OBJECTIVE SOLVED

1. Pick up the correct statements:
   (a) area under a – t graph gives velocity
   (b) area under a – t graph gives change is velocity
   (c) path of projectile as seen by another projectile is parabola
   (d) a body, whatever be its motion, is always at rest in a frame of reference fixed to body itself.

Solution:

$$\frac{dv}{dt} = a$$

$$\Rightarrow \quad \int_{v_1}^{v_2} dv = \int a \; dt$$

$\Rightarrow \quad \Delta v$ = Area under a – t graph

where Δv = magnitude of change in velocity.

The path of a projectile as seen by another projectile is a straight line becomes, the relative velocity between the particles remains constant.

$\therefore$ (b) and (d)

2. The angular acceleration of a particle moving along a circular path with uniform speed is
   (a) uniform but non-zero
   (b) zero
   (c) variable
   (d) such as cannot be predicted from the given information.

Solution:

As angular speed of the particle is constant and hence angular acceleration is zero.

$\therefore$ (b)

3. A particle is projected horizontally from the top of a cliff of height H with a speed $\sqrt{2gH}$. The radius of curvature of the trajectory at the instant of projection will
   (a) H/2 (b) H
   (c) 2H (d) ∞.

Solution:

Since, $\vec{g} \perp \vec{v}$

Radial acceleration $a_r = g$

We know $a_r = v^2 / r$

$\Rightarrow \quad \frac{v^2}{r} = g$ where r is the radius of curvature.

$\Rightarrow \quad \frac{2gH}{r} = g \qquad (\because \; v = \sqrt{2gH})$

$\Rightarrow \quad r = 2H$

Hence (C)

4. A body when projected vertically up, covers a total distance D. During the time of its flight t. If there were no gravity, the distance covered by it during the same time is equal to

(a) 0 (b) D

(c) 2D (d) 4D.

Solution:

The displacement of the body during the time t as it attains the point of projection

$\Rightarrow$ $S = 0$ $\Rightarrow$ $v_0 t - \frac{1}{2}gt^2 = 0$

$\Rightarrow$ $t = \frac{2v_0}{g}$

During the same time t, the body moves in absence of gravity through a distance $D' = v_0 t$, because in absence of gravity g = 0

$\Rightarrow$ $D' = v_0\left(\frac{2v_0}{g}\right) = \frac{2v_0^2}{g}$ ...(i)

In presence of gravity the total distance covered is

$= D = 2H = 2\frac{v_0^2}{2g} = \frac{v_0^2}{g}$ ...(ii)

(i) $\div$ (ii) $\Rightarrow$ $D' = 2D$

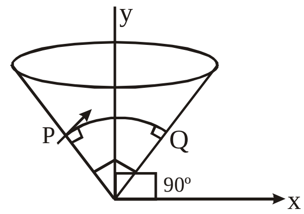
Hence (c)

5. A particle is projected from point P with velocity $5\sqrt{2}ms^{-1}$ perpendicular to the surface of a hollow right angle cone whose axis is vertical. It collides at Q normally. The time of the flight of the particle is

(a) 1 sec. (b) $\sqrt{2}$ sec.

(c) $2\sqrt{2}$ sec (d) 2 sec.

Solution:



$t = \frac{u}{g\sin\theta} = \frac{5\sqrt{2}\times\sqrt{2}}{10} = 1\,\text{sec}.$

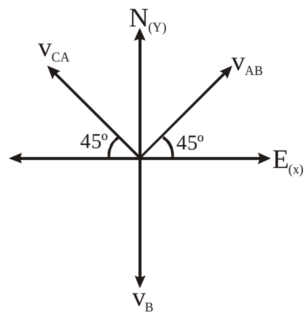
Hence (a)

6. The relative velocity of a car 'A' with respect to car B is $30\sqrt{2}$ m/s due North-East. The velocity of car 'B' is 20 m/s due south. The relative velocity of car 'C' with respect to car 'A' is $10\sqrt{2}$ m/s due North-West. The speed of car C and the direction (in terms of angle it makes with the east).

(a) $20\sqrt{2}$ m/s, 45° (b) $20\sqrt{2}$ m/s, 135°

(c) $10\sqrt{2}$ m/s, 45° (d) $10\sqrt{2}$ m/s, 135°.

