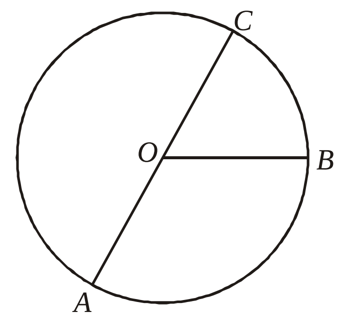
Lesson-7
AREAS RELATED TO CIRCLES

Introduction

A circle is a plane figure bounded by one line (*ABC*) such that the distance of this line from a fixed point within it (point *O*), remains constant throughout



That is $OB = OC = OA =$ constant.

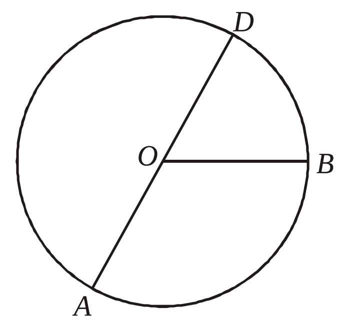
This line is known as the **circumference** of a circle (*C*).

The fixed point (*O*) is the **centre** of a circle.

The constant distance from the centre to the circumference is known as the **radius** of the circle. ($OA = Ob = OC =$ radius) (*r*)

Any line drawn through the centre and terminated both ways by the circumference is called the **diameter** of a circle ($AC =$ diameter)

Circumference and Area



Circumference $\rightarrow C \rightarrow$ the perimeter of the circle.
Area $\rightarrow A \rightarrow$ the space enclosed by the circle.

On-line memory:

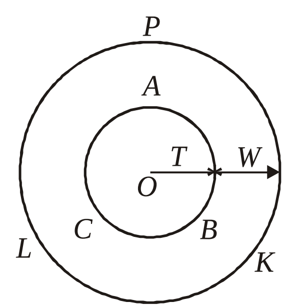
Measurement for	***Formula***
Circumference (*C*)	$C = 2\pi r = \pi d$
Area (*A*)	$A = \pi r^2$

Correlation formula:

$$A = \frac{C^2}{4\pi}$$

Circular Pathway

Let *ABC* be a circle whose radius = *r*.



Outside Pathway

1. There is a pathway *PLK* outside this circle *ABC* whose width = *W*

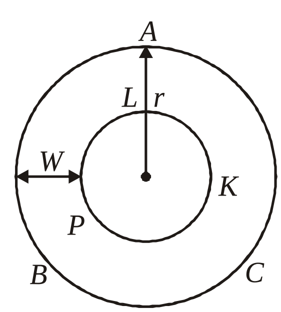
   Area of circular pathway $= \pi \times W(2r + W)$

   (outside)

2. There is a pathway inside the circle *ABC* Width of pathway = *W*

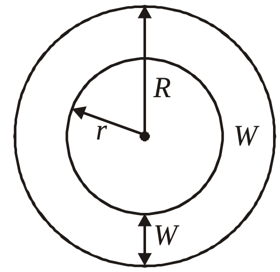
   Area of circular pathway $= \pi \times W(2r - W)$

   (inside)



Inside Pathway

Corollary: If R and r denote the radii of the outer and inner circles respectively, which bound a plane circular ring, it is evident that



$R - r = W$ = width of the ring

Area of ring $= \pi R^2 - \pi r^2$

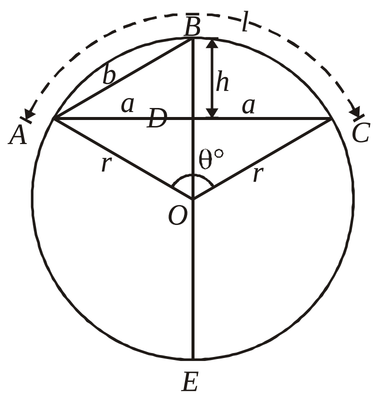
$= \pi(R^2 - r^2)$

$= \pi(R + r)(R - r)$

$= \pi W(R + r)$ [since $W = R - r$]

