## SOLVED EXAMPLES (SUBJECTIVE)

***Example 1.*** An object of mass 5 kg falls from rest through a vertical distance of 20 m and attains a velocity of 10 m/s. How much work is done by the resistance of the air on the object? (g = 10 m/s²).

***Solution:*** Work done by all forces = change in K.E.

$$W_{air} + W_{gravity} = \Delta K.E.$$

$$W_{air} + mgh = \frac{1}{2}mv^2$$

$$W_{air} = \frac{1}{2}mv^2 - mgh$$

$$= \frac{1}{2}\times 5\times 10\times 10 - 5\times 10\times 20$$

$$W_{air} = -750\ J$$

***Example 2.*** A particle of mass m moves along a straight line on smooth horizontal plane, acted upon by a force delivering a constant power P. If the initial velocity of the particle is zero, then find its displacement as a function of time t.

***Solution:*** Work done in time t = change in K.E.

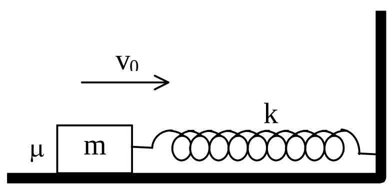
$$P.t = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2P}{m}}\, t^{1/2} \qquad \Rightarrow \qquad \frac{dx}{dt} = \sqrt{\frac{2P}{m}}\, t^{1/2}$$

$$\int_0^x dx = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt$$

$$x = \frac{2}{3}\sqrt{\frac{2P}{m}t^3}$$

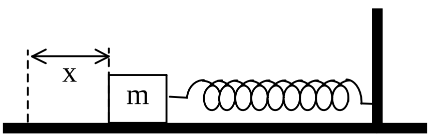
***Example 3.*** A block is projected horizontally on a rough horizontal floor. The coefficient of friction between the block and the floor is $\mu$. The block strikes a light spring of stiffness k with a velocity $v_0$. Find the maximum compression of the spring.




***Solution:*** Since the block slides and the spring is compressed through a distance x the net retarding force acting on it

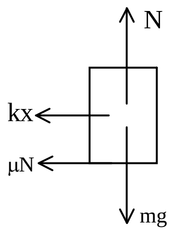
= F = – (kx + $\mu$N) = – ($\mu$ mg + kx)

$\Rightarrow$ Work done by net force for the displacement x, W = ∫F. dx




$$\Rightarrow \qquad W = \int_0^x F\,dx$$

$\Rightarrow \quad \Delta KE = -\int_0^x (\mu mg + kx)\, dx$

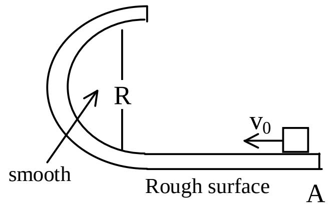



$\Rightarrow \quad \left(0 - \frac{1}{2}mv_0^2\right) = -\left(\mu mgx + \frac{kx^2}{2}\right)$

$\Rightarrow \quad x^2 + \frac{2\mu mg}{k}x - \frac{m}{k}v_0^2 = 0$

$\Rightarrow \quad x = \frac{\frac{-2\mu mg}{k} \pm \sqrt{\frac{4\mu^2 m^2 g^2}{k^2} + \frac{4mv_0^2}{k}}}{2} \quad \Rightarrow \quad x = \frac{\mu mg}{k}\left[\sqrt{1 + \frac{k}{m}\left(\frac{v_0}{\mu g}\right)^2} - 1\right]$

***Example 4.*** A small block is projected with a speed $v_0$ on a horizontal track which turns into a semi circle (vertical) of radius R. Find the minimum value of $V_0$ so that the body will hit the point A after leaving the track at its highest point. The arrangement is shown in the figure, given that the straight part is rough and the curved part is smooth. The coefficient of friction is μ.




***Solution:*** Let the block escape the point at C with a velocity v horizontally. Since it hits the initial spot A after falling through a height 2R we can write $(2R) = (1/2)gt^2$ where t = time of its fall.

