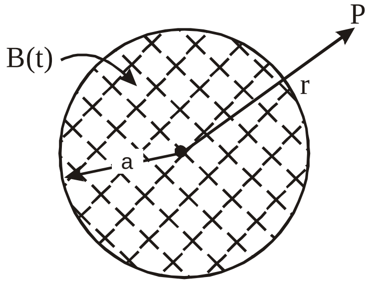
OBJECTIVE SOLVED PROBLEMS

1. A uniform but time-varying magnetic field B (*t*) exists in a circular region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region :



(a) is zero
(b) decreases as 1/r
(c) increases as r
(d) decreases as $1/r^2$.

Ans. ***(b)***

Solution: Magnitude of induced electric field

$$E = \frac{a^2}{2r}\frac{dB}{dt}$$

Thus, the magnitude of the electric field increases linearly from zero at the center of electric field to (a/2) (dB/dt) at the edge of circular region of radius a, and then decreases inversely with distance.

2. A coil of wire having finite inductance and resistance has a conducting ring placed coaxially within it. The coil is connected to a battery at time t = 0. so that a time-dependent current I_1 (t) starts flowing through the coil. If I_2 (t) is the current induced in the ring, and B (t) is the magnetic field at the axis of the coil due to I_1 (t), then as a function of time (t > 0), the product I_2 (t) B (t)

(a) increases with time
(b) decreases with time
(c) does not vary with time
(d) passes through a maximum.

Ans. ***(d)***

Solution : Current through the coil = I_1 (t)

Magnetic field at center B (t) = $\frac{\mu_0}{4\pi}.\frac{2\pi I_1(t)}{a}$

Current induced in the ring I_2 (t) $= \frac{\varepsilon}{R}$

ε = induced emf in the ring

$$I_2(t) = \frac{1}{R}\frac{d\phi}{dt} = \frac{A}{R}\frac{dB}{dt}$$

A = area of the ring

$$I_2(t)B(t) = \frac{A}{R}\frac{dB(t)}{dt}B(t)$$

which passes through the maximum.

3. A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12 V battery. The current in the coil is 1.0 A at approximately the time.

(a) 500 ms (b) 20 ms

(c) 35 ms (d) 1 ms.

Ans. ***(d)***

Solution : Current developed with time in a coil of inductance

$$I = \frac{V}{R}(1 - e^{-t/\tau}) \text{ where } \tau = L/R$$

we have $\tau = \dfrac{8.4\,\text{mH}}{6\Omega} = 1.4\text{ms}$

Hence 1 A = $\left(\dfrac{12V}{6\Omega}\right)(1 - e^{-t/1.4\text{ms}})$

or $\quad e^{-t/1.4\text{ms}} = 1 - \dfrac{1}{2} = \dfrac{1}{2}$

or $\quad -t/1.4\text{ms} = \ln\left(\dfrac{1}{2}\right) = -0.693$

or $\quad t = (1.4 \times 0.693)\text{ms} = 0.97\text{ms} \approx 1 \text{ ms}.$

4. A small square loop of wire of side *l* is placed inside a large square loop of wire of side L (L>>*l*). The loops are co-planar and their centers coincide. The mutual inductance of the system is proportional to

(a) *l*/L (b) l^2/L

(c) L/*l* (d) L^2/l.

Ans. (b)

Solution: Magnetic field produced by a current in a large square loop of wire at its center

$$B = \frac{2\sqrt{2}\mu_0 i}{\pi L}$$

The magnetic flux ϕ_{12} that links big loop with the small square loop of side *l* (*l*<<L) is

$$\phi_{12} = B(l^2) = \frac{2\sqrt{2}\mu_0 i}{\pi}\left(\frac{l^2}{L}\right),$$

$\therefore$ The mutual inductance

$$M_{12} = \frac{\phi_{12}}{i} = \frac{2\sqrt{2}\mu_0 i}{\pi}\left(\frac{l^2}{L}\right)$$

i.e., $\quad M_{12} \propto (l^2/L)$.

5. A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant, uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement (s) from the following :

(a) The entire rod is at the same electric potential

(b) There is an electric field in the rod

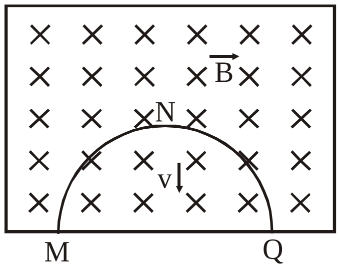
(c) The electric potential is highest at the center of the rod and decreases towards its ends

(d) The electric potential is lowest at the center of the rod and increases towards its ends.

Ans. ***(b)***

Solution: According to Faraday's law, an induced emf is set up on the rod whose magnitude is B/v. Thus, an electric field is generated in the rod. The electric potential varies uniformly along the rod.