{If we put the value of $R(= r + W)$, the formula for circular pathway is obtained. In fact area of ring = area of circular pathway}

Chords and Arcs



$ABC \to$ any arc $= l$ (say)

$AC \to$ chord of the arc $= 2a$

$BD \to$ height of the arc $= h$

$BE \to$ diameter of circle $= d$

$AD = CD$ = half of chord $= a$

Length (arc AB) = half arc = 1/2

$AB \to$ chord of the half-arc $= b$

$\angle AOC \to$ central angle by arc $ABC = \theta°$

Arc $= l$ = any part of the circumference of a circle.

Smaller path is minor arc and larger part is major arc.

chord of the Arc $= 2a$ = the line segment which joins the two ends of the arc.

Height of the Arc $= h$ = the length of the perpendicular drawn from the middle point of an arc upon the chord.

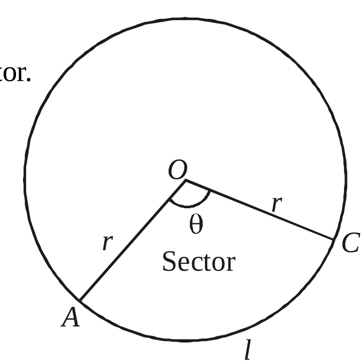
Chord of the Half-Arc $= b$ = the length of the chord of the half of an arc.

Sectors and segments

(a) Sector is a figure bounded by two radii and the arc intercepted between them

Area of sector, A = space enclosed by the sector of a circle.

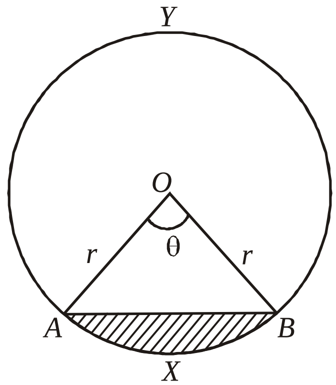
Central angle, θ = angle contained between the two radii of the sector.



Sector area $A = \dfrac{\theta}{360} \times$ area of circle.

Length of the Arc $(l) = \left(\dfrac{\theta}{360}\right) \times$ circumference of circle

(b) Segment is a figure bounded by a chord and the corresponding arc.



If the arc is minor, then it is minor segment (*AXB*) otherwise the segment is a major segment (*AYB*)

AXB = minor segment

AYB = major segment

Area of circle = Area of (minor + major segments)

Central angle = θ°

Area of minor segment = Area of sector – area of Δ*AOB*

Area of major segment = Area of sector – area of Δ*AOB*

SOLVED EXAMPLES

Ex.1: Find the area and the circumference of a circle whose radius is 3.5 m.

Sol.: Using the formula

Area (A) $= \pi r^2$

$= \frac{22}{7} \times (3.5)^2 \text{ m}^2$

$= 38.5 \text{ m}^2$

Circumference (C) $= 2\pi r$

$= 2 \times \frac{22}{7} \times 3.5 \text{ m}$

$= 22 \text{ m}$

Ex.2: How many revolutions will a wheel make in travelling 528 m if its diameter measures 0.7 m.

Sol.: Using the formula,

Circumference of wheel $= \pi d$

where $d = 0.7$ m

$\Rightarrow \quad C = \frac{22}{7} \times 0.7$

= distance covered in one revolution $\frac{\text{Total distance travelled by wheel}}{\text{Circumference of wheel}}$

= No. of revolutions made by wheel

$\Rightarrow \quad \frac{528}{\frac{22}{7} \times 0.7}$ = No. of revolutions

$\Rightarrow \quad 240$ = No. of revolution.

Hence, no. of revolutions the wheel makes is 240.

Ex.3: The diameter of driving wheel of a bus is 1.4 m. How many revolutions per minute (rpm) must the wheel make in order to keep a speed of 66 km/hour.

