SOLVED EXAMPLES

Example 1.

A piston of cross sectional area 100 cm^2 is used in a hydraulic press to exert a force of 10^7 dynes on the water. What is the cross sectional area of the other piston which supports a truck having a mass of 2000 kg?

(1) 18000 cm^2 (2) 18265 cm^2 (3) 19600 cm^2 (4) 18500 cm^2

Solution :

(3) Pressure on the first piston = $\frac{10^7}{100} = 10^5$ dynes/cm^2

Let A be the cross-sectional area of the other piston. Then the pressure on this piston

$$= \frac{2000 \times 1000 \times 980}{A}$$

Since the two pressures are equal, $\frac{2000 \times 1000 \times 980}{A} = 10^5$

or A = 19600 cm^2

Example 2.

Calculate the velocity of efflux of kerosene oil from an orifice of a tank in which pressure is 4 atmosphere. The density of kerosene oil = 720 kg m^{-3} and 1 atmospheric pressure = 1.013 × 10^5 N m^{-2}.

(1) 33.55 m/s (2) 42.42 m/s (3) 54.62 m/s (4) 60.84 m/s

Solution :

(1) Here, P = 4 atm = 4 × 1.013 × 10^5 = 4.052 × 10^5 Nm^{-2}

Density of kerosene oil, ρ = 720 kg m^{-3}

Let h be the depth of the orifice below the free surface of oil in the tank. Then,

$$P = h\rho g \quad \text{or} \quad h = \frac{P}{\rho g}$$

or $h = \frac{4.052 \times 10^5}{720 \times 9.8}$ = 57.43 m

Now, velocity of efflux, $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 57.43} = 33.55$ ms^{-1}

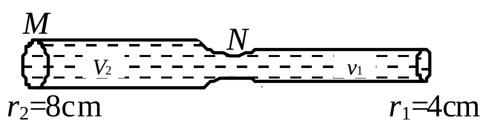
Example 3.

A horizontal pipe having a construction is shown in the figure. The radius at M and N are respectively 8 cm and 4 cm. Calculate the velocity of N if velocity of water at M is 16 cm/sec.

(1) 16 cm/s (2) 32 cm/s (3) 64 cm/s (4) 128 cm/s

Solution :

(3) We have $A_1 v_1 = A_2 v_2 \Rightarrow \pi r_1^2 v_1 = \pi r_2^2 v_2$ $v_1 = \left(\frac{r_2}{r_1}\right)^2 v_2 =$ 64 cm/sec

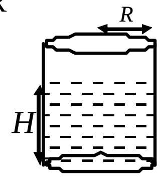


Example 4.	In the above question calculate the pressure at N, if pressure at M is 10^6 dyne/cm^2. (1) 9.76×10^5 dyne/cm^2 (2) 2×10^5 dyne/cm^2 (3) 8.54 dyne/cm^2 (4) 7.26 dyne/cm^2
Solution :	**(1)** From Bernaulli's theorem $P_2 + \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2$ $P_2 - P_1 = \frac{1}{2}\rho\left[v_1^2 - v_2^2\right] = \frac{1}{2}\rho[64-16] = 24\times10^3$ $\therefore \quad P_1 = P_2 - 24\times10^3 = 9.76\times10^5$ dyne/cm^2
Example 5.	A spherical ball of mass 2 gm and having radius 0.6 cm takes 5 seconds to fall steadily through a height of 40 cm inside a long column of liquid of density 1.2. Calculate the coefficient of viscosity of the liquid. (1) 4.22 poise (2) 8.98 poise (3) 5.88 poise (4) 9.65 poise
Solution :	**(3)** Here density of the ball $\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi r^3} = \frac{2\times3}{4\times\frac{22}{7}\times(0.6)^3} = 2.2$ gm/cc Now, $\eta = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{v} = \frac{2}{9}\frac{r^2(\rho-\sigma)gt}{s}$ $[s = vt]$ $= \frac{2}{9}\frac{(0.6)^2(2.2-1.2)980\times5}{40}$ = 5.88 poise
Example 6.	Two equal drops of water each of radius r are falling through air with a steady velocity of 16 cm/sec. The two drops combined to form a big drop. Calculate the terminal velocity of the big drop. (1) 32 cm/s (2) 64 cm/s (3) $2^{1/3} \times 16$ cm/s (4) $2^{2/3} \times 16$ cm/s
Solution :	**(4)** Since volume remains constant, hence $\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3$ where n = number of small drops, r = radius of small drop and R = radius of big drop $\therefore \quad R = n^{1/3}r$ If v_1 and v_2 are the terminal velocities of the small and big drops respectively, then $\frac{v_2}{v_1} = \left(\frac{R}{r}\right)^2 = n^{2/3} \quad \therefore \quad v_2 = n^{2/3}v_1 = 2^{2/3}\times16$ cm/sec

Example 7.

