2 + y^2 - 2px + r^2 = 0$ and $x^2 = y^2 - 2qy + r^2 = 0$ with touch externally if

(a) $\frac{1}{p^2} + \frac{1}{q^2} = \frac{1}{r^2}$ (b) $\frac{1}{p^2} + \frac{1}{r^2} = \frac{1}{q^2}$ (c) $\frac{1}{a^2} + \frac{1}{r^2} = \frac{1}{p^2}$ (d) $p^2 + q^2 = r^2$

2. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = r^2$ intersect each other in two distinct points if

(a) $r < 2$ (b) $r > 8$ (c) $2 < r < 8$ (d) $2 \le r \le 8$

3. The angle between the tangents drawn from origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

4. The circumcentre of the triangle formed by (c, 7), (c, –2) and (5, 7) is

(a) (5, 7) (b) $\left(4, \frac{5}{2}\right)$ (c) (c, 7) (d) (c, –2)

5. The common chord of the circles $x^2 + y^2 - 4x - 4y = 0$ and $x^2 + y^2 = 16$ subtends an angle at the origin equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

6. The shortest distance between the circles $x^2 + y^2 - 10x - 10y + 41 = 0$ and $x^2 + y^2 = 1$ is

(a) $\sqrt{41} - 1$ (b) 0 (c) $\sqrt{41}$ (d) $5\sqrt{2} - 4$

7. A circle passes through the points of intersection of the line $x = 0$ and circle $x^2 + y^2 + 2x = 3$. If this circle passes through $(\sqrt{3}, 0)$ then its centre is

(a) (0, 0) (b) (0, 1) (c) (1, 0) (d) (1, 1)

8. If the circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 + 4x + 3y + 2 = 0$ cut in A and B then the equation of the circle on AB as diameter is

(a) $x^2 + y^2 + x + 3y + 3 = 0$ (b) $x^2 + y^2 + x + 6y + 1 = 0$

(c) $2x^2 + 2y^2 + 2x + 6y + 1 = 0$ (d) none of these

9. A variable chord is drawn through the origin to the circle $x^2 + y^2 - 2ax = 0$. Locus of the centre of the circle drawn on this chord as diameter is

(a) $x^2 + y^2 + ax = 0$ (b) $x^2 + y^2 - ax = 0$

(c) $x^2 + y^2 + ay = 0$ (d) $x^2 + y^2 - ay = 0$

10. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre is

(a) $x^2 + y^2 = \frac{9}{4}$ (b) $x^2 + y^2 = \frac{3}{2}$ (c) $x^2 + y^2 = 1$ (d) $x^2 + y^2 = \frac{27}{4}$

11. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is

(a) $x^2 + y^2 - 2x + 2y - 47 = 0$ (b) $x^2 + y^2 + 2x - 2y - 47 = 0$

(c) $x^2 + y^2 + 2x - 2y - 62 = 0$ (d) $x^2 + y^2 - 2x + 2y - 62 = 0$

12. Consider a family of circles which are passing through the point (–1, 1) and are tangent to x-axis. If (h, k) are the coordinates of the centre of the circles, then the set of values of k is given by the interval

(a) $-\frac{1}{2} \le k \le \frac{1}{2}$ (b) $k \le \frac{1}{2}$ (c) $0 < k < \frac{1}{2}$ (d) $k \ge \frac{1}{2}$

13. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ $(c > 0)$ touch each other if

(a) $2|a| = c$ (b) $|a| = c$ (c) $a = 2c$ (d) $|a| = 2c$

14. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is

(a) 10/3 (b) 3/5 (c) 6/5 (d) 5/3

15. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point

16. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having centre at (0, 3) is

(a) $x^2 + y^2 - 6y - 5 = 0$ (b) $x^2 + y^2 - 6y + 5 = 0$

(c) $x^2 + y^2 - 6y - 7 = 0$ (d) $x^2 + y^2 - 6y + 7 = 0$

BASIC LEVEL ASSIGNMENT

1. Find the equation of the circle whose centre lies on the line $2x - y - 3 = 0$ and which passes through the points $(3, -2)$ and $(-2, 0)$.

2. Show that four points $(0, 0)$, $(1, 1)$, $(5, -5)$ and $(6, -4)$ are concyclic.

3. Find the centre, the radius and the equation of the circle drawn on the line joining $A(-1, 2)$ and $B(3, -4)$ as diameter.