Solution:



Given $|\vec{v}_{AB}| = 30\sqrt{2}ms^{-1}$

$|\vec{v}_B| = 20\,ms^{-1}$

$|\vec{v}_{CA}| = 10\sqrt{2}\,ms^{-1}$

Now

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = 30\sqrt{2}(\cos 45^o\,\hat{i} + \sin 45^o\,\hat{j})$

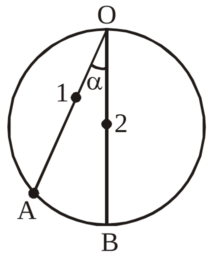
or, $\vec{v}_A - \vec{v}_B = (30\hat{i} + 30\hat{j})ms^{-1}$ ...(i)

and, $\vec{v}_B = (-20\hat{j})ms^{-1}$ ...(ii) and $\vec{v}_{CA} = \vec{v}_C - \vec{v}_A = 10\sqrt{2}(-\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j})$

or $\vec{v}_C - \vec{v}_A = (-10\hat{i} + 10\hat{j})ms^{-1}$ ...(iii)

solving equation (i) (ii) and (iii) we'll get $\vec{v}_C = 20\hat{i} + 20\hat{j}$ (a)

7. Two particles 1 and 2 are allowed to descend on the two frictionless chord OA and OB of a vertical circle, at the same instant from point O. The ratio of the velocities of the particles 1 and 2 respectively, when they reach on the circumference will be (OB is the diameter)



(a) $\sin\alpha$ (b) $\tan\alpha$
(c) $\cos\alpha$ (d) none of these.

Solution:

$OA = d\cos\alpha,\ a_{OA} = g\cos\alpha$

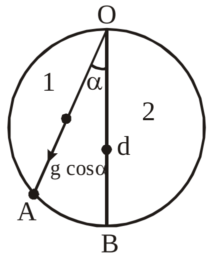
Along OA

$\Rightarrow\ v_A^2 = 2g\cos\alpha . d\cos\alpha$

Along OB

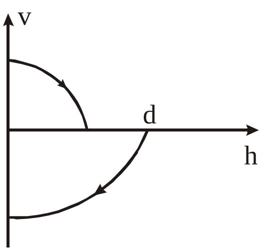
$v_B^2 = 2gd$

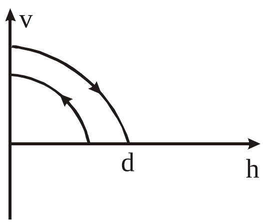
$\Rightarrow\ \frac{v_A}{v_B} = \cos\alpha$

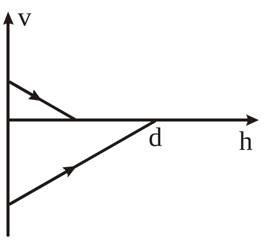


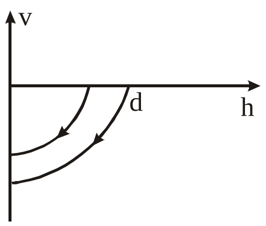
Hence, C is correct.

8. A ball is dropped vertically from a height d above the ground. If it hits the ground and bounces up vertically to a height d/2. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as

(a)


(b)


(c)


(d)


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Solution:

As the ball falls, at height h the velocity of the ball is zero and at any height $h,\ v^2 = u^2 + 2g(d-h)$, with decreasing h, v increases.
When h = 0, v = velocity is maximum.
After the ball collides the floor, its velocity changes in magnitude as wall as direction, as the body goes to a smaller height in bouncing up .The change in velocity takes place within zero height and with no change in time.
Hence, (a) is correct choice.

9. A particle moves along a straight line according to the law $S^2 = at^2 + 2bt + c$. The acceleration of the particle varies as

(a) S^{-3} (b) $S^{2/3}$

(c) S^2 (d) $S^{5/2}$.

Solution:

$S = (at^2 + 2bt + c)^{1/2}$

Differentiating, $\frac{dS}{dt} = \frac{1}{2}(at^2 + 2bt + c)^{-1/2} \times (2at + 2b) = \frac{at + b}{\sqrt{at^2 + 2bt + c}}$

$$\frac{d^2S}{dt^2} = \frac{\left(\sqrt{at^2 + 2bt + c}\right) \times a - \frac{(at + b)(at + b)}{\sqrt{at^2 + 2bt + c}}}{(at^2 + 2bt + c)}$$

$$= \frac{a(at^2 + 2bt + c) - (at + b)^2}{\sqrt{at^2 + 2bt + c} \times \left(at^2 + 2bt + c\right)} = \frac{(ac - b^2)}{S \times S^2}$$

$\therefore$ $\frac{d^2S}{dt^2} \propto \frac{1}{S^3}$ $\Rightarrow$ acceleration $\propto S^{-3}$

$\therefore$ (a)

10. A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t, the maximum velocity acquired by car is

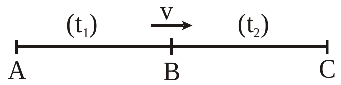
(a) $V = \frac{\alpha\beta}{(\alpha + \beta)} t$ (b) $V = \frac{\alpha\beta}{(\alpha - \beta)} t$

(c) $V = \frac{2\alpha\beta}{(\alpha + \beta)} t$ (d) $V = \frac{2\alpha\beta}{(\alpha - \beta)} t$.