There is cylindrical vessel of radius R. A homogeneous liquid is filled in it. What should be the height of the liquid in the vessel so that the force exerted by the liquid on the sides of the vessel is equal to the force exerted on the bottom of the vessel.

(1) R/2 (2) R (3) 2R (4) 4R



Solution :

(2) Let H be the height of the liquid in the vessel,

then weight of the liquid $=\pi R^2 H\rho g$

Therefore, force on the bottom $=\pi R^2 H\rho g$

Now, area of the wall of the cylinder which is in contact of the liquid

$= 2\pi RH$

Total force on the wall = $\frac{1}{2}\rho gH \times 2\pi RH$

If the two forces are equal then,

$\pi\rho gRH^2 = \pi R^2 H\rho g$ $\therefore$ H = R

Example 8.

A uniform U-tube is kept vertical and it contains mercury in both arms. Now glycerine (density 1.3 gm/cc) and an oil (density 0.8 gm/cc) is poured into the two arms separately till the upper surface of glycerine and oil are at same level. If the glycerine column is of height 16 cm, then calculate the height of oil column [density of mercury = 13.6 gm/cc]

(1) 10.25 cm (2) 20.42 cm (3) 15.38 cm (4) 12.94 cm

Solution :

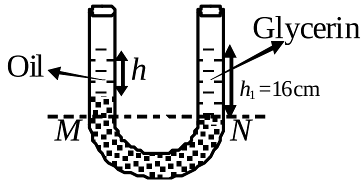
(3) Pressure at M = Pressure due to oil column + pressure due to Hg of height (16–h) = $\rho_o gh + \rho_{Hg} g\left(h_1 - h\right)$

Pressure at N = Pressure of glycerine of column of height 16 cm = $\rho_{gl} gh_1$

Since pressure at M = pressure at N

$\rho_o gh + \rho_{Hg} g\left(h_1 - h\right) = \rho_{gl} gh_1$

$\therefore$ $h = \frac{\left(\rho_{Hg} - \rho_{gl}\right)}{\rho_{Hg} - \rho_o} = 15.38$ cm



Example 9.

The base of a bottle has radius 4 cm and the radius of its neck is 2 cm. A force of 16 N is applied on the cork when the bottle is filled by a liquid, then calculate the force on the base.

(1) 16 N (2) 32 N (3) 64 N (4) 12.94 cm

Solution :

(3) Pressure at the neck = $\frac{F}{\pi r^2} = \frac{16}{\pi\left(0.02\right)^2}$

And pressure on the base, if force F_1 acts on it = $\frac{F_1}{\pi r_1^2} = \frac{F_1}{\pi\left(0.04\right)^2}$

Since pressure is everywhere same, hence

$\frac{F_1}{\pi\left(0.04\right)^2} = \frac{16}{\pi\left(0.02\right)^2}$ $\therefore$ $F_1 = \frac{16\times\left(0.04\right)^2}{\left(0.02\right)^2} = 64\,\text{N}$

MULTIPLE CHOICE QUESTIONS

LEVEL – I

1. If two liquids of same masses but densities ρ_1 and ρ_2 respectively are mixed, then density of mixture is given by

(1) $\rho = \frac{\rho_1 + \rho_2}{2}$ (2) $\rho = \frac{\rho_1 + \rho_2}{2\rho_1\rho_2}$

(3) $\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$ (4) $\rho = \frac{\rho_1\rho_2}{\rho_1 + \rho_2}$

2. The height to which a cylindrical vessel be filled with a homogeneous liquid, to make the average force with which the liquid presses the side of the vessel equal to the force exerted by the liquid on the bottom of the vessel, is equal to

