Lesson-3
PROBABILITY

Random experiment:

It is an experiment in which

(a) all possible outcomes of the experiment are known in advance

(b) the exact outcome of it is not known in advance

Sample space:

The set of all possible outcomes of an experiment is called as its sample space.

Event:

It is a possible out come of an experiment.

Example: Consider the experiment of throwing a dice. The set $S = \{1, 2, ..., 6\}$ is its sample space

(i) The event: "occurence of an even number" is an event

(ii) ϕ - the empty set, consisting of no sample point, is also an event known as the impossible event.

(iii) The event: "Occurence of a no. $x : 1 \le x \le 6, x \in N$" is called as the sure event.

Exhaustive set of events:

A set of outcomes/events is said to be exhaustive if it covers each and every possible outcomes of the sample space of the experiment i.e., at least one of them has to occur in a performance of the experiment.

If $E_1, E_2, ..., E_n$ are events defined on a sample space S and $E_1 \cup E_2 \cup .... \cup E_n = S$, then these events are called exhaustive events.

Mutually exclusive events:

A set of events is said to be mutually exclusive if occurence of any one of them rules out the possibility of occurence of the remaining events. Thus, if A and B are mutually exclusive events then $A \cap B = \phi$.

Equally likely events:

Two or more events are said to be equally likely if none of them can be expected to occur in preference to any of the other events.

Example: (i) In throwing of a fair coin, 'head' and 'tail' are equally likely events.

(ii) In throwing of a dice, the events: $A_1 = \{1, 2\}$, $A_2 = \{3, 4, 5\}$ are not equally likely.

Classical definition of Probability

If an experiment results in n exhaustive, mutually exclusive and equally likely outcomes and m of them are favourable to an event A then the probability of event A is

$$P(A) = \frac{m}{n}$$

$$\therefore \quad 0 \le P(A) \le 1$$

Some results on probability

(i) If ϕ denotes the null event then, $P(\phi) = 0$

(ii) If A is an event then, $P(\bar{A}) = 1 - P(A)$

(iii) For events A, B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(iv) For events A, B and C

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Conditional probability

Given, events A and B, the probability of occurence of A given that B has already occured is called as conditional probability of occurence of A on the condition that B has already occured and is denoted by $P(A/B)$.

$$P(A/B) = \begin{cases} \dfrac{P(A \cap B)}{P(B)} & \text{if} \quad P(B) > 0 \\ \text{not defined} & \text{if} \quad P(B) = 0 \end{cases}$$

Example: Let a dice be tossed once. Define the events.

$A \equiv$ occurence of 1, 2, 4 or 6

$B \equiv$ occurence of 2, 3 or 4

$$P(A) = \frac{2}{3}, \quad P(B) = \frac{1}{2}; \quad P(A/B) = \frac{2}{3}; \quad P(B/A) = \frac{1}{2}$$

Theorem on total probability

If $B_1, \ldots, B_n$ are exhaustive and mutually exclusive events such that $P(B_i) > 0 \;\; \forall \;\; i = 1, \ldots, n$ and A is any event then,

$$P(A) = \sum_{j=1}^{n} P(A/B_j).P(B_j)$$

Example: There are two bags, B_1 and B_2. B_1 has 2 black and 3 green balls. B_2 has 3 black and 2 green balls. A dice is thrown. If an even number appears, a ball is drawn from bag B_1; else, a ball is drawn from bag B_2. What is the probability that ball drawn is black.

Solution: We define the following events:

A : ball drawn is black

A_1 : even number appears on the dice

A_2 : odd number appears on the dice

$P(A) = P(A/A_1).P(A_1) + P(A/A_2).P(A_2)$

$P(A/A_1) = \frac{2}{5}$; $P(A/A_2) = \frac{3}{5}$; $P(A_1) = P(A_2) = \frac{1}{2}$

$\therefore \quad P(A) = \frac{1}{2}$

Baye's theorem:

If A, B are events and $P(A) > 0$, $P(B) > 0$, then,

$$P(A/B) = \frac{P(B/A).P(A)}{P(B)}$$

In other words, if $A_1, \ldots, A_n$ are n mutually exclusive and exhaustive events with $P(A_i) > 0 \ \forall \ i$ and A is any event such that $P(A) > 0$, then,

$$P(A_i/A) = \frac{P(A/A_i).P(A_i)}{\sum_{j=1}^{n} P(A/A_j).P(A_j)}$$

Example: In the situation of the previous example, if the ball chosen is found to be black then, what is the probability that it came from bag B_1.

Solution: $P(A_1/A) = \frac{P(A/A_1).P(A_1)}{P(A/A_1).P(A_1) + P(A/A_2).P(A_2)}$

$$= \frac{\frac{2}{5}.\frac{1}{2}}{\frac{2}{5}.\frac{1}{2} + \frac{3}{5}.\frac{1}{2}}$$

$$= \frac{2}{5}$$

Multiplication rule:

If $A_1, \ldots, A_n$ are events such that $P(A_i) > 0 \ \ \forall \ \ i = 1, \ldots, n$ then,

$$P(A_1 \cap \ldots . \cap A_n) = P(A_1).P(A_2/A_1).P(A_3/A_1 \cap A_2) \ldots .. P(A_n/A_1 \cap A_2 \cap . \ldots .. \cap A_{n-1})$$

Example: A bag has 5 red and 3 green balls. A ball is drawn at random, its colour is noted and is replaced with 2 more balls of the same colour. This process is done 3 times. What is the probability that a red ball is drawn in each of the 3 draws.

Solution: We define the following events:

B_i : ball drawn in i^{th} draw is red : $i = 1, 2, 3$

$$P(B_1 \cap B_2 \cap B_3) = P(B_1).P(B_2/B_1)\ .P(B_3/B_1 \cap B_2)$$

$$= \frac{5}{8} \times \frac{7}{10} \times \frac{9}{12} = \frac{21}{64}$$

Independence of events:

Two events A and B are said to be independent if occurence or non occurence of event A does not depend on the occurence or non occurence of event B. Thus, A and B are independent if one of the following equivalent conditions are true:

(i) $P(A \cap B) = P(A).P(B)$

(ii) $P(A/B) = P(A)$ if $P(B) > 0$

(iii) $P(B/A) = P(B)$ if $P(A) > 0$

Three events A, B, C are said to be (completely) independent if, and only if, they are pairwise independent and

$$P(A \cap B \cap C) = P(A).P(B).P(C)$$

Note: Pair wise independence does not imply complete independence.

Example: From a pack of 52 cards, 2 cards are drawn, one by one, without replacement

We define the following events:

A_1 : first card drawn is spade

A_2 : second card drawn is spade

$$P(A_1) = \frac{13}{52};\ P(\overline{A_1}) = \frac{39}{52}$$

$$P(A_2) = P(A_2/A_1).P(A_1) + P(A_2/\overline{A_1}).P(\overline{A_1})$$

$$= \frac{12}{51} \cdot \frac{13}{52} + \frac{13}{51} \cdot \frac{39}{52}$$

$$= \frac{1}{4}$$

$$P(A_1 \cap A_2) = P(A_1).P(A_2/A_1) = \frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$$

$\therefore$ A_1 and A_2 are not independent

Use of binomial theorem

Let an experiment consist of n trials (like, n tosses of a dice). Let p (respectively q) denote the probability of occurence (respectively non occurence) of an event A in a trial. The probability of occurence of A, exactly r times, in n trials is given by.

$${}^nC_r \cdot p^r \cdot q^{n-r}.$$

Example: A coin is tossed 10 times. Find the probability of coming of (*i*) exactly 6 heads (*ii*) At least six heads.

Solution: The probability of getting a head in one trial $p = 1/2$ the probability of not getting a head in one trial $q = 1/2$.

(*i*) The probability of getting exactly 6 heads. in 10 trials = $^{10}C_6 . \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^4 = \frac{^{10}C_6}{2^{10}}$

(*ii*) The probability of getting at least six heads

$$= {}^{10}C_6 . p^6 q^4 + {}^{10}C_7 p^7 q^3 + {}^{10}C_8 . p^8 q^2 + {}^{10}C_9 p^9 q^1 + {}^{10}C_{10} p^{10}$$

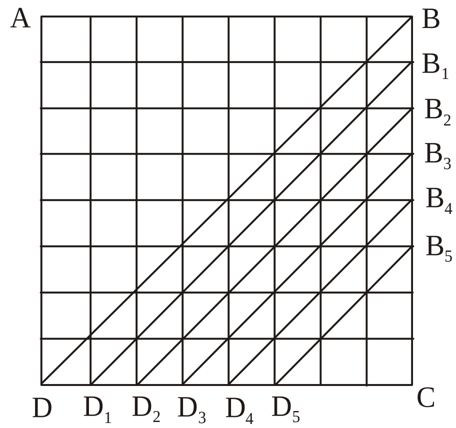
$$= ({}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}) \times \frac{1}{2^{10}}$$

$$= \frac{193}{512} .$$

SOLVED EXAMPLES

Ex.1: If three squares are chosen at random on a chess board find the probability that they lie on a diagonal line.

Sol.: The total number of ways of choosing 3 squares out of 64 is $^{64}C_3$.