4. Find the equation of the tangent and the normal to the circle $x^2 + y^2 = 25$ at the point $P(-3, -4)$.

5. Show that the tangent to $x^2 + y^2 = 5$ at $(1, -2)$ also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$

6. Find the equation of the tangents to the circle $x^2 + y^2 - 2x + 8y = 23$ drawn from an external point $(8, -3)$.

7. Find the equation of the circle whose centre is $(-4, 2)$ and having the line $x - y = 3$ as a tangent

8. Find the equation of the circle through the points of intersections of two given circles $x^2 + y^2 - 8y - 2y + 7 = 0$ and $x^2 + y^2 - 4x + 10y + 8 = 0$ and passing through $(3, -3)$.

9. Find the equation of chord of the circle $x^2 + y^2 - 4x = 0$ which is bisected at the point $(1, 1)$.

10. Find the equation of chord of contact of the circle $x^2 + y^2 - 4x = 0$ with respect to the point $(6, 0)$.

11. Find the length of the tangent drawn from the point $(3, 2)$ to the circle $4x^2 + 4y^2 + 4x + 16y + 13 = 0$.

12. Obtain the equations of common tangents of the circles $x^2 + y^2 = 9$ and $x^2 + y^2 - 12x + 27 = 0$.

13. The centres of the circle passing through the points $(0, 0)$, $(1, 0)$ and touching the circle $x^2 + y^2 = 9$ are $\left(\frac{1}{2}, \pm\sqrt{2}\right)$.

14. The abscissae of two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $x^2 + 2px - q^2 = 0$. Find the equation and the radius of the circle with AB as diameter.

15. A circle touches the line $y = x$ at a point P such that $OP = 4\sqrt{2}$, where O is the origin. The circle contains the point $(-10, 2)$ in its interior and the length of its chord on the line $x + y = 0$ is $6\sqrt{2}$. Determine the equation of the circle.

ADVANCED LEVEL ASSIGNMENT

1. Show that the line $x + y = 2$ touches the circles $x^2 + y^2 = 2$ and $x^2 + y^2 + 3x + 3y - 8 = 0$ at the point where the two circles touch each other.

2. One of the diameters of the circle circumscribing the rectangle $ABCD$ is $4y = x + 7$. If A and B are the points $(-3, 4)$ and $(5, 4)$ respectively, find the area of the rectangle.

3. A circle of radius 2 lies in the first quadrant and touches both the axes of co-ordinates. Find the equation of the circle with centre at (6, 5) and touching the above circle externally.

4. If $\left(m_i, \frac{1}{m_i}\right)$; $i = 1, 2, 3, 4$ are four distinct points on a circle, show that $m_1 m_2 m_3 m_4 = 1$.

5. Show that the circle on the chord $x\cos\alpha + y\sin\alpha - p = 0$ of the circle $x^2 + y^2 = a^2$ as diameter is $x^2 + y^2 - a^2 - 2p\,(x\cos\alpha + y\sin\alpha - p) = 0$

6. Find the length of the chord of the circle $x^2 + y^2 = 16$ which bisects the join of the points (2, 3) and (1, 2) perpendicularly.

7. Find the angle that the chord of circle $x^2 + y^2 - 4y = 0$ along the line $x + y = 1$ subtends at the circumference of the larger segment.

8. Prove that the equation $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$, where λ is a parameter, represents a family of circles passing through two fixed points A and B on the x-axis. Also find the equation of that circle of the family, the tangents to which at A and B meet on the line $x + 2y + 5 = 0$.

9. Find the area of the quadrilateral formed by a pair of tangents from the point (4, 5) to the circle $x^2 + y^2 - 4x - 2y - 11 = 0$ and a pair of its radii.

10. If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ cut the co-ordinate axes in concyclic points, prove that $a_1a_2 = b_1b_2$.

11. Show that the length of the tangent from any point on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + c_1 = 0$ is $\sqrt{c_1 - c}$.

12. Find the point from which the tangents to the three circles

$$x^2 + y^2 - 4x + 7 = 0,\ 2x^2 + 2y^2 - 3x + 5y + 9 = 0 \text{ and } x^2 + y^2 + y = 0$$

are equal in length. Find also this length.

13. The chord of contact of tangents from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$. Show that a, b, c are in G.P.

14. Obtain the equation of the circle orthogonal to both the circles $x^2 + y^2 + 3x - 5y + 6 = 0$ and $4x^2 + 4y^2 - 28x + 29 = 0$ and whose centre lies on the line $3x + 4y + 1 = 0$.

15. From the point $A(0, 3)$ on the circle $x^2 + 4x + (y - 3)^2 = 0$, a chord AB is drawn and extended to a point M such that $AM = 2AB$. Find the equation of the locus of M.

16. From the origin, chords are drawn to the circle $(x - 1)^2 + y^2 = 1$. Find the equation of the locus of the middle points of these chords.

