SOLVED OBJECTIVE PROBLEMS

Problem 1. Two thin rings each of radius R are coaxially placed at a distance R. The rings have a uniform mass distribution and have mass m_1 and m_2 respectively. Then the work done in moving a mass m from centre of one ring to that of the other is

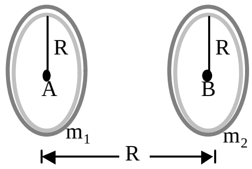
(a) zero

(b) $\dfrac{Gm(m_1 - m_2)(\sqrt{2}-1)}{\sqrt{2}R}$

(c) $\dfrac{Gm(\sqrt{2})(m_1 - m_2)}{R}$

(d) $\dfrac{Gmm_1(\sqrt{2}+1)}{m_2 R}$.

Ans. ***(b)***



Solution:

$$V_A = \begin{pmatrix}\text{Potential at} \\ \text{A due to A}\end{pmatrix} + \begin{pmatrix}\text{Potential at} \\ \text{A due to B}\end{pmatrix}$$

$$\Rightarrow \quad V_A = -\frac{Gm_1}{R} - \frac{Gm_2}{\sqrt{2}R} \text{ and}$$

$$V_B = \begin{pmatrix}\text{Potential at} \\ \text{B due to A}\end{pmatrix} + \begin{pmatrix}\text{Potential at} \\ \text{B due to B}\end{pmatrix}$$

$$\Rightarrow \quad V_B = -\frac{Gm_2}{R} - \frac{Gm_1}{\sqrt{2}R}$$

Since $W_{A\to B} = m(V_B - V_A)$

$$\Rightarrow \quad W_{A\to B} = \frac{Gm(m_1 - m_2)(\sqrt{2}-1)}{\sqrt{2}R}.$$

Problem 2. A simple pendulum has a time period T_1 when on the earth's surface, and T_2 when taken to a height R above the earth's surface, where R is radius of earth. The value of T_2/T_1 is

(a) 1

(b) $\sqrt{2}$

(c) 4

(d) 2.

Ans. ***(d)***

Solution: The acceleration due to gravity at earth's surface is g and at a distance R from earth's surface it is g/4. Hence

$$\frac{T_1}{T_2} = 2 \quad [\because T = 2\pi\sqrt{l/g}].$$

Problem 3. The kinetic energy of a satellite in an orbit close to the surface of the earth is E. What should be its kinetic energy so that it escapes from the gravitational field of the earth ?

(a) $\sqrt{2}E$

(b) 2E

(c) $2\sqrt{2}E$

(d) 4E.

Ans. ***(b)***

Solution: Orbital velocity close to surface of earth is $\sqrt{gR}$. So,

$$E = \frac{1}{2}m(\sqrt{gR})^2$$

$$\Rightarrow \quad E = \frac{1}{2}mgR$$

If the body is to escape, the velocity at surface of earth is $\sqrt{2gR}$. If E' is the kinetic energy corresponding to this velocity then

$$E' = \frac{1}{2}m(\sqrt{2gR})^2$$

$$\Rightarrow \quad E' = 2E$$

Problem 4. A geo-stationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few hundred kilometers above the earth's surface (R_{earth}=6400 km) will approximately be

(a) $\frac{1}{2}$ hr. (b) 1 hr.

(c) 2 hr. (d) 4 hr.

Ans. ***(c)***

Solution: We know that

$$T^2 \propto R^3 \quad \text{or} \quad (T_2/T_1) = (R_2/R_1)^{3/2}$$

$$\text{or} \quad \frac{T_2}{T_1} = \left(\frac{6400}{36000}\right)^{3/2}$$

$$\text{or} \quad T_2 = \left(\frac{6400}{36000}\right)^{3/2} \times 24 \approx 2\,\text{hr.}$$

Problem 5. If the radius of the earth were to shrink by one per cent, its mass remaining the same, the value of g on the earth's surface would

(a) increase by 0.5% (b) increase by 2%

(c) decrease by 0.5% (d) decrease by 2%.

Ans. ***(b)***

Solution: $g = \frac{GM}{R^2}$

$$\Rightarrow \quad \frac{dg}{g} = -2\frac{dR}{R}$$

$$\frac{dR}{R} = -1\% \quad \Rightarrow \quad \frac{dg}{g} = 2\%$$

Problem 6. A satellite is revolving round the earth in an orbit of radius r with time period T. If the satellite is revolving round the earth in an orbit of radius r + Δr (Δr<< r) with time period T + ΔT then,

(a) $\frac{\Delta T}{T} = \frac{3}{2}\frac{\Delta r}{r}$ (b) $\frac{\Delta T}{T} = \frac{2}{3}\frac{\Delta r}{r}$

(c) $\frac{\Delta T}{T} = \frac{\Delta r}{r}$ (d) $\frac{\Delta T}{T} = -\frac{\Delta r}{r}$.