The diagonal DB divides the chess board into 2 triangles BAD and BCD. In triangle BCD, the three squares can be chosen along the line DB, D_1B_1, D_2B_2, D_3B_3, D_4B_4 & D_5B_5. The number of ways of choosing 3 squares in this way is

$$^8C_3 + {^7C_3} + {^6C_3} + {^5C_3} + {^4C_3} + {^3C_3}$$

Similarly we can calculate the number of ways of selecting the 3 squares in the triangle BAD, ADC and ABC. Since we have to count the squares on diagonals BD and AC only once.

$\therefore$ The number of favourable ways is

$$4({^7C_3} + {^6C_3} + {^5C_3} + {^4C_3} + {^3C_3}) + 2\,{^8C_3} = 392.$$

Hence the probability of the required event is $\dfrac{392}{^{64}C_3}$.

Ex.2: An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained, by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4, ..., 12 is picked and the number on the card is noted. What is the probability that noted number is either 7 or 8 ?

Sol.: A : event of getting head

B : event of getting tail

E : event that noted number is 7 or 8.

$$P(A) = P(B) = \frac{1}{2}$$

$P(E|A) = P$(getting 7 on pair of dice) + P (getting 8 on pair of dice)

$$= \frac{6}{36} + \frac{5}{36} = \frac{11}{36}.$$

$P(E|B)$ = P (getting 7 or 8 when a card is picked from the pack of 11 cards)

$$= \frac{2}{11}$$

$\therefore \quad P(E) = P(A)\, P(E|A) + P(B)\, P(E|B)$

$$= \frac{1}{2} \cdot \frac{11}{36} + \frac{1}{2} \cdot \frac{2}{11} = \frac{193}{792}$$

Ex.3: An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6 is thrown n times and the list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5, 6 only three numbers appear in this list.

Sol.: Let $P = \{1, 2, ..., 6\}$. Total number of n-tupules with entries from P is 6^n. There are ${}^6C_3$ ways to choose a subset of 3 elements from P. Let $Q = \{a, b, c\}$ be one such subset. Total number of n-tupules with entries from Q is 3^n. This covers the cases where exactly two elements of Q appear in an n-tupule and cases where exactly one element of Q appears in a tupule. The former case can happen in ${}^3C_2.(2^n - 2)$ ways while the latter case can happen in ${}^3C_1.1.$ ways. Thus number of n-tupules whose entries consititue a subset of 3 elements from P is

$${}^6C_3[3^n - {}^3C_2(2^n - 2) - {}^3C_1.1]$$

Therefore, the required probability is

$$\frac{{}^6\mathrm{C}_3\,[3^n - 3\,(2^n - 2) - 3]}{6^n}.$$

Ex.4: Cards are drawn one-by-one at random from a well shuffled pack of 52 playing cards until 2 aces are obtained for the first time. If the process ends at nth draw show that probability of this event is

$$\frac{(n-1)\,(52-n)\,(51-n)}{50 \times 49 \times 17 \times 13} \quad \text{where } 2 \le n \le 50.$$

Sol.: We must have one ace in first $(n - 1)$ attempts and one ace in the nth attempt.

A : event of drawing one ace in first $n - 1$ draws

B : event of drawing an ace in nth draw

$$P(A) = \frac{{}^4\mathrm{C}_1\ {}^{48}\mathrm{C}_{n-2}}{{}^{52}\mathrm{C}_{n-1}}.$$

$$P(B/A) = \frac{{}^3\mathrm{C}_1}{52 - (n-1)} = \frac{3}{53 - n}.$$

$$\therefore \quad P(A \cap B) = P(A).P(B/A)$$

$$= \frac{{}^4C_1 \; {}^{48}C_{n-2}}{{}^{52}C_{n-1}} \cdot \frac{3}{53-n}$$

$$= 4 \; \frac{48\,!}{(n-2)\,!\,(50-n)\,!} \; \frac{(n-1)!\,(53-n)\,!}{52\,!} \cdot \frac{3}{53-n}$$

$$= \frac{(n-1)\,(52-n)\,(51-n)}{50\,.\,49\,.17\,.13}\,.$$

Ex.5: A set A has n elements. A subset P of A is selected at random. Returning the elements of P, a subset Q is selected from A. Find the probability that P and Q have no common elements.

Sol.: Let $A = \{a_1, a_2, ..., a_n\}$. For each a_i $(1 \le i \le n)$, we have following four choices:

(i) $a_i \in P$ and $a_i \in Q$

(ii) $a_i \in P$ and $a_i \notin Q$

(iii) $a_i \notin P$ and $a_i \in Q$

(iv) $a_i \notin P$ and $a_i \notin Q$.

Thus, the total number of ways of forming P and Q is 4^n. Out of these four choices, (ii), (iii) & (iv) are favourable to the occurrence of $P \cap Q = \phi$. Thus, the number of favourable ways is 3^n. Hence, the probability of given event is

$$P(P \cap Q = \phi) = \left(\frac{3}{4}\right)^n.$$

Ex.6: Two numbers x and y are chosen at random one by one, with replacement, from set of non-negative integers. Find the probability that $x^2 + y^2$ is divisible by 10.

Sol.: By division algorithm., we can write

$x = 10\,q_1 + r_1,$ where q_1 is quotient and r_1 is remainder such that it lies between 0 to 9.

Similarly, $y = 10\,q_2 + r_2$, where $0 \le r_2 \le 9$.

$$\therefore \quad x^2 + y^2 = (10\,q_1 + r_1)^2 + (10\,q_2 + r_2)^2$$

$$= 100\,(q_1^2 + q_2^2) + 20\,(q_1 r_1 + q_2 r_2) + r_1^2 + r_2^2.$$

$$= \text{multiple of } 10 + r_1^2 + r_2^2.$$

If $r_1^2 + r_2^2$ is multiple of 10, then $x^2 + y^2$ is divisible by 10.

Total number of ordered pairs (r_1, r_2) is $= 10 \times 10$.

Ordered pairs $(r_1, r_2) : r_1^2 + r_2^2$ is divisible by 10 are

(0, 0), (1, 3), (1, 7), (2, 4), (2, 6), (3, 1), (3, 9), (4, 2), (4, 8), (5, 5), (6, 2), (6, 8), (7, 1), (7, 9), (8, 4), (8, 6), (9, 3), (9, 7).

Hence, required probability is $\frac{18}{10 \times 10} = \frac{9}{50}$.

Ex.8: Out of 3*n* consecutive positive integers, 3 are chosen at random without replacement. What is the probability that the sum of these numbers is divisible by 3 ?

Sol.: Let the 3*n* positive integers begin with *a*. We write these 3*n* numbers in 3 rows as follows:

$a \quad a+3 \quad a+6 \ \ldots\ldots\ldots\ldots\ldots\ldots \ a+3n-3$

$a+1 \quad a+4 \quad a+7 \ \ldots\ldots\ldots\ldots\ldots\ldots \ a+3n-2$

$a+2 \quad a+5 \quad a+8 \ \ldots\ldots\ldots\ldots\ldots\ldots \ a+3n-1.$

The total number of ways of choosing 3 integers out of the 3*n* integers is

$$^{3n}C_3 = \frac{3n!}{3!(3n-3)!} = \frac{3n(3n-1)(3n-2)}{6}$$

$$= \frac{n(3n-1)(3n-2)}{2}$$

The sum of three numbers will be divisible by 3 if, and only if, either all the three numbers are from the same row or three numbers are chosen one from each row.

Therefore, the number of favourable ways is

$$3.{}^nC_3 + ({}^nC_1)({}^nC_1)\ {}^nC_1 = \frac{3n(n-1)(n-2)}{6} + n^3$$

$$= \frac{3n^3 - 3n^2 + 2n}{2}$$

Therefore, the probability of the required event is $\dfrac{3n^2 - 3n + 2}{(3n-1)(3n-2)}$.

Ex.9: A coin is tossed $(m + n)$ times, $m > n$. Show that the probability of getting (at least) *m* consecutive heads is $\dfrac{(n+2)}{2^{m+1}}$.

Sol.: The sequence of outcomes in which there are atleast *m* consecutive heads & their probabilities are as follows : (where *X* may denote a head or tail)

Different out comes	**Probabilities**
(HH ...... *m* times) XX ...... *n* times	$\left(\frac{1}{2}\right)^m .1$
T(HH ...... *m* times) XX ...... $(n-1)$ times	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^m .1$
XT (HH ...... *m* times) XX ...... $(n-2)$ times	$1.\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^m .1$
....	
....	
(XX ...... $(n-1)$ times) T (HH ...... *m* times)	$1.\frac{1}{2}\left(\frac{1}{2}\right)^m$

Therefore, required probability is $\left(\frac{1}{2}\right)^m + \left\{\left(\frac{1}{2}\right)^{m+1} + \left(\frac{1}{2}\right)^{m+1} + \text{...... } n \text{ times}\right\}$

$$= \left(\frac{1}{2}\right)^m + n\left(\frac{1}{2}\right)^{m+1}$$

$$= \frac{2+n}{2^{m+1}}$$

Ex.10: A man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point

Sol.: The probability of taking a step forward is $p = 0.4$ and the probability of taking a step backward is $q = 0.6$. In 11 steps, he will be one step away from the starting point provided.

