SOLVED OBJECTIVE PROBLEMS

Problem 1. ***A metal bar of length L and area of cross-section A is rigidly clamped between two walls. The Young's modulus of its material is Y and the coefficient of linear expansion is α. The bar is heated so that its temperature increases by $\theta^\circ C$. Then the force exerted at the ends of the bar is given by :***

(a) $YL\alpha\theta$ *(b)* $YL\alpha\theta / A$

(c) $YA\alpha\theta$ *(d)* $Y\theta\alpha / LA$.

Ans. *(c)*

Solution: The coefficient of linear expansion is defined as

$$\alpha = \frac{\text{increse in length}}{\text{original length} \times \text{temp. rise}} = \frac{l}{L\theta}$$

$\therefore$ Increase in length $l = \alpha L\theta$. Now

$$Y = \frac{FL}{Al}$$

or $$F = \frac{YAl}{L} = \frac{YA\alpha L\theta}{L} = YA\alpha\theta$$

Problem 2. ***When a force is applied at one end of an elastic wire, it produces a strain ε in the wire. If Y is the Young's modulus of the material of the wire, the amount of energy stored per unit volume of the wire is given by :***

(a) $Y \times \varepsilon$ *(b)* $\left(\frac{1}{2}\right) Y \times \varepsilon$

(c) $Y \times \varepsilon^2$ *(d)* $\left(\frac{1}{2}\right) Y \times \varepsilon^2$.

Ans. *(d)*

Solution: Energy stored per unit volume $= \frac{1}{2}$(stress × strain). But stress = Young's modulus × strain.

Therefore energy stored per unit volume $= \frac{1}{2} Y\varepsilon^2$.

Problem 3. ***Two springs of equal lengths and equal cross-sectional areas are made of materials whose Young's modulii are in the ratio of 2:3. They are suspended and loaded with the same mass. When stretched and released, they will oscillate with time periods in the ratio of :***

(a) $\sqrt{3} : \sqrt{2}$ *(b)* **3 : 2**

(c) $3\sqrt{3} : 2\sqrt{2}$ *(d)* **9 : 4.**

Ans. *(a)*

Solution: Young's modulus $Y = \frac{F}{A} \cdot \frac{L}{l}$

Force constant $k = \frac{F}{l} = \frac{YA}{L}$

Where l is the extension in the spring of original length L and cross-sectional area A when a force $F = Mg$ is applied. Now, the time period of vertical oscillations is given by :

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{ML}{YA}}$$

$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{Y_2}{Y_1}} = \sqrt{\frac{3}{2}}$

Hence the correct choice is (a).

Problem 4. ***A cubical block of steel of each side equal to l is floating on mercury in a vessel. The densities of steel and mercury are ρ_S and ρ_m. The height of the block above the mercury level is given by:***

(a) $l\left(1+\frac{\rho_S}{\rho_m}\right)$ ***(b)*** $l\left(1-\frac{\rho_S}{\rho_m}\right)$

(c) $l\left(1+\frac{\rho_m}{\rho_S}\right)$ ***(d)*** $l\left(1-\frac{\rho_m}{\rho_S}\right)$.

Ans. ***(b)***

Solution: Volume of block $= l^3$. Let h be the height of the block above the surface of mercury. Volume of mercury displace $= (l-h)l^2$.

$\therefore$ Weight of mercury displaced $= (l-h)l^2\rho_m g$.

This is equal to the weight of the block which is $\rho_s l^3 g$.

Thus $(l-h)l^2\rho_m g = \rho_s l^3 g$

which gives $h = l\left(1-\frac{\rho_s}{\rho_m}\right)$

Problem 5. ***A cube of wood supporting a mass of 200 g just floats in water. When the mass is removed, the cube rises by 2 cm. What is the size of the cube ?***

(a) 6 cm ***(b) 8 cm***
(c) 10 cm ***(d) 12 cm.***

Ans. ***(c)***

Solution: Let the side of the cube be l cm. The volume of the cube above the surface of water = volume of water displaced due to mass of 200 g. Therefore mass of displaced water is 200 g, its volume is 200 cm^3. Hence $2 \times l \times l = 200$ or $l = 10$ cm. Hence the correct choice is (c).

Problem 6. ***Two identical cylindrical vessels, each of base area A, have their bases at the same horizontal level. They contain a liquid of density ρ. In one vessel the height of the liquid is h_1 and in the other $h_2 > h_1$. When the two vessels are connected, the work done by gravity in equalizing the levels is :***

(a) $2\rho Ag(h_2-h_1)^2$ ***(b)*** $\rho Ag(h_2-h_1)^2$

(c) $\frac{1}{2}\rho Ag(h_2-h_1)^2$ ***(d)*** $\frac{1}{4}\rho Ag(h_2-h_1)^2$.