Sol.: Using the formula,

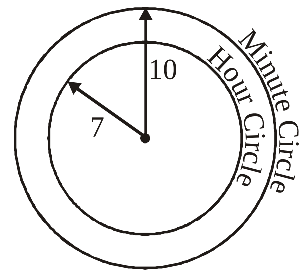
No. of revolutions per minute $= \frac{\text{Total distance travelled per minute}}{\text{Circumference of wheel } (C)}$

$= \frac{\frac{66 \times 100}{60}}{\frac{22}{7} \times 1.4}$

$= \frac{110.0}{4.4} = 250.$

Hence, the wheel makes 250 revolutions/minute.

Ex.4: The hands of a clock are 10 cm and 7 cm respectively. Find the difference between the distance traversed by their extremities in 3 days 5 hours.



Sol.: Total time = 3 days 5 hours = 77 hours.

radius of minute hand = r_M = 10 cm

radius of hour hand = r_H = 7 cm

Total distance traversed by the extremity of hour hand

= Circumference × No. of revolution

$H = 2\pi r_H \times \frac{77}{12}$

$H = 2 \times \frac{22}{7} \times 7 \times \frac{77}{12}$

= 282.33 cm

Total distance traversed by extremity of minute hand

= Circumference × No. of revolutions

$M = 2\pi \times r_M \times 77$

$M = 2 \times \frac{22}{7} \times 10 \times 77$

M = 4840 cm

The required difference = $M - H$

= 4840 – 282.33

= 4557.67 cm

Ex.5: Assuming the circumference of a circle to be $3\frac{1}{7}$ times the diameter, find the circumference of the circle whose area is 1386 m^2.

Sol.: Using the correlation formula

$A = \frac{C^2}{4\pi}$

where A = 1386 m^2

$\Rightarrow \quad 1386 = \frac{C^2}{4\pi}$

$\Rightarrow \quad C^2 = 4 \times \frac{22}{7} \times 1386$

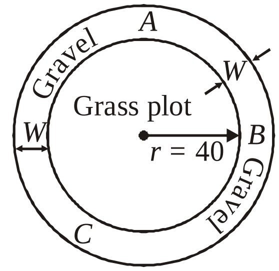
[Since $C = 3\frac{1}{7} \times d = \pi d \Rightarrow \pi = \frac{22}{7}$]

$\Rightarrow \quad C$ = 132 m

Hence, the circumference of the circle is 132 m

Ex.6: A circular grass plot 40 m. in radius is surrounded by a ring of gravel. Find the width of the gravel so that the area of the grass and gravel may be equal?

Sol.: The circular grass plot *ABC* has a ring of gravel *cutside* it.



Radius of grass plot = r = 40 m

Width of gravel = W = ?

It has been given that

area of grass = area of gravel.

$\Rightarrow \quad \pi r^2 = \pi W(2r + W)$

$\Rightarrow \quad W^2 + 2rW - r^2 = 0$

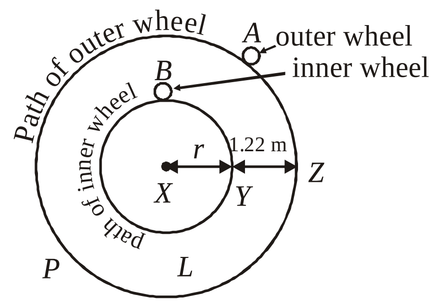
$\Rightarrow \quad W^2 + 80\,W - 1600 =$[Since r = 40]

$$\Rightarrow \quad W = \frac{-80 \pm \sqrt{80^2 + 4 \times 1600}}{2}$$

= 16.57 m (considering the +ve value)

Hence, the width of the gravel is 16.57 m

Ex.7: A two-wheeled carriage, whose axle tree is 1.22 m long, is driven around a circle; the outer wheel makes $1\frac{1}{2}$ revolutions for every 1 revolutions of inner wheel. The wheels are each 0.9 m high. Find the circumference of the circle described by the outer wheel.