(a) the number of steps taken forward is 6,

or (b) the number of steps taken forward is 5

$\therefore$ The required probability $= {}^{11}C_6\, p^6\, q^5 + {}^{11}C_5\, p^5\, q^6$.

$$= {}^{11}C_6\,(0.4)^6(0.6)^5 + {}^{11}C_5\,(0.4)^5(0.6)^6$$

$$= 462 \times \left(\frac{6}{25}\right)^5.$$

Ex.11: Out of m people sitting in a row, n people are chosen at random (where $m - n + 1 \geq n$).. Find the probability that no two of the chosen people are seated together.

Sol.: The total number of ways in which n people may be chosen out of m people is ${}^mC_n$.

Favourable cases are a kin to following situation:

If $m - n$ people are sitting in a row find number of choices of n places for the n people so that no two of the n people are next to each other :

The number of ways = ${}^{m-n+1}C_n$.

Hence, the probability of the required event is $\frac{{}^{m-n+1}C_n}{{}^mC_n}$.

Ex.12: I write a letter to my friend and do not receive a reply. It is known that one out of m letters does not reach its destination. What is the probability that my friend received the letter ? (It is certain that my friend would have replied if he did receive my letter).

Sol.: Let A denote the event that my friend received the letter and B denote the event that I get a reply.

$$P(A) = \frac{m-1}{m} \quad ; \quad P(A') = \frac{1}{m}$$

$$P(B/A) = \frac{m-1}{m} \quad ; \quad P(B/A') = 0$$

$$P(A \cap B) = \left(\frac{m-1}{m}\right)^2.$$

$$P(B) = P(A)\,P(B/A) + P(A')\,P(B/A')$$

$$= \left(\frac{m-1}{m}\right)^2.$$

$$P(B') = 1 - P(B) = 1 - \frac{(m-1)^2}{m^2} = \frac{2m-1}{m^2}$$

$$P(A/B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A) - P(A \cap B)}{P(B')}$$

$$= \frac{\dfrac{m-1}{m} - \dfrac{(m-1)^2}{m^2}}{\dfrac{2m-1}{m^2}}$$

$$= \frac{m-1}{2m-1}$$

Ex.13: A card from a pack of 52 cards is lost. From the remaining cards of the pack two cards are drawn and are found to be spades. Find the probability of the missing card to be spade.

Sol.: Let E_1 : missing card is spade

E_2 : missing card is club

E_3 : missing card is diamond

E_4 : missing card is heart

and E : event of drawing 2 spades from the remaining cards.

Since E_i's are equally likely, we have

$$P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$$

$$P(E|E_1) = \frac{^{12}C_2}{^{51}C_2}.$$

$$P(E|E_2) = \frac{^{13}C_2}{^{51}C_2}$$

Similarly, $P(E|E_3) = P(E|E_4) = \dfrac{^{13}C_2}{^{51}C_2}$.

We have to find P($E_1|E$) i.e., the probability of the missing card being spade when two cards drawn are spades. By Baye's Theorem :

$$P(E_1|E) = \frac{P(E_1)\,P(E|E_1)}{\sum_{i=1}^{4} P(E_i)\,P(E|E_i)}$$ ($\because$ E_i's are exhaustive and mutually exclusive events)

$$= \frac{\frac{1}{4}\cdot\frac{{}^{12}C_2}{{}^{51}C_2}}{\frac{1}{4}\frac{{}^{12}C_2}{{}^{51}C_2} + \frac{3}{4}\frac{{}^{13}C_2}{{}^{51}C_2}}$$

$$= \frac{{}^{12}C_2}{{}^{12}C_2 + 3\,{}^{13}C_2} = \frac{11}{50}$$

Ex.15: A bag contains p white and q red balls. r ($r < p$, q) balls are drawn from the bag, one by one without replacement. Find the probability of the rth ball drawn being white.

Sol.: Let E_i denote the event that out of the first (r – 1) balls drawn, i balls are white. Let A denote the event that rth ball drawn is white.

$$P(E_i) = \frac{{}^{p}C_i\ {}^{q}C_{r-1-i}}{{}^{p+q}C_{r-1}} \quad : \quad 0 \le i \le r-1$$

and $$P(A|E_i) = \frac{p-i}{(p+q)-(r-1)}$$

Now, $$P(A) = \sum_{i=0}^{r-1} \frac{{}^{p}C_i\ {}^{q}C_{r-1-i}}{{}^{p+q}C_{r-1}} \frac{p-i}{p+q-r+1}$$

$$= \frac{1}{p+q-r+1}\left(\sum_{i=0}^{r-1} \frac{p!}{i!(p-i)!} \frac{{}^{q}C_{r-1-i}}{{}^{p+q}C_{r-1}}.(p-i)\right)$$

$$= \frac{1}{p+q-r+1}\left(\sum_{i=0}^{r-1} \frac{p\ {}^{p-1}C_i\ {}^{q}C_{r-1-i}}{{}^{p+q}C_{r-1}}\right)$$

$$= \frac{p}{p+q-r+1}\left(\sum_{i=0}^{r-1} \frac{{}^{p-1}C_i\ {}^{q}C_{r-1-i}}{{}^{p+q}C_{r-1}}\right)$$

$\sum_{i=0}^{r-1} {}^{p-1}C_i\ {}^{q}C_{r-1-i}$ = coefficient of x^{r-1} in expansion of $(1 + x)^{p-1}\ (1 + x)^q$

$= {}^{p+q-1}C_{r-1}$

$$\therefore \quad P(A) = \frac{p}{p+q-r+1}\ \frac{{}^{p+q-1}C_{r-1}}{{}^{p+q}C_{r-1}}$$

$$= \frac{p}{p+q-r+1}\ \frac{(p+q-1)!}{(r-1)!\,(p+q-r)!} \times \frac{(r-1)!(p+q-r+1)!}{(p+q)!} = \frac{p}{p+q}$$

BASIC LEVEL ASSIGNMENT

1. *A* has a 3 shares in a lottery containing 3 prizes and 6 blanks. *B* has one share in a lottery containing one prize and 2 blanks. Compare their chances of success.

2. Five persons entered a lift cabin on the ground-floor of an 8 floor house. Suppose that each of them independently and with equal probability, can leave the cabin at any floor beginning with the first. Find out the probability of all five persons leaving at different floors.

3. A coin is tossed n times; what is the chance that the head will present itself an odd number of times ?

4. A, B, C in order cut a pack of cards, replacing them after each cut, on the condition that the first who cuts a spade shall win a prize ; find their respective chances.

5. In a certain city two newspapers *A* and *B* are published. It is known that 25% of the city population reads *A* and 20% reads *B* while 8% reads both *A* and *B*. It is also known that 30% of those who read *A* but not *B* look into advertisements and 40% of those who read *B* but not *A* look advertisements while 50% of those who read both *A* and *B* look into advertisements. What is the percentage of the population who read an advertisement.

6. There are two bags, one of which contains three black and four white balls while the other contains four black and three white balls. A die is cast, if the face 1 or 3 turns up a ball is taken from the first bag and if any other face turns up, a ball is chosen from the second bag. Find the probability of choosing a black ball.

7. In a given race the odds in favour of four horses *A*, *B*, *C*, *D* are 1 : 4, 1 : 6, 1 : 5, 1 : 3 respectively. Assuming that a dead heat is impossible, find the chance that one of them wins the race.

8. 6 Boys and 6 girls sit in a row at random. Find the probability that the boys and girls sit alternately.

9. What is the probability that *S*'s come consecutively if all the letters of the word "MISSISSIPPI" are rearranged randomly?

10. A card is is drawn at random from a pack of cards. What is the probability that it is a Jack, if it is known that a red card is drawn.

11. *A*, *B* and *C*, in order, toss a coin. The one who gets a head first wins. Find their respective probabilities of winning.

12. A pack of cards has one card missing. Two cards are drawn and are found to be spades. What is the probability that the missing card is not a spade?

13. If two dice are thrown 5 times, what is the probability that sum of the numbers coming up will be 10 in exactly 2 out of these 5 trails.

14. If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} with replacement, determine the probability that the roots of the equation $x^2 + p\,x + q = 0$ are real.

15. A and B are two independent events. The probability that both occur simultaneously is $\frac{1}{6}$ and the probability that neither occurs is $\frac{1}{3}$. Find the probabilities of occurence of the events A and B separately.

16. The probabilities of three events A, B and C are P(A) = 0.6 , P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$ and $P(A \cup B \cup C) \geq 0.85$. Find the possible values of $P(B \cap C)$.

17. An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white.

18. Three players A, B and C, toss a coin cyclically in that order (that is A, B, C, A, B, C, A, B......) till a head shows. Let p be the probability that the coin shows a head. Let α, β and γ be, respectively, the probabilities that A, B and C gets the first head. Prove that $\beta = (1 - p)\,\alpha$. Determine α, β and γ (in terms of p).

19. Three factories A, B and C produce the same product. The factory A produces twice as many as B produces, while the factories B and C produce the same quantity. It is known that 2% of the products of A as well as of C are defective while 4% of the products of B are defective. All the products of the three factories are stocked together. If a product is selected at random from the stock, what is the probability that the chosen product is defective ?

20. Each of three bags A, B, C contains white balls and black balls. A has a_1 white & b_1 black, B has a_2 white & b_2 black and C has a_3 white & b_3 black balls. A ball is drawn from a bag and found to be white. What are the probabilities that the ball is from (i) bag A ; (ii) bag B ; (ii) bag C.