(1) half of the radius of the vessel

(2) radius of the vessel

(3) one-fourth of the radius of the vessel

(4) three-fourth of the radius of the vessel

3. With rise in temperature, density of a given body changing according to one of the following relations

(1) $\rho = \rho_o[1 + \gamma d\theta]$ (2) $\rho = \rho_o[1 - \gamma d\theta]$

(3) $\rho = \rho_o \gamma d\theta$ (4) $\rho = \rho_o / \gamma d\theta$

4. A metallic sphere floats in an immiscible mixture of water ($\rho_w = 10^3$ kg/m^3) and a liquid ($\rho_L = 13.5 \times 10^3$ kg/m^3) such that its 4/5th portion is in water and 1/5th portion in the liquid. The density of metal is

(1) 4.5×10^3 kg/m^3 (2) 4.0×10^3 kg/m^3

(3) 3.5×10^3 kg/m^3 (4) 1.9×10^3 kg/m^3

5. An iceberg is floating partly immersed in sea water, the density of sea water is 1.03 g/cm^3 and that of ice is 0.92 g/cm^3. The fraction of the total volume of the iceberg above the level of sea water is

(1) 8.1% (2) 11%

(3) 34% (4) 0.8%

6. A silver ingot weighing 2.1 kg is held by a string so as to be completely immersed in a liquid of relative density 0.8. The relative density of silver is 10.5. The tension in the string in kg-wt is

(1) 1.6 (2) 1.94

(3) 3.1 (4) 5.25

7. A sphere of solid material of relative density 9 has a concentric spherical cavity and floats completely immersed in water. If the radius of the sphere be R, then the radius of the cavity (r) will be related to R as

(1) $r^3 = \frac{8}{9}R^3$ (2) $r^3 = \frac{2}{3}R^3$

(3) $r^3 = \frac{\sqrt{8}}{3}R^3$ (4) $r^3 = \sqrt{\frac{2}{3}}R^3$

8. The reading of a spring balance when a block is suspended from it in air is 60 N. This reading is changed to 40 N when the block is submerged in water. The specific gravity of the block must be therefore

(1) 3 (2) 2

(3) 6 (4) 3/2

9. An open vessel containing water is given a constant acceleration a in the horizontal direction. Then the free surface of water gets sloped with the horizontal at an angle θ given by

(1) $\theta = \tan^{-1}\left(\frac{a}{g}\right)$ (2) $\theta = \tan^{-1}\left(\frac{g}{a}\right)$

(3) $\theta = \sin^{-1}\left(\frac{a}{g}\right)$ (4) $\theta = \cos^{-1}\left(\frac{g}{a}\right)$

10. A U-tube is partially filled with water. Oil which does not mix with water is next poured into one side, until water rises by 25 cm on the other side. If the relative density of oil is 0.8, the oil level will stand higher than the water level by

(1) 6.25 cm (2) 12.50 cm

(3) 31.75 cm (4) 62.50 cm

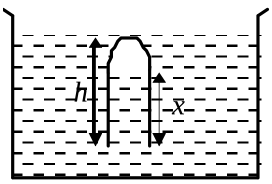
11. A tube of length h (which is wide enough so that surface tension effects can be neglected) filled with air and closed at one end is lowered in a tank of mercury to a depth h as shown, so that mercury rises a distance x into the tube. If the mercury barometer also reads h, then

(1) $h(h - x) = h^2$

(2) $(2h - x)(h - x) = h^2$

(3) $(2h - x)(h + x) = h^2$

(4) $2h(h - x) = h^2$



12. Pressure at the bottom of tank of water is $3P$, where P is atmospheric pressure. If the water is drawn out till the level of water is lowered by one fifth, then the pressure at the bottom of the tank is

(1) $2P$ (2) $13P/5$
(3) $8P/5$ (4) $4P/5$

13. A cylinder is filled with non-viscous liquid of density d to a height h_o and a hole is made at a height h_1 from the bottom of the cylinder. The velocity of liquid issuing out of the hole is

(1) $\sqrt{2gh_o}$ (2) $\sqrt{2g\left(h_o - h_1\right)}$
(3) $\sqrt{dgh_1}$ (4) $\sqrt{dgh_o}$