Sol.: Let *A* and *B* be two wheels - outer and inner respectively.

Diameter of the wheels = height of the wheels

= 0.9 m (given)

Distance between two wheels = length of a axle tree

= *YZ* = 1.22 m

Now, two wheels on turning round a path make two circles - inner (*BYL*) and outer (*AZP*).

If the radius of inner circle = r m

the radius of outer circle = r + 1.22 m

Since, the outer wheel makes $1\frac{1}{2}$ revolution for every 1 revolutions of inner wheel,

$$\frac{\text{Circumference of outer circle}}{\text{Circumference of inner circle}} = \frac{1\frac{1}{2}}{1} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{2\pi(r + 1.22)}{2\pi r} = \frac{3}{2}$$

$\Rightarrow \quad r = 2.44$

So, the circumference of the circle described by outer wheel

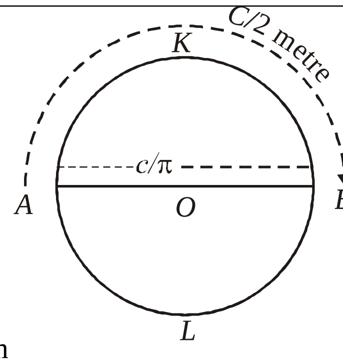
$= 2\pi\,(r + 1.22)$

$= 2 \times \frac{22}{7} \times 3.66$

= 22.98

= 23 m

Ex.8: A man by walking diametrically across a circular grass plot, finds that it has taken him 45 sec. less than if he had kept to the path round the outside. If he walks 90 metres per minute, find the circumference of the grass plot.



Sol.: Let $AKBL$ be the circular grass plot whose circumference = C m

(i) point k (*i.e.,* along circumference) travelling a distance = $AKB = \frac{C}{2}$ metre = S_1

and point O (*i.e.*, along diameter) travelling a distance = $AOB = \frac{C}{\pi}$ metre

(since circumference = π × diameter)

= S_2 (say).

Walking speed of the person

= V = 90 metre/min.

$= \frac{90}{60}$ metre/sec.

Using the formula distance = speed × time

$(S_1 - S_2) = V(t_1 - t_2)$

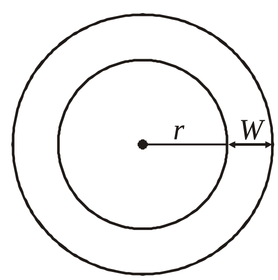
$\Rightarrow \left(\frac{C}{2} - \frac{C}{\pi}\right) = \frac{90}{60} \times 45$ [Since $t_1 - t_2$ = 45 sec.(given)]

$\Rightarrow C \times \frac{2}{11} = \frac{135}{2}$

$\Rightarrow C = 371.25$ cm

Hence, the circumference is 371.25

Ex.9: The areas of two concentric circles are 154 m^2 and 308 m^2 respectively, find the breadth of the ring.



Sol.: Let the radius of inner circle = r m

$$r = \sqrt{\frac{154}{\pi}} = 7 \text{ m.}$$

Another circle is outside this circle.

If the width of the ring = W

Area of ring = $\pi W = (2r + W)$

$\Rightarrow$ Outer circle – Inner circle = $\pi W(2 \times 7 + W)$

$\Rightarrow 308 - 154 = \pi W(14 + W)$

$\Rightarrow 154 = \frac{22}{7} \times W(14 + W)$

$W = 2.89$ m.

Ex.10: The circumferences of two concentric circles are 62.832 m and 37.6992 m. Find the area between the circles.

Sol.: Let the circumference of outer circle C_1 = 62.832 m

circumference of inner circle C_2 = 37.6992 m

The area between two circles = $A_1 - A_2$ = ?