ADVANCE LEVEL ASSIGNMENT

1. Chards are drawn one by one at random from well shuffled full pack of 52 playing cards until 2 aces are obtained for the frist time. If N is the numebr of aces required to be drawn, then show that $P(N=n)=\frac{(n-1)(52-n)(5-n)}{50\times 49\times 17\times 13}$ where $2\le n\le 50$.

2. Suppose the probability for A to win game against B is 0.4. If A has an option of playing either "best of 3 games" or a best of 5 gams match against B, which option should A choose so that his probability of winnign is higher? (No game ends in a draw).

3. A box contains 2 fifty paise coins, 5 twenty five paise coins and certain fixed number N(≥ 2) of ten box at random. Find the probability that the total value of these 5 coins is less than one rupee and fifty paise.

4. A man takes a step forward with probability 0.4 and backwards with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point.

5. A lot containts 20 articles. The probability that the lot contins exactly two defective articles is 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one by one, withot replacement and are tested till all the defective articles are found. What is the probability that the testing procedure ends at the twelfth testing.

6. An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white, it is not replaced into the urn, otherwise it is replaced along with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black.

7. From a pack of 52 cards an even number of cards is drawn. Find the probability that half of these cards will be red and the other half black.

8. Eight players P_1, P_2, ..., P_8 play a knock out tournament. It is known that whenever the players P_i and P_j play, the player p_i will win if $i < j$. Assuming that the players are paired at random in each round, what is the probability that the player P_4 reaches the final. 21. $\frac{4}{35}$

9. There is 30% chance that it rains on any particular day. What is the probability that there is at least one rainy day within a period of 7 days ? Given that there is at least one rainy day in a week, what is the probability that there are at least two rainy days in a week ?

10. There are four six faced dies such that each of two dies bears the numbers 0, 1, 2, 3, 4 and 5 and the other two dies are ordinary dies bearing numbers 1, 2, 3, 4, 5 and 6. If all the four dies are thrown, find the probability that the total of numbers coming up on all the dies is 10.

11. Let p be the probability that a man aged x years will die with in a year. Let $A_1, A_2, ..., A_n$ be n men each aged x years. Find the probability that out of these n men A_1 will die with in a year and is the first to die.

12. Let $X = \{x \mid 1 \leq x \leq 50, x \in N\}$. A member of the set X is selected at random. Find the probability that the selected number is a solution of the inequation $\frac{x^2 - 30x + 200}{x - 15} < 0$.

13. A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair?

14. Suppose a sample space consists of the integers 1, 2, ... , 2 n. The probability of choosing an integer α is proportional to log α. Show that the conditional probability of choosing the integer n, given that an even integer is chosen is $\frac{\log n}{n \log 2 + \log (n!)}$

OBJECTIVE ASSIGNMENT

Choose the correct option in the following :

1. The probability of India winning a test match against West Indies is $\frac{1}{2}$. Assuming independence of winning from match to match, the probability that in a 5 match series India's win in series occurs at third test is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

2. 3 mangoes and 3 apples are in a box. If 2 fruits are chosen at random, the probability that one is a mango and the other is an apple is

(a) $\frac{3}{5}$ (b) $\frac{5}{6}$ (c) $\frac{1}{36}$ (d) none of these

3. The probability that a marksman will hit a target is $\frac{1}{5}$. Then the probability of at least one hit in 10 shots is

(a) $\frac{1}{5^{10}}$ (b) $1-\left(\frac{4}{5}\right)^{10}$ (c) $1-\frac{1}{5^{10}}$ (d) $\left(\frac{4}{5}\right)^{10}$

4. If two events A and B are such that $P(A^C) = 0.3$, $P(B) = 0.4$ and $P(AB^C) = 0.5$. Then, $P[B|(A \cup B^C)]$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) none of these

5. A pack of cards contains 4 aces, 4 king, 4 queens and 4 jacks. Two cards are drawn at random. The probability that at least one of them is an ace is

(a) $\frac{3}{5}$ (b) $\frac{3}{16}$ (c) $\frac{9}{20}$ (d) $\frac{1}{9}$

6. Fifteen coupons are numbered 1, 2, ..., 15 respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is

(a) $\left(\frac{9}{10}\right)^{6}$ (b) $\left(\frac{8}{15}\right)^{7}$ (c) $\left(\frac{3}{5}\right)^{7}$ (d) none of these

7. The probability of a problem being solved by two students is $\frac{1}{2}$ and $\frac{1}{3}$ respectively. The probability that the problem is solved is

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{1}{3}$ (d) 1

8. The probability that an event A happens in one trial of an experiment is 0.4. Three independent trials of the experiment are formed. The probability that the event A happens at least once is

(a) 0.936 (b) 0.784 (c) 0.904 (d) none of these

9. An urn contains 6 white and 4 black balls. A fair dice is rolled and as many balls as the number on the dice are chosen from the urn. The probability that the balls selected are white is

(a) $\frac{1}{5}$ (b) $\frac{1}{6}$ (c) $\frac{1}{7}$ (d) $\frac{1}{8}$

10. A purse contains 4 copper coins and 3 silver coins. A second purse contains 6 copper coins and 2 silver coins. A coin is taken out from one of the purses. The probability that it is a copper coin is

(a) $\frac{4}{7}$ (b) $\frac{37}{56}$ (c) $\frac{3}{7}$ (d) $\frac{3}{4}$

11. The probability that a man can hit a target is $\frac{3}{4}$. He tries 5 times. The probability that he will hit the target at least three times is

(a) $\frac{291}{364}$ (b) $\frac{371}{464}$ (c) $\frac{471}{502}$ (d) $\frac{459}{512}$

12. A fair coin is tossed 100 times. The probability of getting tail an odd number of times is

(a) $\frac{1}{2}$ (2) $\frac{1}{8}$ (c) $\frac{3}{8}$ (d) none of these

13. A and B are two events, odd against A are 2 : 1 odds in favour of $A \cup B$ are 3 : 1. If $x \le P(B) \le y$ then the ordered pair (x, y) is

(a) $\left(\frac{5}{12}, \frac{3}{4}\right)$ (b) $\left(\frac{2}{3}, \frac{3}{4}\right)$ (c) $\left(\frac{1}{3}, \frac{3}{4}\right)$ (d) none of these

14. A_1, A_2 ....., A_n are n independent events with $P(A_i) = \frac{1}{1+i}$ $(1 \le i \le n)$. The probability that none of A_1, A_2, ..., A_n occurs is

(a) $\frac{n!}{(n+1)!}$ (b) $\frac{n}{n+1}$ (c) $\frac{1}{(n+1)!}$ (d) $\frac{1}{n+1}$

15. Four numbers are multiplied together. Then, the probability that the product will be divisible by 5 or 10 is

(a) $\frac{369}{625}$ (b) $\frac{399}{625}$ (c) $\frac{123}{625}$ (d) $\frac{133}{625}$

16. The probability of the simultaneous occurence of two events A and B is p. If the probability that exactly one of A, B occurs is q then

(a) $P(\overline{A}) + P(\overline{B}) = 2 + 2q - p$ (b) $P(\overline{A}) + P(\overline{B}) = 2 - 2p - q$

(c) $P(A \cap B | A \cup B) = \frac{p}{p+q}$ (d) $P(\overline{A} \cap \overline{B}) = 1 - p - q$

17. Two integers x and y are chosen with replacement out of the set {0, 1, 2, 3,....., 10}. Then the probability that $|x - y| > 5$ is

(a) $\frac{81}{121}$ (b) $\frac{30}{121}$ (c) $\frac{25}{121}$ (d) $\frac{20}{121}$

18. Let A be a set containing n elements. A subset P of the set A is chosen at random. The set A is reconstructed by replacing the elements of P, and another subset Q of A is chosen at random. The probability that $P \cap Q$ contains exactly m ($m < n$) elements is

(a) $\frac{3^{n-m}}{4^n}$ (b) $\frac{{}^nC_m\, 3^m}{4^n}$ (c) $\frac{{}^nC_m\, 3^{n-m}}{4^n}$ (d) none of these

19. A car is parked by an owner amongst 25 cars in a row not at either end. On his return he finds that exactly 15 places are still occupied. The probability that both the neighbouring places are empty is

(a) $\frac{97}{276}$ (b) $\frac{15}{154}$ (c) $\frac{15}{92}$ (d) none of these

20. A man alternately tosses a coin and throws a die beginning with the coin. The probability that he gets a head on the coin before he gets 5 or 6 in the dice is

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) none of these

21. The probability that a function with domain and codomain $A = \{1, 2, 3, ..., n\}$ selected at random is neither one-one nor onto is $\frac{29}{32}$. The value of n is

(a) 3 (b) 4 (c) 5 (d) none of these

22. A bag A contains two white and two red balls and another bag B contains 4 white and 5 red balls. A ball is drawn and is found to be red. The probability that it was drawn from bag B is

(a) $\frac{25}{52}$ (b) $\frac{1}{2}$ (c) $\frac{10}{19}$ (d) $\frac{13}{18}$

23. From a set of four positive and four negative numbers, four numbers are chosen at random without replacement and are multiplied. What is the probability that the product is positive?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{19}{35}$ (d) $\frac{23}{35}$

24. If A and B are two mutually exclusive events, then

(a) $P(A) \leq P(\overline{B})$ (b) $P(A) > P(\overline{B})$ (c) $P(A) < P(B)$ (d) none of these

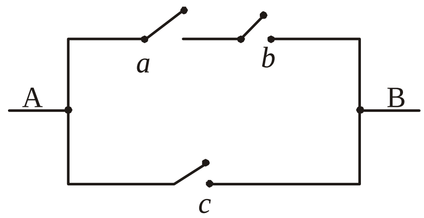
25. In a box containing 100 bulbs, 10 are defective. What is the probability that out of a sample of 5 bulbs, none is defective ?