6. A thin semi-circular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$ (see figure). At the position MNQ the speed of the ring is v and the potential difference developed across the ring is



(a) zero

(b) $Bv\pi R^2/2$ and M is at higher potential

(c) πRBv and Q is at higher potential

(d) 2RBv and Q is at higher potential.

Ans. ***(d)***

Solution: The induced emf as given by Faraday's law of induction is

$$E = -B\,l\,v$$

$$l = 2R$$

= projection of ring perpendicular to the direction of v

$$= -B \times 2R \times v$$

$$= -2BvR.$$

7. Two different coils have self inductances $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then

(a) $\frac{i_1}{i_2} = \frac{1}{4}$ (b) $\frac{i_1}{i_2} = 4$

(c) $\frac{W_1}{W_2} = 4$ (d) $\frac{W_1}{W_2} = \frac{1}{4}$.

Ans. ***(a) and (d)***

Solution : $e_1 = L_1\frac{di_1}{dt}$ and $e_2 = L_2\frac{di_2}{dt}$

$$\therefore \quad \frac{di_1}{dt} = \frac{di_2}{dt}$$

$$\therefore \quad \frac{e_1}{e_2} = \frac{L_1}{L_2} = \frac{8}{2}$$

$$\Rightarrow \quad \frac{e_2}{e_1} = \frac{1}{4}.$$

Power given to the coils are same.

So $e_1 i_1 = e_2 i_2$

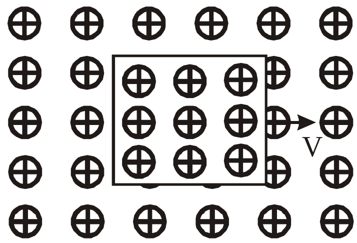
$$\therefore \quad \frac{i_1}{i_2} = \frac{e_2}{e_1} = \frac{1}{4}$$

$$\text{Energy} = \frac{1}{2} L i^2$$

$$\therefore \quad \frac{W_1}{W_2} = \frac{\frac{1}{2} L_1 i_1^2}{\frac{1}{2} L_2 i_2^2}$$

$$= \frac{L_1}{L_2}\left(\frac{i_1}{i_2}\right)^2 = 4 \times \frac{1}{16} = \frac{1}{4}.$$

8. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular and into the plane of the loop exists everywhere, see figure. The current induced in the loop is



(a) BLv/R clockwise
(b) BLv/R anticlockwise
(c) 2 BLv/R anticlockwise
(d) zero.

Ans. ***(d)***

Solution: Since the magnetic induction is uniform, the flux, ϕ, through the square loop at any time t, is

$$\phi = B \times A = B \times L^2 = \text{constant}$$

Hence, $e = \frac{-d\phi}{dt}$ = zero.

9. The inductance of a closed-packed coil of 400 turns is 8 mH. A current of 5 mA is passed through it. The magnetic flux through the coil is approximately

(a) $0.1\ \mu_0$ Wb
(b) $0.2\ \mu_0$ Wb
(c) $1.0\ \mu_0$ Wb
(d) $2.0\ \mu_0$ Wb.

Ans. (a)

Solution: $L = \frac{N\phi}{i}$

$$8 \times 10^{-3} = \frac{400 \times \phi}{5 \times 10^{-3}}$$

$$\Rightarrow \quad \phi = \frac{40 \times 10^{-6}}{400} \text{Wb} = 10^{-7} \text{Wb}$$

$$\Rightarrow \quad \phi = \frac{4\pi \times 10^{-7}}{4\pi} \text{Wb}$$

$$\Rightarrow \quad \phi = \frac{\mu_0}{4\pi} \text{Wb}$$

$$\Rightarrow \quad \phi \approx 0.1\mu_0 \text{Wb} .$$

10. The current in an L – R circuit builds up to $3/4^{th}$ of its steady state value in 4 seconds. The time constant of this circuit is

(a) $\frac{1}{\ln 2}\text{sec}$ (b) $\frac{2}{\ln 2}\text{sec}$

(c) $\frac{3}{\ln 2}\text{sec}$ (d) $\frac{4}{\ln 2}\text{sec}$.

Ans. ***(b)***

Solution : $I = I_0(1 - e^{-t/\tau})$ where $\tau \rightarrow$ time constant

$$\therefore \quad \frac{3}{4}I_0 = I_0(1 - e^{-t/\tau})$$

$$\Rightarrow \quad \frac{3}{4} = 1 - e^{-t/\tau}$$

$$\Rightarrow \quad e^{-t/\tau} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{-t}{\tau}\ln e = \ln\frac{1}{4}$$

$$\Rightarrow \quad \frac{-4}{\tau} = -2\ln 2$$

$$\Rightarrow \quad \tau = \frac{2}{\ln 2} .$$

11. The magnetic flux through each turn of a 100 turn coil is $(t^3 - 2t) \times 10^{-3}$ Wb, where t is in second. The induced emf at t = 2 s is

(a) -4V (b) -1V

(c) +1V (d) +4V.