Ans. ***(d)***

Solution: After the levels in the two vessels become equal, the increase in height of the liquid in one vessel is $\frac{1}{2}(h_2-h_1)$ with the same decrease in height in the other. Thus, effectively a slab of liquid $\frac{1}{2}(h_2-h_1)$ in thickness falls a vertical distance equal to its thickness under the action of gravity. Therefore,

Work done by the gravity is

$$W = mg \times \frac{1}{2}(h_2-h_1)$$

where mass of the slab m is given by

$$m = \rho \times V = \rho \times \frac{1}{2}(h_2 - h_1) \times A$$

Therefore $W = \frac{1}{2}\rho(h_2 - h_1) \times A \times g \times \frac{1}{2}(h_2 - h_1) = \frac{1}{4}\rho A g (h_2 - h_1)^2$.

Problem7. ***A soap bubble of radius r is blown up to form a bubble of radius $2r$ under isothermal conditions. If σ is the surface tension of soap solution, the energy spent in doing so is :***

(a) $3\pi\sigma r^2$ ***(b)*** $6\pi\sigma r^2$

(c) $12\pi\sigma r^2$ ***(d)*** $24\pi\sigma r^2$.

Ans. ***(d)***

Solution: Surface area of bubble of radius $r = 4\pi r^2$. Surface area of bubble of radius $2r = 4\pi(2r)^2 = 16\pi r^2$. Therefore, increase in surface area $= 16\pi r^2 - 4\pi r^2 = 12\pi r^2$. Since a bubble has two surfaces, the total increase in surface area $24\pi\sigma r^2$.

$\therefore$ Energy spent = work done = $24\pi\sigma r^2$

Problem 8. ***The time period of a simple pendulum is T. The pendulum is oscillated with its bob immersed in a liquid of density σ. If the density of the bob is ρ and viscous effect is neglected, the time period of the pendulum will be :***

(a) $\left(\frac{\rho}{\rho - \sigma}\right)^{1/2} T$ ***(b)*** $\left(\frac{\sigma}{\rho - \sigma}\right)^{1/2} T$

(c) $\left(\frac{\rho}{\sigma}\right)^{1/2} T$ ***(d)*** $\left(\frac{\sigma}{\rho}\right)^{1/2} T$.

Ans. ***(a)***

Solution: The net downward force acting on the bob is

$$ma = F = (\rho - \sigma)Vg \Rightarrow \left(\frac{\rho - \sigma}{\rho}\right) g = a \qquad (\because\ m = \rho V)$$

This is the effective acceleration due to gravity.

$$\therefore \qquad T' = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{\rho l}{(\rho - \sigma)g}} = \left(\frac{\rho}{\rho - \sigma}\right)^{1/2} T \quad \left(\because T = 2\pi\sqrt{\frac{l}{g}}\right).$$

Problem 9. ***A concrete sphere of radius R has a cavity of radius r which is packed with sawdust. The relative densities of concrete and sawdust are 2.4 and 0.3 respectively. For this sphere to float with its entire volume submerged under water, the ratio of the mass of concrete to the mass of sawdust will be :***

(a) 8 ***(b) 4***

(c) 3 ***(d) zero.***

Ans. ***(b)***

Solution: Let m be the mass of concrete and ρ its density and let m' be the mass of sawdust and ρ' its density. Then

$$m = \frac{4\pi}{3}(R^3 - r^3)\rho \quad and \quad m' = \frac{4\pi}{3} r^3 \rho'$$

$$\therefore \qquad \frac{m}{m'} = \frac{R^3 - r^3}{r^3} \cdot \frac{\rho}{\rho'} \qquad \dots \text{(i)}$$

Since the entire volume $V = \frac{4\pi}{3}R^3$ of the sphere is submerged under water, we have, from the principle of floatation,

Weight of concrete + weight of sawdust = weight of volume V water displaced

or $\quad mg + m'g = V\rho_0 g \; or \quad m + m' = V\rho_0$

where ρ_0 is the density of water. Thus

$$\frac{4\pi}{3}(R^3 - r^3)\rho + \frac{4\pi}{3}r^3\rho' = \frac{4\pi}{3}R^3\rho_0$$

or $\quad (R^3 - r^3)d + r^3 d' = R^3 \qquad \ldots$ (ii)

where $d = \rho/\rho_0$ and $d' = \rho'/\rho_0$ are the relative densities of concrete and sawdust respectively. Equation (ii), on simplification, gives

$$\frac{R^3}{r^3} = \frac{(d - d')}{(d-1)} \quad \text{or} \quad \frac{R^3}{r^3} - 1 = \frac{(d-d')}{(d-1)} - 1$$

or $\quad \frac{R^3 - r^3}{r^3} = \frac{(1-d')}{(d-1)} \qquad \ldots$ (iii)