Here, using the correlation formula for circumference and area.

$$A = \frac{C^2}{4\pi}$$

$$\Rightarrow \quad A_1 - A_2 = \frac{C_1^2}{4\pi} - \frac{C_2^2}{4\pi}$$

$$\Rightarrow \quad A_1 - A_2 = \frac{C_1^2 - C_2^2}{4\pi}$$

$$\Rightarrow \quad A_1 - A_2 = \frac{(C_1 + C_2)(C_1 - C_2)}{4\pi}$$

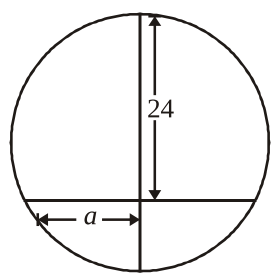
$$\Rightarrow \quad A_1 - A_2 = \frac{(62.832 + 37.6992)\,(62.832 - 37.6992)}{4 \times 3.1416}$$

$$\Rightarrow \quad A_1 - A_2 = \frac{100.5 \times 25.13}{12.5664}$$

$$\Rightarrow \quad A_1 - A_2 = 201 \text{ m}^2$$

Hence, the area between the circles is 201 m^2.

Ex.11: Find the chord of an arc whose height is 24 m, in a circle of radius 15 m



Sol.: Here,

the height of an arc = h = 24 m (known)

the diameter of circle = d = $2r$

=2 × 15 = 30 m

the chord of the arc = $2a$ = ?

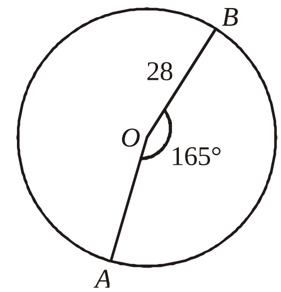
Using the formula,

the chord of the arc = $2\sqrt{h(d-h)}$

$$\Rightarrow \quad 2a = 2\sqrt{24(30-24)}$$

$$\Rightarrow \quad 2a = 24$$

Ex.12: In a circle of radius 28 m, find the area of a sector whose angle measures 165°.



Sol.: Here, the radius of circle $= r = 28$ m

sector angle $= \theta° = 165°$

sector area $= A = ?$

Using the formula

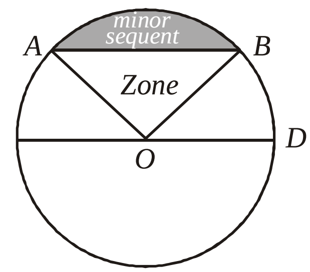
$$A = \left(\frac{\theta}{360}\right) \times \pi r^2$$

$$\Rightarrow \quad A = \frac{165}{360} \times \frac{22}{7}\ (28)^2$$

$$\Rightarrow \quad A = 1129.33 \text{ m}^2$$

Here, the required area of a sector is 1129.33 m^2.

Ex.13: The radius of a circle is 75 cm. A zone of that circle has one of its parallel chords coinciding with the diameter and the other equal to the radius. Find the area of the zone.



Sol.: Let *AB* and *CD* be two parallel chords of the circle.

Then AB = radius of circle $= r = 75$ cm

CD = diameter of circle = 150 cm

If *O* is centre of circle, then

$OA = OB = AB = 75$ and *OAB* is an equilateral Δ, so, using the formula

Area of minor segment $= 0.09\ r^2$ (central angle = 60°)

$= .09 \times (75)^2 = 509.54 \text{ cm}^2$

Required area between *AB* and *CD* = Area of semicircle *CAD* – Area of minor segment

$$= \frac{\pi r^2}{2} - 509.54$$

$$= \frac{22}{7} \times \frac{75 \times 75}{2} - 509.54$$

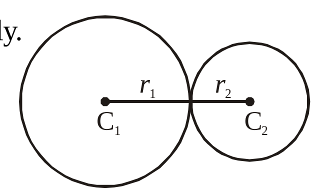
$$= 8326 \text{ cm}^2$$

Hence, the area of the required, zone = 8326 cm^2

Ex.14: Two circles touch externally. The sum of their areas is 130π sq. cm. and the distance between their centres is 14 cm. Find the radii of the circles.