Ans. ***(b)***

Solution: $\phi = (t^3 - 2t) \times 10^{-3}$

$$\frac{d\phi}{dt} = (3t^2 - 2) \times 10^{-3}$$

$$\left.\frac{d\phi}{dt}\right|_{t=2} = (3\times 4 – 2) \times 10^{-3} \text{ . Wb/s}$$

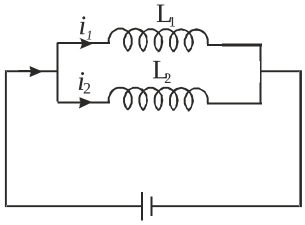
$$= 10^{-2}\text{ Wb/s}$$

$$e = -N\frac{d\phi}{dt}$$

$$= -100\times 10^{-2}\text{ V}$$

= - 1 V.

12. Two inductances L_1 and L_2 are placed far apart and in parallel. Their combined inductance is



(a) $\frac{L_1 L_2}{L_1 + L_2}$ (b) $(L_1 + L_2)$

(c) $(L_1 + L_2)\frac{L_1}{L_2}$ (d) $(L_1 + L_2)\frac{L_2}{L_1}$.

Ans. ***(a)***

Solution:

$$E = L_e\frac{di}{dt}$$

$$E = L_1\frac{di_1}{dt} = L_2\frac{di_2}{dt}$$

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{E}{L_e} = \frac{E}{L_1} + \frac{E}{L_2}$$

$$\frac{1}{L_e} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$L_e = \frac{L_1 L_2}{L_1 + L_2}.$$

13. The magnetic flux through a coil varies with time as $\phi = 5t^2+6t+9$. The ratio of emf at t = 3s to t = 0s will be

(a) 1 : 9 (b) 1 : 6

(c) 6 : 1 (d) 9 : 1.

Ans. ***(c)***

Solution:

$$\frac{d\phi}{dt} = 10t + 6$$

$$e = -\frac{d\phi}{dt} = -(10t + 6)$$

$$e|_{t=3} = -(10 \times 3 + 6) = -36$$

$$e|_{t=0} = -(10 \times 0 + 6) = -6$$

$$\frac{e_{t=3}}{e_{t=0}} = \frac{-36}{-6} = \frac{6}{1}.$$

14. An air-plane with 20m wing spread is flying at 250 ms^{-1} straight south parallel to the earth's surface. The earth's magnetic field has a horizontal component of $2 \times 10^{-5}\,Wb\,m^{-2}$ and the dip angle is 60º. Calculate the induced emf between the plane tips is:

(a) 0.174 V (b) 0.173 V

(c) 1.173 V (d) 0.163 V.

Ans. ***(b)***

Solution: As the plane is flying horizontally it will cut the vertical component of earth's field B_V. So the emf induced between its tips,

$$e = B_V\, v l$$

But as by difinition of angle of dip,

$$\tan\phi = \frac{B_V}{B_H} \quad \text{i.e.,} \quad B_V = B_H \tan\phi$$

So $e = (B_H \tan\phi) v l = 2 \times 10^{-5} \times \sqrt{3} \times 250 \times 20$

i.e., $e = (\sqrt{3}) \times 10^{-1}\,V = 0.173\,V$.

15. A wire in the form of a circular loop of radius 10 cm lies in a plane normal to a magnetic field of 100 T. If this wire is pulled to take a square shape in the same plane in 0.1 s, average induced emf in the loop is:

(a) 6.70 volt (b) 5.80 volt

(c) 6.75 volt (d) 5.75 volt.

Ans. ***(c)***

Solution: According to Faraday's law of electromagnetic induction,

$$E_{induced} = \frac{-\Delta\phi}{\Delta t} = -\frac{B(A_f - A_i)}{\Delta t}$$

Let r be the radius of circle; then side of square formed $= \frac{2\pi r}{4} = \frac{\pi r}{2}$

Change is area of loop $= A_i - A_f = \pi r^2 - \left(\frac{\pi r}{2}\right)^2 = \frac{\pi(4-\pi)r^2}{4}$

Hence average emf induced $= \frac{\pi(4-\pi)r^2}{4} \cdot \frac{B}{t}$

$$= \frac{\pi(4-\pi) \times (0.1)^2 \times 100}{4 \times 0.1} = 6.75 \text{ volt.}$$