(a) 10^{-5} (b) $\left(\frac{1}{2}\right)^5$ (c) $\left(\frac{9}{10}\right)^5$ (d) $\left(\frac{9}{10}\right)$

26. For n independent events A_i's, $P(A_i) = 1/(1+i),\ i = 1, 2....n$. The probability that atleast one of the events occurs is

(a) $1/n$ (b) $1/(n+1)$ (c) $n/(n+1)$ (d) none of these

27. Consider the circuit



If the probability that each switch is closed in p, then find the probability of current flowing through AB

(a) $p^2 + p$ (b) $p^2 + p - 1$ (c) $p^3 + p$ (d) $p^2 + p + 1$

28. A dice is thrown $(2n+1)$ times. The probability that faces with even numbers appear odd number of times is

(a) $\frac{2n+1}{2n+3}$ (b) $\frac{n+1}{2n+1}$ (c) $\frac{n}{2n+1}$ (d) none of these

29. The probability that a teacher will give an inanounced test during any class meeting is 1/5. If a student is absent twice, the probability that he will miss atleast line test, is

(a) 7/25 (b) 9/25 (c) 16/25 (d) 24/25

30. A body is thrown stones at a targer. The probability of hitting the target at any trial is 1/2. The probability of hitting the garget 5th time at the 10th throw is

(a) $\frac{5}{2^{10}}$ (b) $\frac{63}{2^9}$ (c) $\frac{{}^{10}C_5}{2^{10}}$ (d) $\frac{{}^{10}C_4}{2^{10}}$

MORE THAN ONE CORRECT ANSWERS

31. A and B are two events, the probability that exactly one of them occurs is given by

(a) $P(A) + P(B) - 2(A \cap B)$ (b) $P(A \cup B) - P(A \cap B)$

(c) $P(A') + P(B') - 2P(A' \cap B')$ (d) $P(A \cap B') + P(A' \cap B)$

32. A bag contains four tickets marked with 112, 121, 211, 222 one ticket is drawn at random from the bag. Let $E_i (i = 1, 2, 3)$ denote the event that ith digit on the ticket is 2. then

(a) E_1 and E_2 are independent (b) E_2 and E_3 are independent

(c) E_3 and E_1 are independent (d) E_1, E_2, E_3 are independent

33. If A and B are independent events such ath $0 < P(A) < 1$, $0 < P(B) < 1$, then

(a) A, B are mutually exclusive (b) A and $\bar{B}$ are independent

(c) $\bar{A}, \bar{B}$ are independent (d) $P(A/B) + P(A/B) = 1$

34. If A and B are two events such that $P(A) = 1/2$ and $P(B) = 2/3$, then

(a) $P(A \cup B) \geq 2/3$ (b) $P(A \cap B') \leq 1/3$

(c) $1/6 \leq P(A \cap B) \leq 1/2$ (d) $1/6 \leq P(A' \cap B) \leq 1/2$

35. For two events A and B, if P(A) = P(A/B) = 1/4 and P(B/A) = 1/2, then

(a) A and B are independent (b) A and B are mutually exclusivent

(c) P(A′/B) = 3/4 (d) P(B′/A′) = 1/2

36. Let $0 < P(A) < 1, 0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, then

(a) $P(B/A) = P(B) - P(A)$ (b) $P(\bar{A} - \bar{B}) = P(\bar{A}) - P(\bar{B})$

(c) $P(\overline{A \cup B}) = P(\bar{A})P(\bar{B})$ (d) $P(A/B) = P(A)$

37. Let X be a set containing n elements. if two subsets A and B of X are picked at random, the probability that A and B have the same number of elements is

(a) $\frac{{}^{2n}C_n}{2^{2n}}$ (b) $\frac{1}{{}^{2n}C_n}$ (c) $\frac{1.3.5...(2n-1)}{2^n.n!}$ (d) $\frac{3^n}{4^n}$

38. If $\bar{E}$ and $\bar{F}$ are the complementary events of the events E and F respectively, then

(a) $P(E/F) + P(\bar{E}/F) = 1$ (b) $P(E/F) + P(E/\bar{F}) = 1$

(c) $P(\bar{E}/F) + P(E/\bar{F}) = 1$ (d) $P(E/\bar{F}) + P(\bar{E}/\bar{F}) = 1$

39. For any two events A and Bin a sample space

(a) $P(A/B) \geq \frac{P(A) + P(B) - 1}{P(B)}, P(B) \neq 0$ is always true

(b) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$ does not hold

(c) $P(A \cup B) = 1 - P(\bar{A}).P(\bar{B})$ if A and B are independent

(d) $P(A \cup B) = 1 - P(\bar{A})P(\bar{B})$ if A and B are disjoint

40. Three numbers are chosen at random without replacement from {1, 2, 3,....10}. The probability that minimum of the chosen nummber is 3 or their maximum is 7, cannot exceed

(a) $\frac{11}{30}$ (b) $\frac{11}{40}$ (c) $\frac{11}{50}$ (d) $\frac{11}{60}$

MISCELLANEOUS ASSIGNMENT

Comprehension-1

If points of the closed interval $[\alpha, \beta]$ are sample point and points of the closed interval $[a, b] \leq [\alpha, \beta]$ be a favourable points for the event E then the probability for the event E is defined by

$$P(E) = \frac{\text{length of } [a, b]}{\text{length of } [\alpha, \beta]}$$

1. The set of values of $P \in R$ for which $x^2 + px + \frac{1}{4}(p + 2) \geq 0$ for all x Î R is

(a) $(-2, 1)$ (b) $(2, \infty)$ (c) $[-1, 2]$ (d) $[1, 2]$

2. The set of values of $p \in R$ for which the equation $x^2 + px + \frac{1}{4}(p + 2) = 0$ will have a real roots is

(a) [2, ¥] (b) (2, ¥) (c) (–¥, 2) (d) $R - (-1, 2)$

3. If p is chosen at random from the interval [0, 6] then the probability that the roots of the equation $x^2 + px + \frac{1}{4}(p + 2) = 0$ will be real is

(a) $\frac{3}{5}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

Comprehension-2

Suppose E_1, E_2, E_3 be three exclusive event such that $P(E_i) = P_i$; for $i = 1, 2, 3$

4. If P_1, P_2, P_3 are the roots of $27x^3 - 27x^2 + ax - 1 = 0$ then value of a is

(a) 9 (b) 6 (c) 3 (d) none of these

5. P (none of E_1, E_2, E_3 occurs) equals

(a) 0 (b) $P_1 + P_2 + P_3$

(c) $(1 - P_1)(1 - P_2)(1 - P_3)$ (d) none of these

6. $P(E_1 \cap E_2') + P(E_2 \cap E_3') + P(E_3 \cap E_1')$ equals

(a) $P_1(1 - P_2) + P_2(1 - P_3) + P_3(1 - P_1)$ (b) $P_1P_2 + P_2P_3 + P_3P_1$

(c) $P_1 + P_2 + P_3$ (d) none of these

Match the following:

7. A is a set containing n elements. A subset P of A is chosen at random. The set A is reconstructed by replacing the elements of the subset P. A subset Q of A is again chosen at random. The probability that

A. $P \cap Q = \phi$	(p) $n(3^{n-1})/4^n$
B. $P \cap Q$ is a singleton	(q) $(3/4^n)$
C. $P \cap Q$ contains 2 elements	(r) ${}^{2n}C_n/4^n$
D. $\lvert P\rvert = \lvert Q\rvert$	(s) $9n(n-1)/2(4^n)$

8. Suppose n (> 6) people are asked a question successively in a random order and exactly 3 out of n people know the answer. Let pr denote the probability that rth person asked is the first to know the answer, then

A. probability first four do not know the answer	(p) $\frac{3(n-3)}{n(n-1)}$
B. p_2	(q) $\frac{(n-4)(n-5)(n-6)}{n(n-1)(n-2)}$
C. p_{n-2}	(r) $1/{}^nC_c$
D. p_r	(s) $\frac{3(n-r)(n-r-1)}{n(n-1)(n-2)}$

INTEGER TYPE QUESTIONS

9. A bag contains 2 red and 2 white balls two balls are drawn one by one without replacement of which at least one is red. If the probability that in the next two draws exactly one red ball and one white ball is drawn is ‘P’. Then find [4P], where [.] = *g.i.f.*

10. The probability that any rectangle chosen on a chess board is a square is x. Then find [1/x]. Where [.] = *g.i.f.*

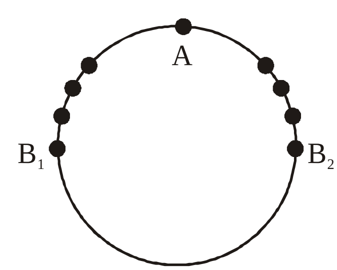
11. A is targeting to B, B and C are targeting to A. Probability of hitting to target by A, B and C are 2/3, 1/2 and 1/3 respectively. If A is hit then the probability that B hits the target but C doesn’t is ‘p’. Find 1/p.