$\Rightarrow \quad t = 2\sqrt{R/g}$

$\therefore \quad$ the distance $AB = 2v\sqrt{R/g}$

$\Rightarrow \quad d = 2v\sqrt{R/g} \qquad \ldots\ldots \text{(i)}$

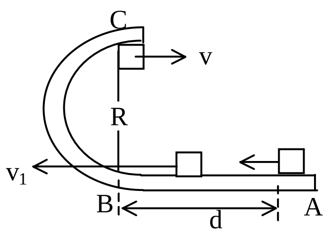
work energy theorem is applied to the motion of the body from A to B leads

$\Delta KE = W_f$

$\Rightarrow \quad \frac{1}{2}mv_0^2 - \frac{1}{2}mv_1^2 = \mu mgd$

$\Rightarrow \quad v_0 = \sqrt{v_1^2 + 2\mu gd} \qquad \ldots\text{(ii)}$

Energy conservation between B and C yields




$\frac{1}{2}mv_1^2 - \frac{1}{2}mv^2 = mg(2R)$

$\Rightarrow \quad v_1 = \sqrt{v^2 + 4gR} \qquad \ldots\text{(iii)}$

when the bead escapes C, its minimum speed v can be given as

$\frac{mv^2}{R} = mg \qquad (\because \text{ the normal contact force} = 0)$

$\Rightarrow \quad v = \sqrt{gR}$ .... (iv)

By using (iii) and (iv) we obtain

$v_1 = \sqrt{5gR}$ ..... (v)

using (i) and (iv) we obtain

$d = \left(\sqrt{gR}\right)2\left(\sqrt{\frac{R}{g}}\right) = 2R$ .....(vi)

Putting the values of $v_1$ and d in (ii) we obtain

$v_0 = \sqrt{5gR + 2\mu g(2R)}$

$\Rightarrow \quad v_0 = \sqrt{(5+4\mu)gR}$

***Example 5.*** Two smooth balls of mass $m_1$ and $m_2$ connected by a light inextensible string are at the opposite points of horizontal diameter of a smooth semi cylindrical surface of radius R. If $m_1$ is released, find its speed at any angular distance θ moved by $m_2$.

***Solution:*** Let the ball $m_2$ moves through an angle θ, the mass m will fall through a distance

$h_1 = R\theta$.

The ball $m_2$ rises through a height $h_2$ as, $h_2 = R \sin\theta$

The change in gravitational potential energy of $m_1$ is

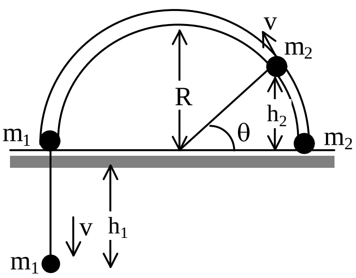
$\Delta PE_1 = -m_1gh_1 = = -m_1 gR\theta$

(since $m_1$ loses its potential energy as it falls down).

The change in gravitational potential energy of $m_2$ is

$\Delta PE_2 = m_2gh_2 = m_2gR\sin\theta$

(Since $m_2$ gains potential energy as it rises up)




$\Rightarrow$ The total change in gravitational potential energy

$= \Delta PE = \Delta PE_1 + \Delta PE_2$

$\Rightarrow \Delta PE = -m_1gR\theta + m_2gR \sin\theta = gR(m_2 \sin\theta - m_1\theta)$ .....(i)

$$= \Delta KE = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 = \frac{(m_1+m_2)v^2}{2}$$ .....(ii)

where v = speed of $m_1$ and $m_2$ at the positions as shown in the figure.