Ans. ***(a)***

Solution: Since, $T^2 = kr^3$

Differentiating the above equation

$$\Rightarrow \quad 2\frac{\Delta T}{T} = 3\frac{\Delta r}{r}$$

$$\Rightarrow \quad \frac{\Delta T}{T} = \frac{3}{2}\frac{\Delta r}{r}$$

Problem 7. Mass of moon is 1/81 times that of earth and its radius is 1/4 the earth's radius. If escape velocity at surface of earth is 11.2 km/sec, then its velue at surface of moon is:

(a) 0.14 km/sec (b) 0.5 km/sec

(c) 2.5 km/sec (d) 5 km/sec.

Ans. ***(c)***

Solution: $V_e = 11.2 = \sqrt{\frac{2GM}{R}}, \quad V_m = \sqrt{\frac{2GM_m}{R_m}}$

$$V_m = \sqrt{\frac{G \times M \times 4}{81 \times R}} = \frac{11.2 \times 2}{9} = 2.5\,km/sec.$$

Problem 8. A satellite of mass m_s revolving in a circular orbit of radius r_s round the earth of mass M has a total energy E. Then its angular momentum will be

(a) $(2Em_sr_s^2)^{1/2}$ (b) $(2Em_sr_s^2)$

(c) $(2Em_sr_s)^{1/2}$ (d) $(2Em_sr_s)$.

Ans. ***(a)***

Solution: The velocity υ_s of the satellite is given by

$$\frac{GMm_s}{r_s^2} = \frac{m_s\upsilon_s^2}{r_s}$$

$$\therefore \quad \upsilon_s = \sqrt{\left(\frac{GM}{r_s}\right)} \quad \ldots(i)$$

$$K.E. = \frac{1}{2}m_s\upsilon_s^2 = \frac{1}{2}m_s\left(\frac{GM}{r_s}\right)$$

$$P.E. = -\frac{GMm_s}{r_s}$$

$\therefore$ Total energy E=K.E. + P.E.

$$= -\frac{GMm_s}{2r_s} \qquad \ldots(ii)$$

The angular momentum L is given by

$$L = m_s \upsilon_s r_s$$

$$= m_s\left(\frac{GM}{r_s}\right)^{1/2} r_s = (GMm_s{}^2 r_s)^{1/2} \qquad \ldots(iii)$$

From eqs. (ii) and (iii), we get

$$L = (2\,E\,m_s r_s{}^2)^{1/2}.$$

Problem 9. A straight rod of length extends L from x = a to x = L + a. The gravitational force exerted on a point mass m at x = 0 if the mass per unit length of the rod is $A + Bx^2$, is

(a) $GmA\left(\frac{1}{a+L} - \frac{1}{a} + BL\right]$ (b) $Gm\,A\left[\frac{1}{a} - \frac{1}{a+L} + BL\right]$

(c) $Gm\,A\left[\left(\frac{1}{a+L} - \frac{1}{a}\right) - BL\right]$ (d) $Gm\,A\left[\left(\frac{1}{a} - \frac{1}{a+L}\right) - BL\right]$.

Ans. ***(b)***

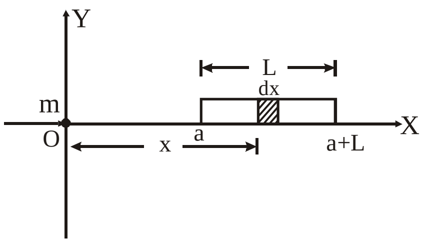
Solution: Mass per unit length = $A + Bx^2$. So the mass of length dx is $dM = dx(A + Bx^2)$

$$dF = \frac{Gm(dM)}{x^2} = Gm\left[\frac{A + Bx^2}{x^2}\right]$$

$$F = \int_a^{a+L} Gm\left(\frac{1}{x^2}\right)dx\,\left(A + Bx^2\right)$$

$$= \int_a^{a+L} Gm\left(\frac{A}{x^2} + B\right)dx$$

$$= Gm\left[\frac{A}{a} - \frac{A}{a+L} + BL\right].$$



Problem 10. An artificial satellite moving in a circular orbit around the earth has a total (K.E. + P.E.) = E_0. Its potential energy is

(a) $-E_0$ (b) $1.5\,E_0$

(c) $2\,E_0$ (d) E_0.

Ans. ***(c)***

Solution: Total energy = kinetic energy + Potential energy

$$E_0 = \frac{1}{2}mv^2 - \frac{GMm}{r} \qquad ...(i)$$

Further, $\frac{mv^2}{r} = \frac{GMm}{r^2}$

or $\frac{1}{2}mv^2 = \frac{GMm}{2r}$ ...(ii)

Substiuting the value of $\frac{1}{2}mv^2$ in equation (i) from equation (ii), we get

$$E_0 = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

Therefore, $P.E. = -\frac{GMm}{r} = 2E_0$.