12. A box contains 4 balls which are either red or black 2 balls are draw and found to be red. If these are replaced. If the probability that the next drawn will result in a red ball is p. Find 8p.

13. If the numbers $x \in R$ and $y \in R$ are selected such that $x \in [0, 4]$ and $y \in [0, 4]$. If the probability that selected number satisfies the inequality $y^2 \le x$ be p. Find 1/p.

14. Numbers are selected at random, one at a time from the two digit numbers 00, 01, 02, ...,99 with replacement. An event E occurs if and only if the product of two digits of a selected no is 18. If four numbers are selected the probability that event E occurs exactly 3 times be p. Find the value of $\frac{(25)^4 p}{24}$

15. Fifteen persons, among whom are A and B, sit down at random at a round table. It p is the probability that there are exactly 4 persons between A and B find 7 p.



16. A locker can be opened buy dialing a fixed three-digit code (between 000 and 999). A stranger, who does not know the code, tries to open the locker by dialing three digits at randm. If p is the probability that the stranger succeeds at the kth trial, find 1000 p. (Assume that the stranger does not repeat unsuccefful combinations.)

17. The probability of a bomb hitting a bridge is 1/2 and two direct hits are needed to destory it. Find the least numebr of bombs required so that the probability of the bridge being destroyed is greater than 0.9.

18. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. If p denoted the probability that 7 before 5, find 15*p*.

PREVIOUS YEAR QUESTIONS

IIT-JEE/JEE-ADVANCE QUESTIONS

1. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is

(a) $\frac{11}{3^5}$ (b) $\frac{10}{3^5}$ (c) $\frac{17}{3^5}$ (d) $\frac{13}{3^5}$

2. A student appears for test I, II and III. The student is successful if he passes either in tests I and II or test I and III. The probabilities of the student passing in test I, II and III are p, q and $\frac{1}{2}$, respectively. If the probability that the student is successful is $\frac{1}{2}$, then

(a) $p = q = 1$ (b) $p = q = 1/2$ (c) $p = 1, q = 0$ (d) $p = 1, q = 1/2$

3. The probability that at least one of the events A and B occur is 0·6, if A and B occur simultaneously with probability 0·2, then $P(\bar{A}) + P(\bar{B})$ is

(a) 0·4 (b) 0·8 (c) 1·2 (d) 1·4

4. One hundred identical coins, each with probability, p, of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on fifty coins is equal to that of heads showing on 51 coins, then the value of p is

(a) $\frac{1}{2}$ (b) $\frac{49}{101}$ (c) $\frac{50}{101}$ (d) $\frac{51}{101}$

5. For two given events A & B, $P(A \cap B)$is

(a) not less than $P(A) + P(B) - 1$ (b) not greater than $P(A) + P(B)$

(c) equal to $P(A) + P(B) - P(A \cup B)$ (d) equal to $P(A) + P(B) + P(A \cup B)$

6. If E and F are independent events such that $0 < P(E) < 1$ & $0 < P(F) < 1$ then

(a) E & F mutually exclusive

(b) E & $\bar{F}$ (complement of the event F) are independent

(c) $\bar{E}$ & $\bar{F}$ are independent

(d) $P(E/F) + P(\bar{E}/F) = 1$

7. India plays two matches each with West-Indies & Australia. In any match the probabilities of India getting points 0, 1 and 2 are 0.45, 0.05 & 0.50 respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points is

(a) 0.8750 (b) 0.0875 (c) 0.0625 (d) 0.0250

8. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained, the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5 is

(a) $\frac{16}{81}$ (b) $\frac{1}{81}$ (c) $\frac{80}{81}$ (d) $\frac{65}{81}$

9. Let E and F be two independent events. The probability that both E and F happen is 1/12 and the probability that neither E nor F happens is 1/2. Then

(a) $p(E) = \frac{1}{3}, p(F) = \frac{1}{4}$ (b) $p(E) = \frac{1}{2}, p(F) = \frac{1}{6}$

(c) $p(E) = \frac{1}{6}, p(F) = \frac{1}{2}$ (d) $p(E) = \frac{1}{4}, p(F) = \frac{1}{3}$

10. For the three events A, B & C, P (exactly one of the events A or B occurs) = P (exactly one of the events B or C occurs) = P (exactly one of the events C or A occurs) = p & P (all the three events occur simultaneously) = p^2, where $0 < p < \frac{1}{2}$. Then the probability of at least one of the three events A, B & C occurring is

(a) $\frac{3p + 2p^2}{2}$ (b) $\frac{p + 3p^2}{4}$ (c) $\frac{p + 3p^2}{2}$ (d) $\frac{3p + 2p^2}{4}$

11. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals

(a) $\frac{1}{2}$ (b) $\frac{7}{15}$ (c) $\frac{2}{15}$ (d) $\frac{1}{3}$

12. If $\overline{E}$ and $\overline{F}$ are complementary events of events E and F respectively and if $0 < P(F) < 1$, then

(a) $P(E/F) + P(\overline{E}/F) = 1$ (b) $P(E/F) + P(E/\overline{F}) = 1$

(c) $P(\overline{E}/F) + P(E/\overline{F}) = 1$ (d) $P(E/\overline{F}) + P(\overline{E}/\overline{F}) = 1$

13. There are four machines and it is known that exactly two of them are faulty machines are identified. Then the probability that only two tests are needed is

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

14. If E and F are events with $P(E) \leq P(F)$ and $P(E \cap F) > 0$, then

(a) occurrence of $E \Rightarrow$ occurrence of F (b) occurrence of $F \Rightarrow$ occurrence of E

(c) non occurrence of $E \Rightarrow$ non occurrence of F (d) none of the above implications holds

15. A fair coin is tossed repeatedly. If the tail appears on first four tosses, then the probability of the head appearing on the fifth toss equals

(a) $\frac{1}{2}$ (b) $\frac{1}{32}$ (c) $\frac{31}{32}$ (d) $\frac{1}{5}$

16. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals

(a) $\frac{1}{4}$ (b) $\frac{1}{7}$ (c) $\frac{1}{8}$ (d) $\frac{1}{49}$

17. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in at least two and a 40% chance of passing in exactly two. Which of the following relations are true?

(a) $p + m + c = \frac{19}{20}$ (b) $p + m + c = \frac{27}{20}$ (c) $pmc = \frac{1}{10}$ (d) $pmc = \frac{1}{4}$

18. If $P(B) = \frac{3}{4}$, $P(A \cap B \cap \overline{C}) = \frac{1}{3}$ and $P(\overline{A} \cap B \cap \overline{C}) = \frac{1}{3}$, then $P(B \cap C)$ is

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{15}$ (d) $\frac{1}{9}$

19. Two numbers are selected randomly from the set $S = \{1, 2, 3, 4, 5, 6\}$ without replacement one by one. The probability that minimum of the two numbers is less than 4 is

(a) $\frac{1}{15}$ (b) $\frac{14}{15}$ (c) $\frac{1}{5}$ (d) $\frac{4}{5}$

20. If three distinct numbers are choosen at random from the first 100 natural numbers, then the probability that all three of them are divisible by 2 and 3 both is

(a) $\frac{4}{55}$ (b) $\frac{4}{35}$ (c) $\frac{4}{33}$ (d) $\frac{4}{1155}$

21. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one all is drawn at random, then the probability that 2 white and 1 black ball will be drawn is

(a) $\frac{13}{32}$ (b) $\frac{1}{4}$ (c) $\frac{1}{32}$ (d) $\frac{3}{16}$

22. A six faced fair dice is thrown until 1 comes, then the probability that 1 comes in even no. of trials is

(a) 5/11 (b) 5/6 (c) 6/11 (d) 1/6

23. Mr. *A* gave his telephone number to Mr. *B*. Mr. *B* remembers that the first two digits were 40 and the remaining four digits were two 3's, one 6 and one 8. He is not certain about the order of the digits. Mr. *B* dials 403638. The probability that he will get *A*'s house is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{12}$

24. There are four letters and four envelopes bearing addresses at random. The probability that the letters are placed in the correct envelope is

(a) $\frac{23}{24}$ (b) $\frac{9}{24}$ (c) $\frac{1}{16}$ (d) $\frac{1}{24}$

25. The chance that the vowels are separated in an arrangement of the letters of the word HORROR is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{3}{8}$

Comprehension:

There are n urns each containing $(n + 1)$ balls such that the i^{th} urn contains i white balls and $(n + 1 - i)$ red balls. Let U_i be the event of selecting i^{th} urn, $i = 1, 2, 3, ..., n$ and W denotes the event getting a white ball. Using this information, solve the following questions

26. $P(U_i) \alpha\, i$ and find $\lim_{n \to \infty} P(W)$

(a) 1 (b) 1/3 (c) 1/2 (d) 2/3

27. If $P(u_i) = c$, where c is a constant, then $P\left(\frac{U_n}{W}\right)$ is equal to

(a) $\frac{2}{n+1}$ (b) $\frac{1}{n+1}$ (c) $\frac{n}{n+1}$ (d) $\frac{1}{2}$

28. If n is even and E denotes the event of choosing even numbered urn $\left(P(u_i) = \frac{1}{n}\right)$, then the value of $P\left(\frac{W}{E}\right)$ is

(a) $\frac{n+2}{2n+1}$ (b) $\frac{n+2}{2(n+1)}$ (c) $\frac{n}{n+1}$ (d) $\frac{1}{n+1}$

29. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

(a) 1/2 (b) 1/3 (c) 2/5 (d) 1/5

30. Let $H_1, H_2, ..., H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, $i = 1, 2, ..., n$. Let E be any other event with $0 < P(E) < 1$.