Sol.: If two circles touch externally, then the distance between their centres is equal to the sum of their radii.

Let the radii of the two circles be r_1 cm and r_2 cm respectively.



Let C_1 and C_2 be the centres of the given circles. Then,

$$C_1C_2 = r_1 + r_2$$

$\Rightarrow \quad 14 = r_1 + r_2$ [$\because$ C_1C_2 = 14 cm (given)]

$\Rightarrow \quad r_1 + r_2 = 14$ ... (i)

It is given that the sum of the areas of two circles is equal to 130p cm2.

$\therefore \quad \pi r_1^2 + \pi r_2^2 = 130\pi$

$\Rightarrow \quad r_1^2 + r_2^2 = 130$ ...(ii)

Now, $(r_1+r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$

$\Rightarrow \quad 14^2 = 130 + 2r_1r_2$

$\Rightarrow \quad 196 - 130 = 2r_1r_2$ ...(iii)

$\Rightarrow \quad r_1r_2 = 33$

Now,

$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$

$\Rightarrow \quad (r_1 - r_2)^2 = 130 - 2 \times 33$

$\Rightarrow \quad (r_1 - r_2)^2 = 64$ ...(iv)

$\Rightarrow \quad r_1 - r_2 = 8$

Solving (i) and (iv), we get r_1 = 11 cm and r_2 = 3 cm.

Hence, the radii of the two circles are 11 cm and 3 cm.

Ex.15: Find the area of the shaded region in figure where ABCD is a square of side 10 cm. (use: π = 3.14)

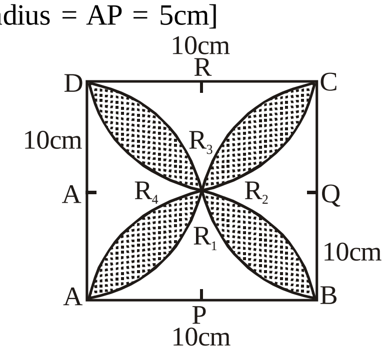
Sol.: Let us mark the four unshaded regions as R_1, R_2, R_3 and R_4.

We have,

Area of R_1 + Area of R_3

= Area of square ABCD – Area of two semi-circles having centres at Q and S.

$= \left(10 \times 10 - 2 \times \frac{1}{2} \times 3.14 \times 5^2\right) \text{cm}^2$ [$\because$ Radius = AP = 5cm]



$= (100 - 3.14 \times 25)\text{cm}^2 = (100 - 78.5)\ \text{cm}^2 = 21.5\ \text{cm}^2$

Similarly, we have

Area of R_2 + Area of R_4 = 21.5 cm^2

$\therefore$ Area of the shaded region

= Area of square ABCD – (Area of R_1 + Area of R_2 + Area of R_3 + Area of R_4)

$= (100 - 2 \times 21.5)\ \text{cm}^2 = 57\ \text{cm}^2$.

BASIC LEVEL ASSIGNMENT

1. Find the area and the circumference of a circle whose diameter is 14 m.

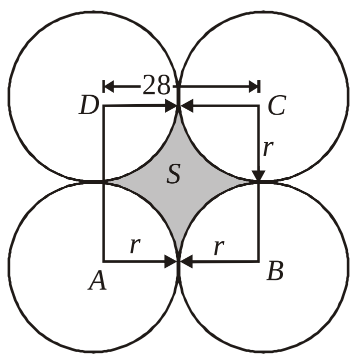
2. A piece of wire is bent in the shape of an equilateral triangle of each side 6.6 m. reshaped circular ring. Find the diameter of the ring.

3. The area of a circle is equal to the area of a square. Compare their perimeters.

4. Two men, A and B, purchase a grindstone 30 cm in diameter for Rs. 12, of which A pays Rs. 7, and B pays Rs. 5. Now supposing the inner most 10 cm of the diameter as useless, how many centimeters of the radius may A grind down before sending the grindstone to B?