14. With increase in temperature the viscosity of

(1) both gases and liquids increases
(2) both gases and liquids decreases
(3) gases increases and liquids decreases
(4) gases decreases and of liquids increases

15. If V_1 and V_2 be the volumes of the liquids flowing out of the same tube in the same interval of time, η_1 and η_2 their coefficients of viscosity respectively, then

(1) $\frac{\eta_1}{\eta_2} = \frac{V_2}{V_1}$ (2) $\frac{\eta_1}{\eta_2} = \frac{V_1}{V_2}$
(3) $\frac{\eta_1}{\eta_2} = \frac{V_1^2}{V_2^2}$ (4) $\frac{\eta_1}{\eta_2} = \frac{V_2^2}{V_1^2}$

16. Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. the retarding viscous force acting on the spherical ball is

(1) directly proportional to R but inversely proportional to v
(2) directly proportional to both radius R and velocity v
(3) inversely proportional to both radius R and velocity v
(4) inversely proportional to R but directly proportional to velocity v

17. Two tubes A and B are in series. Radius of A is R and that of B is $2R$. If water flows through A with velocity v then velocity of water through B is

(1) $v/2$ (2) v
(3) $v/4$ (4) $v/8$

18. Two rain drops falling through air have radii in the ratio 1 : 2. They will have terminal velocity in the ratio

(1) 4 : 1 (2) 1 : 4
(3) 2 : 1 (4) 1 : 2

19. A water barrel stands on a table of height h. If a small hole is punched in the side of the barrel at its base, it is found that the resultant stream of water strikes the ground at a horizontal distance R from the table. What is the depth of water in the barrel?

(1) $\frac{R^2}{h}$ (2) $\frac{R^2}{2h}$
(3) $\frac{R^2}{4h}$ (4) $\frac{4R^2}{h}$

20. Water flows steadily through a horizontal pipe of variable cross-section. If the pressure of water is P at a point where flow speed is v, the pressure at another point where the flow speed is $2v$, is (Take density of water as ρ)

(1) $P - \frac{3\rho v^2}{2}$ (2) $P - \frac{\rho v^2}{2}$
(3) $P - \frac{3\rho v^2}{4}$ (4) $P - \rho v^2$

LEVEL – II

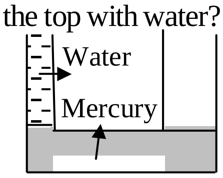
1. A wooden cube just floats inside water, when a 200 g mass is placed on it. When the mass is removed the cube is 2 cm above the water level. The size of the cube is

(1) 5 cm (2) 10 cm
(3) 15 cm (4) 20 cm

2. The fraction of a floating object of volume V_o and density d_o above the surface of a liquid of density d will be

(1) $\frac{d_o}{d}$ (2) $\frac{dd_o}{d + d_o}$
(3) $\frac{d - d_o}{d}$ (4) $\frac{dd_o}{d - d_o}$

3. A U-tube in which the cross-sectional area of the limb on left is one quarter, the limb on the right contains mercury (density 13.6 g/cm^3). The level of mercury in the narrow limb is at a distance of 36 cm from the upper end of the tube. What will be the rise in the level of mercury in the right limb if the left limb is filled to the top with water?



(1) 1.2 cm
(2) 2.35 cm
(3) 1.32 cm
(4) 0.8 cm

4. A sphere of solid material of specific gravity 8 has a concentric spherical cavity and just sinks in water. Then the ratio of radius of cavity to that of outer radius of the sphere must be

(1) $\frac{(3)^{1/3}}{2}$ (2) $\frac{(5)^{1/3}}{2}$

(3) $\frac{(7)^{1/3}}{2}$ (4) $\frac{(9)^{1/3}}{2}$

5. A stream-lined body falls through air from a height h on the surface of a liquid. Let d and D denote the densities of the materials of the body and the liquid respectively. If $D > d$, then the time after which the body will be instantaneously at rest, is

(1) $\sqrt{\frac{2h}{g}}$ (2) $\sqrt{\frac{2h}{g}\frac{D}{d}}$

(3) $\sqrt{\frac{2h}{g}\frac{d}{D}}$ (4) $\sqrt{\frac{2h}{g}}\left(\frac{d}{D-d}\right)$