Using (iii) in (i) and noting that $\frac{\rho}{\rho'} = \frac{d}{d'}$, we have

$$\frac{m}{m'} = \frac{(1-d')}{(d-1)} \times \frac{d}{d'} = \frac{(1-0.3)}{(2.4-1)} \times \frac{2.4}{0.3} = 4$$

Problem 10. ***A closed compartment containing liquid is moving with some acceleration in horizontal direction. Neglect the effect of gravity. Then the pressure in the compartment is :***

(a) ***same everywhere*** ***(b)*** ***lower in the front side***
(c) ***lower in the rear side*** ***(d)*** ***lower in the upper side.***

Ans. ***(b)***

Solution: Due to frictional force (which acts in a direction opposite to the direction of acceleration) on the rear face, the pressure in the rear side will be increased. Hence the pressure in the front side will be lowered.

Problem 11. ***A vessel contains oil (density 0.8 g cm^{-3}) over mercury (density 13.6 g cm^{-3}). A homogeneous sphere floats with half volume immersed in mercury and the other half in oil. The density of the material of the sphere in g cm^{-3} is***

(a) ***3.3*** ***(b)*** ***6.4***
(c) ***7.2*** ***(d)*** ***12.8.***

Ans. ***(c)***

Solution: Weight of sphere = weight of mercury displaced + weight of oil displaced

or $\quad V\rho g = \frac{V}{2} \times 13.6 \times g + \frac{V}{2} \times 0.8 \times g$

or $\quad \rho = \frac{13.6 + 0.8}{2} = 7.2 \quad g \quad cm^{-3}$.

Problem 12. ***Two rain drops of radii r_1 and r_2 reaching the ground with terminal velocities have their linear momenta p and $32p$. The ratio r_2 / r_1 will be***

(a) ***2 : 1*** ***(b)*** ***1 : 2***
(c) ***2 : 3*** ***(d)*** ***3 : 2.***

Ans. ***(a)***

Solution: The terminal velocity $v \propto r^2$ and the mass of the drop $m \propto r^3$. Hence the momentum $p = mv \propto r^5$.

Given that $p_2 / p_1 = \frac{32}{1}$

$\therefore$ $\left(\frac{r_2}{r_1}\right)^5 = (2)^5$

or, $r_2 / r_1 = 2:1$.

Problem 13. ***Two parallel glass plates are held vertically at a small separation d and dipped in a liquid of surface tension T, angle of contact $\theta = 0$ and density ρ. The height of water that climbs up in the gap between the plates is given by***

(a) $2T / d\rho g$

(b) $T / 2d\rho g$

(c) $T / d\rho g$

(d) ***None of these.***

Ans. ***(a)***

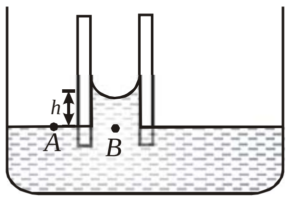
Solution: Upward force due to surface tension is balanced by the weight of the liquid which rises in the gap, so

$$2T.b = b\,dh\rho g\ ,$$

where, $b =$ width of the plates

$$\therefore \quad h = \frac{2T}{d\rho g}\ .$$

Alternative Method:



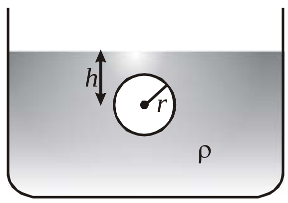
The meniscus between the plates has cylindrical shape with radius $r = \frac{d}{2}$. The pressure just inside the meniscus is $p_0 - T\left(\frac{1}{r} + \frac{1}{\infty}\right) = p_0 - \frac{2T}{d}$,

Now, $p_A = p_B$

or, $p_0 = \left(p_0 - \frac{2T}{d}\right) + h\rho g$

$$\Rightarrow \quad h = \frac{2T}{d\rho g}\ .$$

Problem 14. ***An air bubble rises uniformly through a liquid column of density ρ. At some instant it is at a depth h below the free surface of water and its radius r. If T be the surface tension of the liquid, the pressure of enclosed air in the bubble exceeds the atmospheric pressure by an amount***



(a) $\frac{2T}{r}$

(b) $\frac{2T}{r} + h\rho g$

(c) $h\rho g$

(d) ***None of these.***

Ans. ***(b)***

Solution: Pressure outside the bubble, $p_0 = p_{atm} + h\rho g$

Pressure inside the bubble

$$p_i = p_0 + \frac{2T}{r} = p_{atm} + h\rho g + \frac{2T}{r}$$

$$\therefore \qquad p_i - p_{atm} = h\rho g + \frac{2T}{r}\,.$$