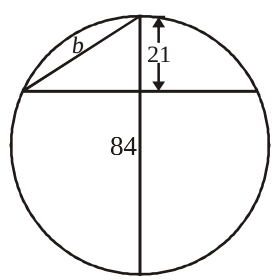
5. A circular grass plot, whose diameter is 70 m, contains a gravel walk 5 m wide round it, 15 m from the edge. Find what it will cost to turf the grass plot at Rs. 2 per m^2.

6. Four equal circles are described about the four corners of a square so that each touches two of the others. Find the area of the space enclosed between the circumferences of the circles; each side of the square measuring 28 m.

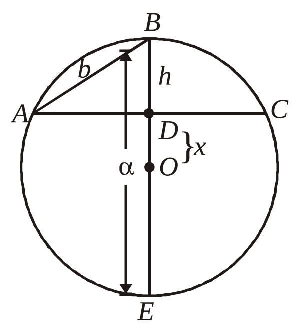


7. The height of an arc is 9 cm, and the chord of the arc is 0.3 m. Find the diameter of the circle.

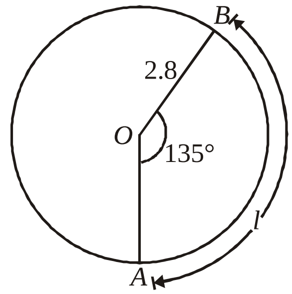
8. The height of an arc is 21 m and the diameter of the circle is 84 m. Find the chord of half the arc.



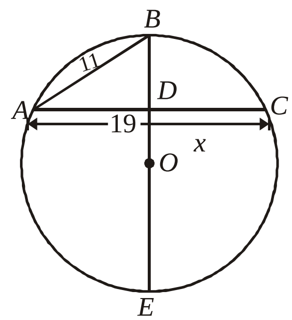
9. The height of an arc is 27 m and the chord of half the arc is 63 m. Find the distance of the chord of the arc from the centre of the circle.



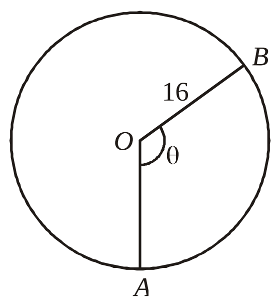
10. The radius of the circle is 2.8 m. Find the length of an arc which subtends an angle of 135° at the centre.



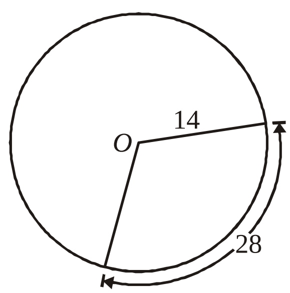
11. The chord of an arc in 19 m and the chord of half the arc is 11 m. Find the length of the arc.



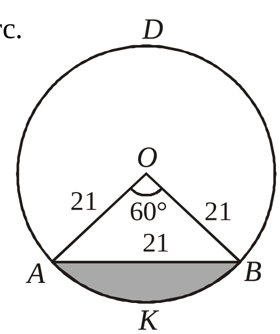
12. The area of a sector is 80 m^2, the radius of the circle is 16 m. Find the angle of the sector



13. Find the area of a sector of a circle, whose radius is 14 cm, and the length of the arc of the sector is 28 cm.

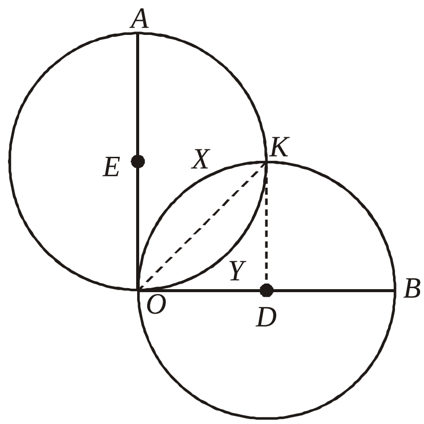


14. In a circle of radius of 21 cm, an arc subtends an angle of 60° at the centre. Find the area of the minor and major segment made by this arc.