6. A U-tube containing a liquid is accelerated horizontally with a constant acceleration a. If the separation between the two vertical limbs is l, then the difference in the heights of the liquid in the two arms is

(1) zero (2) l

(3) $\frac{la}{g}$ (4) $\frac{lg}{a}$

7. A wooden block is floating in a water tank. The block is pressed to its bottom. During the process, work done is equal to

(1) work done against upthrust exerted by the water

(2) work done against upthrust plus loss of gravitational potential energy of the block

(3) work done against upthrust minus lost of gravitational potential energy of the block

(4) none of the above

8. A wooden block is floating in a liquid. 50% of its volume is inside the liquid when the vessel is stationary. Percentage of volume immersed when the vessel moves upwards with an acceleration $a = \frac{g}{2}$ is

(1) 75% (2) 25%

(3) 50% (4) 33.33%

9. Mercury is poured in a U-tube. Temperature of one side is 50°C and the level of mercury on this side is h_1. Temperature of the outer side is 100°C and the level of mercury on this side is h_2. Then

(1) $h_1 = h_2$ (2) $h_2 < h_1$

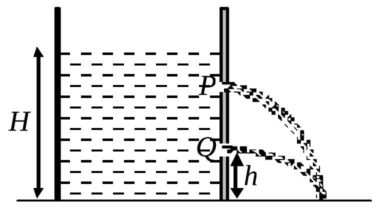
(3) $h_2 > h_1$ (4) $h_2 = 2h_1$

10. A ball floats on the surface of water in a container exposed to the atmosphere. Volume V_1 of its volume is inside the water. If the container is now covered and the air pumped out, let V_2 be the volume now immersed in water. Then

(1) $V_1 = V_2$ (2) $V_1 > V_2$

(3) $V_2 > V_1$ (4) $V_2 = 0$

11. As shown in the following figure, water squirts horizontally out of two small holes in the side of the cylinder and the two streams strike the ground at the same point. If the hole Q is at a height h above the ground and the level of water stands at height H above the ground, then the height of P above ground level is



(1) $2h$ (2) H/h

(3) $H - h$ (4) $H/2$

12. A vessel of area of cross-section A has liquid to a height H. There is a hole at the bottom of vessel having area of cross-section a. The time taken to decrease the level from H_1 to H_2 will be

(1) $\frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H_1}-\sqrt{H_2}\right]$

(2) $\sqrt{2gh}$

(3) $\sqrt{2gh(H_1-H_2)}$

(4) $\frac{A}{a}\sqrt{\frac{g}{2}}\left[\sqrt{H_1}-\sqrt{H_2}\right]$

13. A hole is in the bottom of the tank having water. If total pressure at the bottom is 3 atm (1 atm = 10^5 Nm^{-2}), then velocity of water flowing from hole is

(1) $\sqrt{400}$ ms^{-1} (2) $\sqrt{600}$ ms^{-1}

(3) $\sqrt{60}$ ms^{-1} (4) none of these

14. A wide vessel with a small hole in the bottom is filled with water and kerosene. Neglecting viscosity, the velocity of water flow *v* if the thickness of water layer is h_1 and that of kerosene layer is h_2 is (density of water is ρ_1 g/cc and of kerosene is ρ_2 gm/cc)

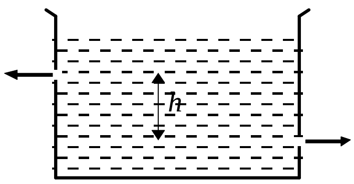
(1) $v = \sqrt{2g(h_1 + h_2)}$

(2) $v = \sqrt{2g\left(h_1 + h_2\frac{\rho_2}{\rho_1}\right)}$

(3) $v = \sqrt{2g(h_1\rho_1 + h_2\rho_2)}$

(4) $v = \sqrt{2g\left(h_1\frac{\rho_1}{\rho_2} + h_2\right)}$

15. There are two identical small holes of area of cross-section *a* on the opposite sides of a tank containing a liquid of density ρ. The difference in height between the holes is *h*. The tank is resting on a smooth horizontal surface. Horizontal force which will have to be applied on the tank to keep it in equilibrium is