From the principle of conservation of energy we obtain,

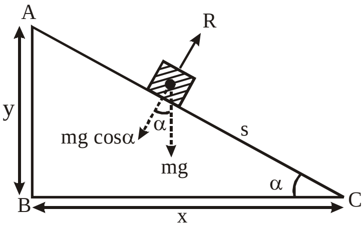
$\Delta KE + \Delta PE = 0$ ..... (iii)

Using (i), (ii) and (iii), we obtain,

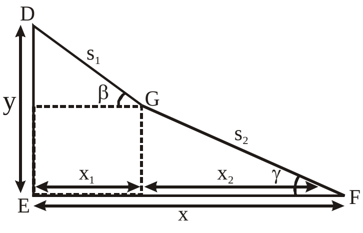
$$\frac{1}{2}(m_1+m_2)v^2 - gR(m_1\theta - m_2 \sin\theta) = 0$$

$$\Rightarrow \quad v = \sqrt{\frac{2gR(m_1\theta - m_2\sin\theta)}{(m_1+m_2)}}.$$

***Example 6:*** In Figure (a) and (b) AC, DG and GF are fixed inclined planes. BC = EF = x and AB = DE = y. A small block of mass m is released from rest from the point A. It's slides down AC and reaches C with a speed $v_C$. The same blocked is released from rest from the point D; it slides down DGF and reaches the point F with speed $v_F$. The coefficient of kinetic friction between the block and the surfaces AC and DGF is $\mu$. Calculate $v_C$ and $v_F$.



(a)



(b)

***Solution:*** (a) ME at A = mgy + 0 and if $v_C$ is the velocity at C,

So ME at C = 0 + (1/2) $mv_C^2$

Loss in ME = mgy – (1/2) $mv_C^2$.

This loss in ME is equal to work done against friction i.e.,

$$mgy - (1/2)mv_C^2 = \mu mg\cos\alpha \times s$$

$$(1/2)mv_C^2 = mgy - \mu mx\,g \qquad [\text{as } \cos\alpha = x/s]$$

or $$v_c = \sqrt{2g(y-\mu x)} \qquad \text{...(i)}$$

(b) In this situation,

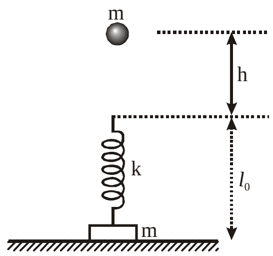
$$mgy - (1/2)mv_F^2 = f_1 s_1 + f_2 s_2$$

or $$(1/2)mv_F^2 = mgy - \mu mg\cos\beta s_1 - \mu mg\cos\gamma\, s_2$$

or $$(1/2)v_F^2 = g[y - \mu x_1 - \mu x_2][\text{as } \cos\beta = (x_1/s_1) \text{ and } \cos\gamma = (x_2/s_2)]$$

or $$v_F = \sqrt{2g(y-\mu x)} = v_C \qquad [\text{as } x_1 + x_2 = x] \qquad \text{...(ii)}$$

***Example 7.*** In the figure shown a massless spring of stiffness k and natural length $l_0$ is rigidly attached to a block of mass 'm' and is in vertical position. A wooden ball of mass m is released from rest to fall under gravity. Having fallen a height h the ball strikes the spring and gets stuck up in the spring at the top. What should be the minimum value of h so that the lower block will just lose contact with the ground later on? Find also the corresponding maximum compression in the spring. Assume that $l_0 >> \frac{4mg}{k}$. Neglect any loss of energy.




***Solution:*** The minimum force needed to lift the lower block is equal to its weight. During upward motion the spring will get elongated. If elongation in the spring for just lifting the block is $x_0$ then

$$kx_0 = mg$$

$$\Rightarrow \qquad x_0 = \frac{mg}{k} \qquad \text{...(i)}$$

From COE

$$mg(l_0 + h) = mg(l_0 + x_0) + \frac{1}{2}kx_0^2$$

$$\Rightarrow \quad mgh = mgx_0 + \frac{1}{2}kx_0^2$$

$$\Rightarrow \quad mgh = \frac{(mg)^2}{k} + \frac{1}{2}\frac{m^2g^2}{k}$$

$$\Rightarrow \quad h = \frac{3mg}{2k}$$

During downward motion, suppose maximum compression in the spring is x. From COE.