Solution:

From motion from A to B $V = \alpha t_1$ or $t_1 = \frac{V}{\alpha}$

From motion from B to C $0 = V - \beta t_2$ or $t_2 = \frac{V}{\beta}$



$\therefore$ $t = t_1 + t_2 = \frac{V}{\alpha} + \frac{V}{\beta} = \frac{V(\alpha + \beta)}{\alpha\beta}$

or $V = \frac{\alpha\beta}{(\alpha + \beta)} t$

$\therefore$ (a)

11. A stone A is dropped from rest from a height h above the ground. A second stone B it simultaneous y thrown vertically up from a point on the ground with velocity v. The line of motion of both the stones is same. The values of v which would enable the stone B to meet the stone A midway between their initial positions is

(a) 2 gh (b) $2\sqrt{gh}$

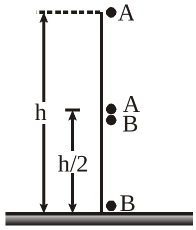
(c) $\sqrt{gh}$ (d) $\sqrt{2gh}$.

Solution:

Time of travel of each stone =

Distance travelled by each stone $= \frac{h}{2}$

For stone A, $\frac{h}{2} = \frac{1}{2}gt^2$ i.e., $t = \sqrt{\frac{h}{g}}$



For stone B, $\frac{h}{2} = ut - \frac{1}{2}gt^2 = u\sqrt{\frac{h}{g}} - \frac{1}{2}g\left(\frac{h}{g}\right)$

$\Rightarrow \quad \frac{h}{2} = u\sqrt{\frac{h}{g}} - \frac{h}{2}$

or, $\quad u\sqrt{\frac{h}{g}} = h$

$\therefore \quad u = h\sqrt{\frac{g}{h}} = \sqrt{gh}$

The correct option is (c)

12. A stone is dropped from rest from the top of a cliff. A second stone is thrown vertically down with a velocity of 30 m/s two seconds later. At what distance from the top of a cliff do they meet?

(a) 60 m (b) 120 m
(c) 80 m (d) 44 m.

Solution:

The two stones meet at distance S from top of cliff t seconds after first stone is dropped.

For 1st stone $S = \frac{1}{2}gt^2$; For 2nd stone $S = u(t-2) + \frac{1}{2}g(t-2)^2$.

i.e., $\quad \frac{1}{2}gt^2 = ut - 2u + \frac{1}{2}gt^2 - 2gt + 2g$

$$0 = (u-2g)t - 2(u-g);\ t = \frac{2(u-g)}{u-2g} = \frac{2(30-10)}{30-20} = 4s$$

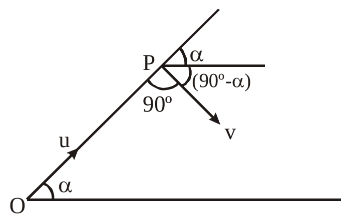
Distance S at which they meet $= \frac{1}{2} \times gt^2 = \frac{1}{2} \times 10 \times 16$

= 80 m from top of cliff

$\therefore$ (c)

13. A particle is projected from a point O with velocity u in a direction making an angle α upward with the horizontal. At P, it is moving at right angles to its initial direction of projection. Its velocity at P is

(a) $u \tan\alpha$ (b) $u \cot\alpha$
(c) $u \operatorname{cosec}\alpha$ (d) $u \sec\alpha$.



Solution:

$v\cos(90-\alpha) = v\sin\alpha = u\cos\alpha;\ v = u\cot\alpha$

$\therefore$ (b)

14. A man running at 6 km/hr on a horizontal road observes that the rain hits him at an angle of 30º from the vertical. The actual velocity of rain has magnitude

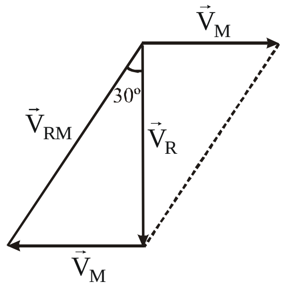
(a) 6 km/hr (b) $6\sqrt{3}$ km/hr
(c) $2\sqrt{3}$ km/hr (d) 2 km/hr

Solution:

$\vec{V}_R \rightarrow$ velcoity of rain

$\vec{V}_{RM} \rightarrow$ velocity of rain relative to man

$\vec{V}_M \rightarrow$ velocity of man



Given, $V_M = 6 \text{ kmh}^{-1}$, $V_{RM} \rightarrow$ at angle 30º from vertical

here $\vec{V}_R = \vec{V}_{RM} + \vec{V}_M$

from figure,

$$\tan 30^o = \frac{V_M}{V_R}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{6}{V_R}$$

$$\Rightarrow \quad V_R = 6\sqrt{3} \text{ km/hr}$$

$\therefore$ (b)

15. The angular acceleration of a particle moving along a circular path with uniform speed is
(a) uniform but non-zero
(b) zero
(c) variable
(d) such as cannot be predicted from the given information.

Solution:

As angular speed of the particle is constant and hence angular acceleration is zero.

$\therefore$ (b)