17. Tangent at any point on the circle $x^2 + y^2 = a^2$ meets the circle $x^2 + y^2 = b^2$ at P and Q. Find the condition on a and b such that tangents at P and Q meet at right angles.

18. The tangent from a point to the circle $x^2 + y^2 = 1$ is perpendicular to the tangent from the same point to the circle $x^2 + y^2 = 3$. Show that the locus of the point is a circle.

19. A variable circle passes through the point $A(a, b)$ and touches the x-axis. Show that the locus of the other end of the diameter through A is $(x - a)^2 = 4by$.

20. AB is a diameter of a circle. CD is a chord parallel to AB and $2CD = AB$. The tangent at B meets the line AC produced at E. Prove that $AE = 2AB$.

ANSWERS

Objective Assignment

1.	(b)	2.	(a)	3.	(c)	4.	(b)	5.	(b)
6.	(c)	7.	(c)	8.	(c)	9.	(c)	10.	(b)
11.	(a)	12.	(a)	13.	(a)	14.	(b)	15.	(d)
16.	(b)	17.	(a)	18.	(c)	19.	(a)	20	(a)
21.	(b)	22.	(b)	23.	(b)	24.	(c)	25.	(a)
26.	(b)	27.	(c)	28.	(b)	29.	(d)	30	(a)
31.	(a,b,c)	32.	(a,b,c,d)	33.	(b,d)	34.	(a,b,c,d)	35.	(b,c)
36.	(b,d)	37.	(a,c,d)	38.	(a,c)	39.	(a,d)	40	(a,b)

Miscellaneous Assignment

1. (c) 2. (b) 3. (c) 4. (d) 5. (b)

6. (d) 7. A-(p); B-(p); C-(r); D)-(q)

8. A-(p),(q); B-(p),(q),(r),(s),(t); C-(s),(t) 9. (1) 10. (6)

11. (1) 12. (2) 13. (7) 14. (5) 15. (1)

16. (1) 17. (8) 18. (2)

Previous Year Questions

IIT-JEE/JEE-ADVANCE QUESTIONS

1. (d) 2. (b) 3. (a) 4. (b) 5. (a)

6. (b) 7. (c) 8. (d) 9. (a) 10. (a)

11. (c) 12. (d) 13. (c) 14. (a) 15. (a)

16. (a) 17. (b) 18. (b) 19. (c) 20. (c)

21. (c) 22. (b) 23. (d) 24. (d) 25. (a)

26. (d) 27. A-(p),(q); B-(p),(q); C-(q),(r) D-(q),(r)

28. 9 29. (d) 30. (a) 31. (d) 32. (a)

DCE QUESTIONS

1.	(c)	2.	(d)	3.	(b)	4.	(b)	5.	(a)
6.	(c)	7.	(a)	8.	(a)	9.	(d)	10.	(c)
11.	(c)	12.	(b)	13.	(c)	14.	(c)		

-MAINS QUESTIONS

1.	(a)	2.	(c)	3.	(b)	4.	(b)	5.	(d)
6.	(d)	7.	(a)	8.	(c)	9.	(b)	10.	(a)
11.	(a)	12.	(d)	13.	(b)	14.	(a)	15.	(a)
16.	(c)								

Basic Level Assignment

1. $x^2 + y^2 + 3x + 12y + 2 = 0$

3. $(1, -1)$, $\sqrt{3}$, $x^2 + y^2 - 2x + 2y - 11 = 0$

4. $3x + 4y + 25 = 0$, $4x - 3y = 0$

6. $13x + 9y = 77$, $3x - y - 27 = 0$

7. $2x^2 + 2y^2 + 16x - 8y - 41 = 0$

8. $23x^2 + 23y^2 - 156x + 38y + 168 = 0$.

9. $y = x$

10. $x = 3$

11. $\dfrac{\sqrt{109}}{2}$

12. $x = 3$ and $y = \pm 3$.

14. $x^2 + y^2 + 2ax + 2py - (b^2 + q^2) = 0$; $\sqrt{a^2 + b^2 + p^2 + q^2}$

15. $x^2 + y^2 + 18x - 2y + 32 = 0$

Advanced Level Assignment

2. 32 sq. units

3. $x^2 + y^2 - 12x - 10y + 52 = 0$

6. $4\sqrt{2}$

7. $\cos^{-1}\dfrac{1}{2\sqrt{2}}$

8. $x^2 + y^2 - 2x - 6y - 8 = 0$.

9. 8 sq. units.

12. $(2, -1)$; 2

14. $4(x^2 + y^2) + 2y - 29 = 0$.

15. $x^2 + y^2 + 8x - 6y + 9 = 0$.

16. $x^2 + y^2 - x = 0$

17. $2a^2 = b^2$