Statement-1: $P(H_i | E) > P(E | H_i) \cdot P(H_i)$ for $i = 1, 2, ..., n$.

because

Statement-2: $\sum_{i=1}^{n} P(H_i) = 1$.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

31. Let E^c denote the complement of an event E. Let E, F, G be pair wise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$. Then $P(E^c \cap F^c | G)$ equals

(a) $P(E^c) + P(F^c)$ (b) $P(E^c) - P(F^c)$ (c) $P(E^c) - P(F)$ (d) $P(E) - P(F^c)$

Paragraph for Question Nos.

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

32. The probability that $X = 3$ equals

(A) $\frac{25}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{125}{216}$

33. The probability that $X \geq 3$ equals

(A) $\frac{125}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{25}{216}$

34. The conditional probability that $X \geq 6$ given $X > 3$ equals

(A) $\frac{125}{216}$ (B) $\frac{25}{216}$ (C) $\frac{5}{36}$ (D) $\frac{25}{36}$

35. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is

(a) $\frac{3}{5}$ (b) $\frac{6}{7}$ (c) $\frac{20}{23}$ (d) $\frac{9}{20}$

Paragraph for Question

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

36. The probability of the drawn ball from U_2 being white is

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$ (C) $\frac{19}{30}$ (D) $\frac{11}{30}$

37. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$ (C) $\frac{15}{23}$ (D) $\frac{12}{23}$

38. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true ?

(A) $P\left[X_1^c \mid X\right] = \frac{3}{16}$

(B) P[Exactly two engines of the ship are functioning | X] = $\frac{7}{8}$

(C) $P\left[X \mid X_2\right] = \frac{5}{16}$

(D) $P\left[X \mid X_1\right] = \frac{7}{16}$

39. Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1, 2, 3, 4, 5 and 6, are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

40. Let X and Y be two events such that $P(X|Y)=\frac{1}{2}, P(Y|X)=\frac{1}{3}$ and $P(X\cap Y)=\frac{1}{6}$. Which of the following is (are) correct ?

(A) $P(X\cup Y)=\frac{2}{3}$ (B) X and Y are independent

(C) X and Y are not independent (D) $P(X^c\cap Y)=\frac{1}{3}$

paragraph for questions

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls

41. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is (a) $\frac{82}{648}$

(b) $\frac{90}{648}$ (c) $\frac{558}{648}$ (d) $\frac{566}{648}$

42. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

(a) $\frac{116}{181}$ (b) $\frac{126}{181}$ (c) $\frac{65}{181}$ (d) $\frac{55}{181}$

DCE QUESTIONS

1. If A & B are two events, the probability that only one of A and B occur is

(a) $P(A) + P(B) - 2P(A\cap B)$ (b) $P(A) + P(B) - P(A\cap B)$

(c) $P(A) + P(B)$ (d) none of these

2. If A and B are two independent events, the probability that both A and B occur is 1/8 and the probability that neither of them occurs is 3/8. The probability of occurrence of A is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{5}$

3. The probability that at least one of the events A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A}) + P(\bar{B})$ is

(a) 0.4 (b) 0.8 (c) 1.2 (d) 1.4

4. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral, equals.

(a) $\frac{1}{2}$ (b) $\frac{1}{5}$ (c) $\frac{1}{10}$ (d) $\frac{1}{20}$

5. If A and B are such events that $P(A) > 0$ and $P(B) \neq 1$. then $P(\overline{A}/\overline{B})$ is equal to

(a) $1 - P(A/\overline{B})$ (b) $1 - P(\overline{A}/B)$

(c) $1 - P(A \cup B)/P(\overline{B})$ (d) $P(\overline{A})/P(\overline{B})$

6. Out of 15 tickets numbered from 1 to 15, three are drawn at random. What is the chance that the numbers on them are in A.P.

(a) $\frac{7}{65}$ (b) $\frac{9}{15}$ (c) $\frac{13}{261}$ (d) none of these

7. Four persons are selected at random from a group of 3 men, 2 women and 4 children, what is the chance that exactly two of them are children

(a) $\frac{9}{21}$ (b) $\frac{10}{23}$ (c) $\frac{11}{24}$ (d) $\frac{10}{21}$

8. The probability that a student is not a swimmer is 1/5. What is the probability that out of 5 students 4 are swimmers?

(a) ${}^5C_4\left(\frac{4}{5}\right)^4 \frac{1}{5}$ (b) $\left(\frac{4}{5}\right)^4 \frac{1}{5}$ (c) ${}^5C_1 \frac{1}{5}\left(\frac{4}{5}\right)^4 \times {}^5C_4$ (d) none of these

9. A six faced die is so biased that it is twice to show an even number as an odd number when thrown. If it is thrown twice, the probability that the sum of two numbers thrown is even, is

(a) $\frac{4}{9}$ (b) $\frac{5}{9}$ (c) $\frac{1}{9}$ (d) none of these

10. In shuffling a pack of cards, four cards are accidently dropped. The probability that the missing cards should be one from each suit is

(a) $\frac{1}{256}$ (b) $\frac{4}{20825}$ (c) $\frac{2197}{20825}$ (d) none of these.

11. For a biased die, the probabilities for the different faces to turn up are

Face:	1	2	3	4	5	6
P :	0.1	0.32	0.21	0.15	0.05	0.17

the die is tossed and you are told that either face 1 and face 2 has turned up, then the probability that it is face 1 is

(a) $\frac{16}{21}$ (b) $\frac{1}{10}$ (c) $\frac{5}{16}$ (d) $\frac{5}{21}$

12. The probability that any of the men A_1, A_2, A_3, A_4 is alive after 95 years of age is 1/2. The probability that A_1 will die at the age of 95 and will be the first to die is

(a) $\frac{15}{16}$ (b) $\frac{15}{64}$ (c) $\frac{1}{4}$ (d) $\frac{8}{15}$

13. Given a throw of three unbiased dice shows different faces, the probability that at least one face shows 6 is

(a) $\frac{5}{6}$ (b) $\frac{5}{18}$ (c) $\frac{1}{2}$ (d) $\frac{13}{18}$

14. A bag A contains 2 white and 2 red balls and another bag contains 4 white and 5 red balls. A ball is drawn and found to be red. The probability that it was drawn from the bag B is

(a) $\frac{25}{52}$ (b) $\frac{1}{2}$ (c) $\frac{10}{19}$ (d) $\frac{5}{19}$

15. Let A and B be two events such that $P(A) = 7/20$, $P(B) = 9/20$ $P(A \cup B) = 11/20$, $P(A \cap \overline{A}) = 1$, then the value of $P(\overline{A} \cap B)$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{10}$ (d) none of these

16. For a party 8 guests are invited by a husband and his wife. They sit around a circular table for dinner. The probability that the husband and his wife sit together is

(a) $\frac{2}{7}$ (b) $\frac{2}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4}{9}$

17. If in a trial the probability of success is twice the probability of failure. In six trials the probability of at least four successes is

(a) $\frac{496}{729}$ (b) $\frac{400}{729}$ (c) $\frac{500}{729}$ (d) $\frac{600}{729}$

18. The value of $(A \cup B \cup C) \cap (A \cap B^C \cap C^C)^C \cap C^C$ is

(a) $B \cap C^C$ (b) $B^C \cap C^C$ (c) $B \cap C$ (d) $A \cap B \cap C$

AIEEE/JEE-MAINS QUESTIONS

1. The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is

2. A random variable X has the probability distribution:

X :	1	2	3	4	5	6	7	8
$p(X)$:	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, the probability $P(E \cup F)$ is

(a) 0.87 (b) 0.50 (c) 0.35 (d) 0.77

3. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is

(a) $\frac{37}{256}$ (b) $\frac{28}{256}$ (c) $\frac{128}{256}$ (d) $\frac{219}{256}$

4. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

(a) $\frac{8}{9}$ (b) $\frac{7}{9}$ (c) $\frac{2}{9}$ (d) $\frac{1}{9}$

5. Let A and B be two events such that $P\left(\overline{A \cup B}\right) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P\left(\overline{A}\right) = \frac{1}{4}$, whree $\overline{A}$ stands for component of event A. Then events A and B are

(a) independent but not equally likely (b) mutually exclusive and independent

(c) equally likely and mutually exclusive (d) equally likely but not independent

6. The probabilities of two events are 0.25 and 0.50. The probability of both happening together is 0.14. Which of the following is the probability of none of the events happening?