$$mg(l_0 + h) = mg(l_0 - x) + \frac{1}{2}kx^2$$

$$\Rightarrow \quad mgh = -mgx + \frac{1}{2}kx^2 \qquad \Rightarrow \quad mg\frac{3mg}{2k} = -mgx + \frac{1}{2}kx^2$$

$$\Rightarrow \quad 3(mg)^2 = -2mg\,kx + k^2x^2$$

$$\Rightarrow \quad k^2x^2 - 2mgkx - 3(mg)^2 = 0$$

$$\Rightarrow \quad x = \frac{2mgk \pm \sqrt{4(mgk)^2 + 12k^2(mg)^2}}{2k^2}$$

$$= \frac{2mgk \pm 4mgk}{2k^2}$$

$$\Rightarrow \quad x = \frac{3mg}{k}.$$

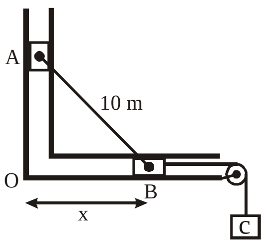
***Example 8.*** Two bodies A and B connected by a light rigid bar 10 m long move in two frictionless guides as shown in the Figure. If B starts from rest when it is vertically below A, find the velocity of B when x = 6m. Assume $m_A = m_B = 200$ kg and $m_c$ = 100kg.

***Solution:*** At the instant, when the bar is as shown in the figure,

$x^2 + y^2 = l^2$

$$\therefore \quad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \qquad \text{...(i)}$$

$$\therefore \quad x\frac{dx}{dt} = -y\frac{dy}{dt} \qquad \text{...(ii)}$$




where $\frac{dx}{dt}$ = velocity of B and $\frac{dy}{dt}$ = velocity of A.

Applying the law of conservation of energy,
loss of potential energy of A, if it is going down when the rod is vertical to the position shown in the Figure

$= m_A g(10 - 8) = 2 \times 200 \times 9.8$

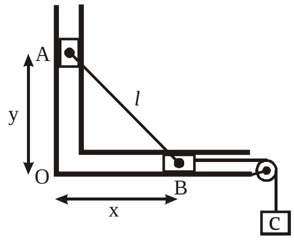
C moves down 6 m since B moves 6 m along x-axis

Loss of potential energy of $C(= mgh) = 100 \times 9.8 \times 6$

Total loss of potential energy = $200 \times 9.8 \times 2 + 100 \times 9.8 \times 6$

$= 100 \times 9.8 \times 10 = 9800$ J.

This must be equal to kinetic energy gained




Kinetic energy gained $=\frac{1}{2}m_A(v_A)^2+\frac{1}{2}m_B(v_B)^2+\frac{1}{2}m(v_C)^2$

$$=\frac{1}{2}\times 200\left(\frac{dy}{dt}\right)^2+\frac{1}{2}\times 200\left(\frac{dx}{dt}\right)^2+\frac{1}{2}\times 100\left(\frac{dx}{dt}\right)^2$$

$$=100\left(\frac{dy}{dt}\right)^2+150\left(\frac{dx}{dt}\right)^2 \qquad =100\left[\frac{x}{y}\frac{dx}{dt}\right]^2+150\left(\frac{dx}{dt}\right)^2$$

from (ii)

$$=100\left[\frac{6}{8}\frac{dx}{dt}\right]^2+150\left(\frac{dx}{dt}\right)^2 \qquad =\left[100\times\frac{9}{16}+150\right]\left(\frac{dx}{dt}\right)^2 \qquad =\frac{3300}{16}v_B^{\ 2}$$

$\therefore$ $\qquad \frac{3300}{16}v_B^2=9800$

$\therefore$ $\qquad v_B=\sqrt{\frac{98\times 16}{33}}=7\times 4\sqrt{\frac{2}{33}}=6.9\,ms^{-1}$

$\therefore$ $\qquad$ velocity of B at the required moment = 6.9 $ms^{-1}$.

***Example 9.*** A locomotive of mass m starts moving so that its velocity varies according to the law $v=\alpha\sqrt{s}$, where $\alpha$ is constant and s is the distance covered. Find the total work done by all the forces acting on the locomotive during the first t seconds after the beginning of motion.