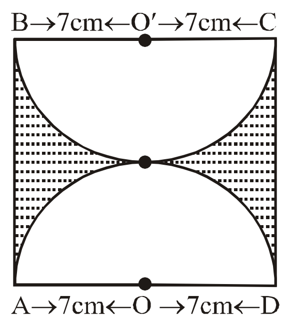


15. A 36° sector of a circle has area of 3.85 cm^2. What is the length of the arc of the sector?

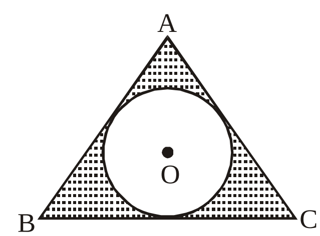
16. If two circles be described on the bounding radii of a quadrant of a circle whose radius is 10 m, as diameters, find the area of the figure common to both the circles.



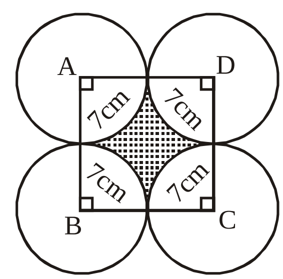
17. Find the area of the shaded region in figure, if ABCD is a square of side 14 cm and APD and BPC are semi-circles.



18. A circle is inscribed in an equilateral triangle ABC is side 12 cm. touching its sides. Find the radius of the inscribed circle and the area of the shaded part.

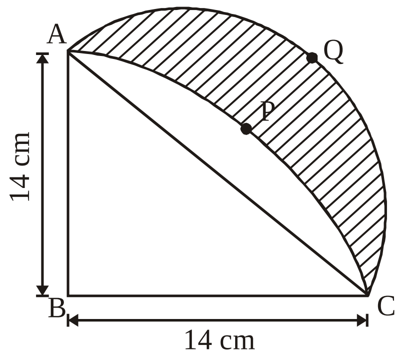


19. Four equal circles are described about the four corners of a square so that each touches two of the others as shown in figure. Find the area of the shaded region, each side of the square measuring 14cm.



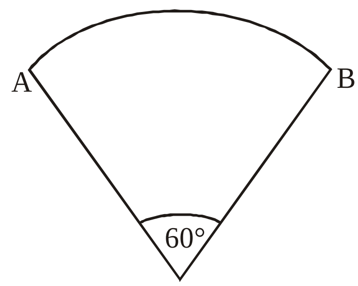
20. The diagram shows two arcs, A and B. Arc A is part of the circle with centre O and radius OP. Arc B is part of the circle with centre M and radius PM, where M is the mid-point of PQ. Show that the area enclosed by the two arcs is equal to $25\left(\sqrt{3}-\frac{\pi}{6}\right)$ cm^2.

21. ABCP is a quadrant of a circle of radis 14 cm. With AC as diameter, a semicircle is drawn. Find the area of the shaded portier.



22. The minute hand of a clock is $\sqrt{21}$ cm long. Find the area described by the minute hand on the face of the clock between 7.0 AM and 7.05 AM.

23. If the adjoining figure is a sector of a circle of radius 10.5 cm. What is the perimeter of the sector? [Take $\pi = 22/7$]



ANSWERS

Basic Level Assignment

1.	44 m	**2.**	6.3 m	**3.**	$\frac{\sqrt{\pi}}{2}$	**4.**	4.59 cm	**5.**	Rs. 6600
6.	168 m^2	**7.**	34 cm	**8.**	42 m	**9.**	46.5 m	**10.**	6.6 m
11.	23 m	**12.**	35.8°	**13.**	196 m^2	**14.**	40 cm^2, 1346 cm^2		
15.	2.2 cm	**16.**	14.26 m^2	**21.**	98 cm^2	**22.**	5.5 cm^2	**23.**	32 cm