(a) 0.39 (b) 0.25 (c) 0.11 (d) none of these

7. If P (A) = $\frac{2}{3}$, P (B) = $\frac{1}{2}$ and P(A $\cup$ B) = $\frac{5}{6}$ then the events A and B are

(a) mutually exclusive (b) independent

(c) independent as well as mutually exclusive (d) noneof these

8. Three identical dice are rolled. The probability that the same number will appear on each of them is

(a) $\frac{1}{6}$ (b) $\frac{1}{36}$ (c) $\frac{1}{18}$ (d) $\frac{3}{28}$

9. The probability of the event A occurring is 0.5 and of B occurring is 0.3. If A and B are mutually exclusive events then the probability of neither A nor B occurring is

(a) 0.6 (b) 0.5 (c) 0.7 (d) none of these

10. A and B are two independent events such that P(A′ $\cap$ B) = $\frac{2}{15}$ and P(A $\cap$ B′) = $\frac{1}{6}$ then P(B) is equal to

(a) $\frac{3}{5}$ (b) $\frac{1}{6}$ (c) $\frac{1}{5}$ (d) $\frac{5}{6}$

11. The probability that the wife will be alive 10 years hence is 7/15 and that of the husband is 7/10.

What is the probability that at least one of them will be alive 10 years hence?

(a) $\frac{129}{150}$ (b) $\frac{126}{150}$ (c) $\frac{101}{150}$ (d) $\frac{94}{150}$

12. A solider is firing at a moving target. He fires four shots. The probability of hitting the target at the first, second, third and fourth shots are 0.6, 0.4, 0.2 and 0.1 respectively. What is the probability that he hits the target?

(a) $\frac{517}{625}$ (b) $\frac{3}{625}$ (c) $\frac{105}{625}$ (d) none of these

13. A and B are two independent events such that P(A) = 0.3 and $P(A \cup \overline{B}) = 0.8$ Then

(a) $P(B) = \frac{1}{2}$ (b) $P(B) = \frac{2}{7}$ (c) $P(A \cap B) = \frac{3}{7}$ (d) $P(A \cap B) = \frac{3}{20}$

14. The probability that a rectangle picked up from a chessboard has the area 6 cm^2 where the distance between consecutive parallel lines on the board is 1 cm, is

(a) $\frac{3}{56}$ (b) $\frac{3}{28}$ (c) $\frac{11}{108}$ (d) none of these

15. Two persons A and B throw two dice each. If A throws a sum of 9 then the probability of Y throwing a sum greater than that of X is

(a) $\frac{1}{6}$ (b) $\frac{1}{9}$ (c) $\frac{1}{54}$ (d) none of these

16. A natural number is selected at random from the set $X = \{x, 1 \le x \le 100\}$. The probability that the number satisfies the inequation $x^2 - 13x \le 30$, is

(a) $\frac{9}{50}$ (b) $\frac{3}{20}$ (c) $\frac{2}{11}$ (d) none of these

17. If E and F are independent events such that 0 < P(E) < 1 and 0 < P(F) < 1 then

(a) E, F are mutually exclusive (b) E, $\overline{F}$ are independent

(c) $\overline{E}$, $\overline{F}$ are not independent (d) $P(E/F) + P(\overline{E}/\overline{F}) = 1$

18. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2 respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is

(a) 0.2 (b) 0.7 (c) 0.06 (d) 0.14

19. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is

(a) $\frac{8}{729}$ (b) $\frac{8}{243}$ (c) $\frac{1}{729}$ (d) $\frac{8}{9}$

20. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ..., 20}.

Statement - 1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$

Statement - 2 : If the four chosen numbers form an AP, then the set of all possible values of common difference is {±1, ±2, ±3, ±4, ±5}.

(a) Statement-1 is false, Statement-2 is true.

(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

(d) Statement-1 is true, Statement-2 is false.

21. Consider 5 independent bernoulli's trials each with probability of success *p*. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then *p* lies in the interval:

(a) $\left(\frac{1}{2},\frac{3}{4}\right]$ (b) $\left(\frac{3}{4},\frac{11}{12}\right]$ (c) $\left[0,\frac{1}{2}\right]$ (d) $\left(\frac{11}{12},1\right]$

22. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is:

(a) $P(C|D) = P(C)$ (b) $P(C|D) \geq P(C)$ (c) $P(C|D) < P(C)$ (d) $P(C|D)=\frac{P(D)}{P(C)}$

23. Three numbers are chosen at random without replacement from {1, 2, 3, ..., 8}. The probability that their minimum is 3, given that their maximum is 6, is

(a) $\frac{3}{8}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{2}{5}$

ANSWERS

Basic level Assignment

1. $\frac{P(A)}{P(B)} = \frac{16}{7}$ 2. ${}^7P_5/7^5$ 3. $\frac{1}{2}$

4. $A = \frac{16}{37}, \frac{12}{37}, \frac{9}{37}$ 5. 13.9% 6. $\frac{11}{21}$

7. $\frac{319}{420}$ 8. $\frac{1}{462}$ 9. $\frac{4}{165}$ 10. $\frac{1}{13}$

11. $P(A) = \frac{4}{7}, P(B) = \frac{2}{7}, P(C) = \frac{1}{7}$ 12. $\frac{39}{50}$ 13. 0.053

14. .62 15. $P(A) = \frac{1}{3}, \ P(B) = \frac{1}{2}$ 16. $0.2 \le P(B \cap C) \le 0.35$

17. $\frac{m}{m+n}$ 18. $\alpha = \frac{p}{1-(1-p)^3}$ $\beta = \frac{p(1-p)}{1-(1-p)^3}$ $\gamma = \frac{(1-p^2)^2 p}{1-(1-p)^3}$

19. $\frac{1}{40}$

20. (i) $\frac{p_1}{p_1 + p_2 + p_3}$; (ii) $\frac{p_2}{p_1 + p_2 + p_3}$; (iii) $\frac{p_3}{p_1 + p_2 + p_3}$

where $p_1 = \frac{a_1}{a_1 + b_1}$; $p_2 = \frac{a_2}{a_2 + b_2}$; $p_3 = \frac{a_3}{a_3 + b_3}$

Advanced Level Assignment

2. Best of 3 games 3. $1 - \frac{10(N+2)}{{}^{N+7}C_2}$ 4. $462\,(0.24)^5$ 5. 99/1900

6. $\frac{23}{30}$ 7. $\frac{\frac{52}{(26!)^2} - 1}{2^{51} - 1}$ 8. $\frac{\frac{52}{(26!)^2} - 1}{2^{51} - 1}$ 9. $1 - \left(\frac{7}{10}\right)^7$; $\frac{10}{21}\left\{1 - \left(\frac{7}{10}\right)^6\right\}$

10. $\frac{125}{1296}$ 11. $\frac{1}{n}\{1-(1-p)^n\}$ 12. $\frac{13}{50}$ 13. $\frac{9m}{8N + m}$

Objective Assignment

1.	(a)	2.	(a)	3.	(b)	4.	(a)	5.	(c)
6.	(c)	7.	(a)	8.	(b)	9.	(a)	10.	(b)
11.	(d)	12.	(a)	13.	(a)	14.	(a, d)	15.	(a)
16.	(b, c, d)	17.	(b)	18.	(c)	19.	(c)	20.	(a)
21.	(b)	22.	(c)	23.	(c)	24.	(a)	25.	(c)
26.	(c)	27.	(a)	28.	(d)	29.	(b)	30.	(b)
31.	(a,b,c,d)	32.	(a,b,c)	33.	(b,c,d)	34.	(a,b,c,d)	35.	(a,c,d)
36.	(c,d)	37.	(a,c)	38.	(a,d)	39.	(a,c)	40.	(a,b)

Miscellaneous Assignment

1.	(c)	2.	(d)	3.	(d)	4.	(a)	5.	(b)
6.	(c)								
7.	A-(q); B-(p); C-(s); D-(r)			8.	A-(q), B-(p), C-(s), D-(r)			9.	(3)
10.	(5)	11.	(2)	12.	(7)	13.	(3)	14.	(4)
15.	(1)	16.	(1)	17.	(8)	18.	(9)		

Previous Year Questions

IIT-JEE/JEE-ADVANCE QUESTIONS

1.	(a)	2.	(b,c)	3.	(b)	4.	(d)	5.	(a,b,c)
6.	(b,c,d)	7.	(b)	8.	(a)	9.	(a,d)	10.	(a)
11.	(b)	12.	(a)	13.	(a,d)	14.	(a)	15.	(d)
16.	(a)	17.	(b,c)	18.	(a)	19.	(d)	20.	(d)
21.	(a)	22.	(a)	23.	(d)	24.	(d)	25.	(b)
26.	(d)	27.	(a)	28.	(b)	29.	(c)	30.	(d)
31.	(c)	32.	(a)	33.	(b)	34.	(d)	35.	(c)
36.	(b)	37.	(d)	38.	(b,d)	39.	(a)	40.	(a,b)
41.	(a)	42.	(d)						

DCE QUESTIONS

1.	(a)	2.	(a)	3.	(c)	4.	(c)	5.	(a)
6.	(a)	7.	(d)	8.	(a)	9.	(b)	10.	(c)
11.	(b)	12.	(c)	13.	(c)	14.	(c)	15.	(b)
16.	(b)	17.	(a)	18.	(a)				

MAINS QUESTIONS

1.	(c)	2.	(d)	3.	(b)	4.	(d)	5.	(a)
6.	(a)	7.	(b)	8.	(b)	9.	(d)	10.	(b)
11.	(b)	12.	(a)	13	(b)	14.	(c)	15.	(a)
16.	(b)	17.	(b)	18.	(d)	19.	(b)	20.	(d)
21.	(c)	22.	(b)	23.	(b)				