***Solution:*** Given v = $\alpha\sqrt{s}$

Differentiating w.r.t. ‘t’, we get

$$\frac{dv}{dt}=\frac{1}{2}\alpha s^{-1/2}\frac{ds}{dt}=\frac{\alpha}{2\sqrt{s}}v$$

$$=\frac{\alpha}{2\sqrt{s}}\times\alpha\sqrt{s}=\frac{\alpha^2}{2}$$

$\therefore$ $\qquad$ acceleration, $a=\frac{\alpha^2}{2}$

Now force acting on the Locomotive,

$$F=ma=m\frac{\alpha^2}{2}.$$

Here u = 0

Now, using $s=ut+\frac{1}{2}at^2$, we have

$$s=0+\frac{1}{2}\frac{\alpha^2}{2}t^2=\frac{\alpha^2t^2}{4}$$

$\therefore$ Work done, $W = Fs = m\frac{\alpha^2}{2} \times \frac{\alpha^2 t^2}{4}$

$$= \frac{m\alpha^4 t^2}{8}.$$

***Example 10.*** The kinetic energy of a particle moving along a circle of radius R depends on the distance covered S as T $= \alpha S^2$, where $\alpha$ is constant. Find the force acting on the particle as a function of S.

***Solution:*** K.E., T = $\alpha S^2$ or $\frac{1}{2}mv^2 = \alpha S^2$

$$\therefore \quad v^2 = \frac{2\alpha S^2}{m} \qquad \text{...(i)}$$

Differentiating both sides w.r.t. 't', we have

$$2v\frac{dv}{dt} = \frac{4\alpha S}{m}\frac{dS}{dt} = \frac{4\alpha S}{m}v \qquad \left[\because \frac{dS}{dt} = v\right]$$

or $\frac{dv}{dt} = \frac{2\alpha S}{m} = a_t$

(Tangential component of acceleration)

Now, centripetal acceleration is

$$a_c = \frac{v^2}{R} = \frac{2\alpha S^2}{mR} \qquad \text{[Using equation (i)]}$$

$\therefore$ Net acceleration of the particle is given by

$$a = \sqrt{a_t^2 + a_c^2}$$

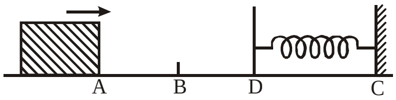
$$= \sqrt{\left(\frac{2\alpha S}{m}\right)^2 + \left(\frac{2\alpha S^2}{mR}\right)^2} = \frac{2\alpha S}{m}\sqrt{1 + \left(\frac{S}{R}\right)^2}$$

Now, force acting on the particle is given by,

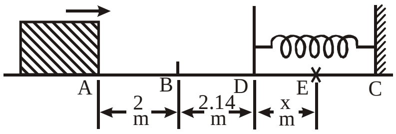
$$F = ma = m \times \frac{2\alpha S}{m}\sqrt{1 + \frac{S^2}{R^2}}$$

or $F = 2\alpha S\sqrt{1 + \frac{S^2}{R^2}}$.

***Example 11.*** A 0.5 kg block slides from the point A (see figure) on a horizontal track with an initial speed of 3m/s towards a weightless horizontal spring of length 1 m and force constant 2 Newton/m. The part AB of the track is frictionless and the part BC has the coefficients of static and kinetic friction as 0.22 and 0.2 respectively. If the distances AB and BD are 2 m and 2.14 m respectively find the total distance through which the block moves before it comes to rest completely (Take g = 10 m/s$^2$).




***Solution:*** Suppose the block comes to rest at the point E i.e. let DE = x. The kinetic energy of the block is spent in overcoming friction and compressing the spring through a distance DE = x.




Kinetic energy of the block

$$=\frac{1}{2}mv^2$$

$$=\frac{1}{2}\times 0.5\times 3^2 = 2.25\text{J} \qquad ...(i)$$

As the part AB of the track is frictionless, work done in moving from A to B is zero

Let normal reaction of the block = mg

Coefficient of friction = $\mu$

$\therefore$ Force due to friction along the track BC = $\mu$mg

$= 0.2\times 0.5\times 10 = 1$ N

Distance through which the block moves against the frictional force

= 2.14 + x m

$\therefore$ Work done by block against friction before it comes to rest

$= \mu\ mg(2.14 + x)$

= ( 2.14 + x )J ...(ii)

Let the spring constant = k

$\therefore$ Work done by the block in compressing the spring through distance x

$$=\frac{1}{2}kx^2$$

$$=\frac{1}{2}\times 2\times x^2 = x^2 \text{ J} \qquad ...(iii)$$

Adding (ii) and (iii) and equating it to (i), we get

$2.14 + x + x^2 = 2.25$

or $x^2 + x - 0.11 = 0$

or $100x^2 + 100x - 11 = 0$

or $(10x + 11)(10x - 1) = 0$

$\therefore$ $x = -\frac{11}{10}$ or $x = \frac{1}{10}$

Since $x \neq -\frac{11}{10}$,

$\therefore$ $x = \frac{1}{10} = 0.1$ m

Restoring force of the spring = kx

$= 2 \times 0.1 = 0.2$ N ...(iv)

Static frictional force of the block

$\mu_{static}\, mg = 0.22 \times 0.5 \times 10$

= 1.1 N ...(v)

From (iv) and (v) it is clear that the static frictional force is grater than the restoring force of the spring. Therefore the block will not move in the backward direction. Hence the total distance through which the block moves before it comes to rest completely 2.00 + 2.14 + 0.10 = 4.24 metres.

***Example 12.*** In a spring gun having spring constant 100 N/m, a small ball of mass 0.1 kg is put in its barrel by compressing the spring through 0.05 m as shown in the figure.

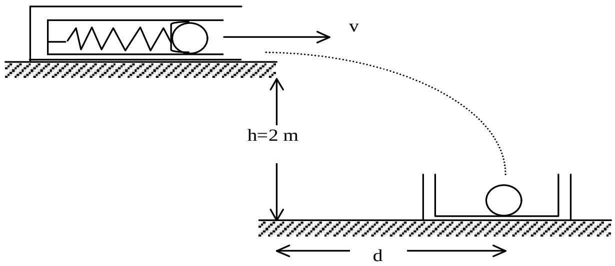
(a) Find the velocity of the ball when spring is released.

(b) Where should a box be placed on ground so that ball falls in it, if the ball leaves the gun horizontally at a height of 2 m above the ground ($g = 10\ m/s^2$)

***Solution:*** (a) When the spring is released its elastic potential energy is converted into kinetic energy

$\frac{1}{2}mv^2$, so $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$\Rightarrow \quad v = \sqrt{\frac{5}{2}}$ m/s.




(b) As vertical component of velocity of ball is zero. Time taken by the ball to reach the ground, $h = \frac{1}{2}gt^2$

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2}{5}}$ seconds

So, the horizontal distance travelled by the ball in this time

$d = v\,.\,t = \sqrt{\frac{5}{2}} \times \sqrt{\frac{2}{5}} = 1$ m

**Example 13.** A smooth, light horizontal rod AB can rotate about a vertical axis passing through its end A. The rod is fitted with a small sleeve of mass m attached to the end A by a weightless spring of length $\ell_0$ and stiffness k. What works must be performed to slowly get this system going and reach the angular velocity ω.

***Solution:*** The mass m rotates in a circle of radius $\ell$, which is the extended length of the spring.

Centripital force on $m = k(\ell - \ell_0) = m\,\omega^2\,\ell$

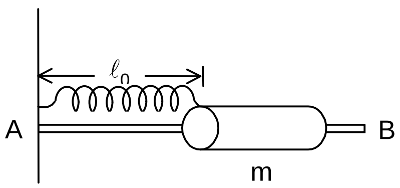
or, $\ell = \frac{\ell_0}{1-n}$ where $n = \frac{m\omega^2}{k}$

$\therefore$ W = change in KE of m + energy stored in the spring

$= \frac{1}{2}m\omega^2\, l^2 + \frac{1}{2}k(\ell - \ell_0)^2$

$$= \frac{1}{2} m \frac{l_0^2 \omega^2}{(1-n)^2} + \frac{1}{2} k \left[ \frac{l_0}{1-n} - l_0 \right]^2$$

$$W = \frac{1}{2} \frac{k\ell^2}{(1-n)^2} \left[ \frac{m\omega^2}{k} + n^2 \right]$$




***Example 14.*** A particle is suspended by a string of length '*l*'. It is projected with such a velocity v along the horizontal such that after the string becomes slack it flies through its initial position. Find v.

***Solution:*** Let the velocity be v′ at B where the string become slack and the string makes angle θ with horizontal by the law of conservation of energy.

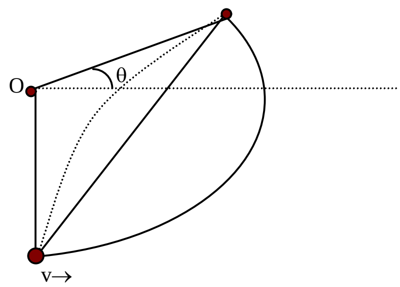
$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + mg\ell(1+\sin\theta) \qquad \text{...(i)}$$

or, $v'^2 = v^2 - 2g\ell\,(1+\sin\theta)$ ...(ii)

By the dynamics of circular motion

$$mg\sin\theta = \frac{mv'^2}{\ell}$$

$\Rightarrow$ $v'^2 = g\ell\sin\theta$ ...(iii)




from equation (ii) and (iii) we get

$\therefore$ $g\,\ell\sin\theta = v^2 - 2g\ell\,(1+\sin\theta)$ ...(iv)

At B the particle becomes a projectile of velocity v′ at 90 - θ with the horizontal.

Here, $u_x = v'\sin\theta$ & $u_y = v'\cos\theta$

$a_x = 0$ & $a_y = -g$

$\therefore$ $\ell\cos\theta = v'\sin\theta\, t$ ...(v)

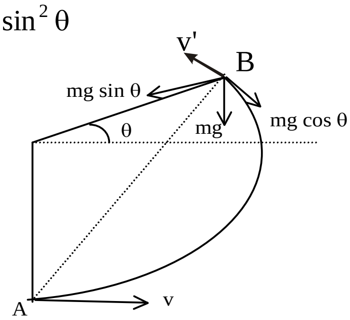
$$\therefore\ t = \frac{\ell\cos\theta}{v'\sin\theta} \quad \& -\ell\,(1+\sin\theta) = v'\cos\theta\,\frac{l\cos\theta}{v'\sin\theta} - \frac{1}{2}g\frac{l^2\cos^2\theta}{v'^2\sin^2\theta}$$

$\Rightarrow$ $2\sin^3\theta + 3\sin^2\theta - 1 = 0$

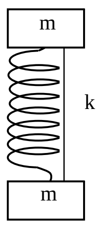
$\therefore$ $\sin\theta = \frac{1}{2}$ is the acceptable solution

$$\therefore\ v^2 = 2g\ell + 3g\ell \times \frac{1}{2} = \frac{7g\ell}{2} \Rightarrow v = \sqrt{\frac{7g\ell}{2}}$$

(from equation (iv))

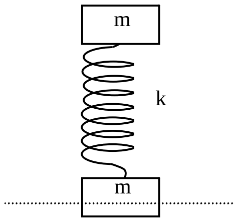



***Example 15.*** A system consists of two identical cubes each of mass m linked together by a compressed massless spring of stiffness 'k'. the cubes are connected by a thread which is burnt at a certain moment. At what value of initial compression 'ε' of the spring will the lower cube bounce up after the thread is burnt.



***Solution :*** Let us take horizontal line through the C.G. of lower cube as reference level.

Let '$\ell$' be the natural length of the spring.




$$T.E._{initial} = \frac{1}{2}k\varepsilon^2 + mg(\ell - \varepsilon) \qquad \ldots(i)$$

Let the spring extend by 'x' after the thread is burnt

$$T.E._{final} = \frac{1}{2}kx^2 + mg(\ell + x)$$

$\Rightarrow$ kx = -kε or kε – 2mg (-kε not acceptable)

$\therefore$ kx = kε – 2mg.

Cube will bounce up when $kx \geq mg$

$\Rightarrow$ $K\varepsilon - 2mg \geq mg$

$\Rightarrow K\varepsilon \geq 3mg \Rightarrow \varepsilon = 3mg / k$.