(1) $gh\rho a$ (2) $\frac{2gh}{\rho a}$

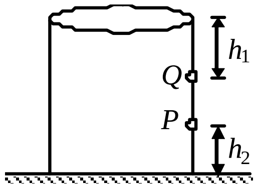
(3) $2\rho agh$ (4) $\frac{\rho gh}{a}$

16. A light cylinder vessel is kept on a horizontal surface. Its base area is *A*. A hole of cross-sectional area *a* is made just at its bottom side. The minimum coefficient of friction necessary for sliding of the vessel due to the impact force of the emerging liquid is ($a << A$)

(1) varying (2) a/A

(3) $2a/A$ (4) none of these

17. In a cylindrical water tank there are two small holes *Q* and *P* on the wall at a depth h_1 from the upper level of water and at a height of h_2 from the lower end of the tank respectively as shown in the figure. Water coming out from both the holes strikes the ground at the same point. The ratio of h_1 and h_2 is



(1) 1

(2) 2

(3) > 1

(4) < 1

18. If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of a sphere of silver (density = 10.5 kg/m^3) on the same size in the same liquid

(1) 0.2 m/s (2) 0.4 m/s

(3) 0.133 m/s (4) 0.1 m/s

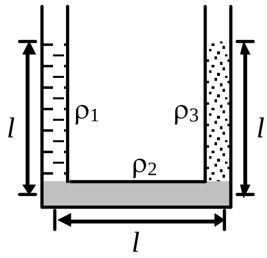
19. A body of density D_1 and mass *M* is moving downwards in glycerine of density D_2 with constant velocity. What is the viscous force acting on it?

(1) $Mg\left(1 - \frac{D_2}{D_1}\right)$ (2) $Mg\left(1 - \frac{D_1}{D_2}\right)$

(3) MgD_1 (4) MgD_2

LEVEL – III

1. Three liquids having densities ρ_1, ρ_2 and ρ_3 are filled in a U-tube. Length of each liquid column is equal to l. $\rho_1 > \rho_2 > \rho_3$ and liquids remain at rest (relative to the tube) in the position shown in figure. It is possible that



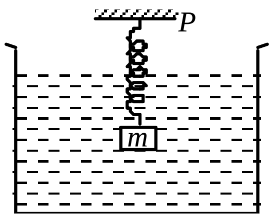
(1) U-tube is accelerating leftwards

(2) U-tube is accelerating upwards with acceleration g

(3) U-tube is moving with a constant velocity

(4) none of the above

2. A cube of mass m and density D is suspended from the point P by a spring of stiffness K. The system is kept inside a beaker filled with a liquid of density d. The elongation in the spring, assuming $D > d$, is



(1) $\frac{mg}{K}\left(1-\frac{d}{D}\right)$ (2) $\frac{mg}{K}\left(1-\frac{D}{d}\right)$

(3) $\frac{mg}{K}\left(1+\frac{d}{D}\right)$ (4) none of these

3. A vessel of water is placed on the floor of an elevator. How does the pressure at the bottom of the vessel change if the elevator moves up with uniform acceleration a?

(1) Increases by $h\rho a$

(2) decreases by $h\rho a$

(3) No change in pressure

(4) None of these

4. A solids sphere of volume v and density ρ floats at the interface of two immiscible liquid of densities ρ_1 and ρ_2 respectively. If $\rho_1 < \rho < \rho_2$, the ratio of volume of the parts of the sphere in upper and lower liquid is

(1) $\frac{\rho-\rho_2}{\rho_2-\rho}$ (2) $\frac{\rho_2-\rho}{\rho-\rho_1}$

(3) $\frac{\rho+\rho_1}{\rho+\rho_2}$ (4) $\frac{\rho+\rho_2}{\rho+\rho_1}$

5. A body weighs m_1 in a fluid of density d_1 and m_2 in a fluid of density d_2. What would be the weight in a fluid of density d_3?

(1) $\frac{m_1(d_3-d_1)-m_2(d_2-d_3)}{(d_2-d_1)}$

(2) $\frac{m_1(d_2-d_3)-m_1(d_1-d_3)}{(d_2-d_1)}$

(3) $\frac{m_2(d_3-d_1)-m_1(d_3-d_2)}{(d_2-d_1)}$

(4) $\frac{m_1(d_2-d_3)+m_2(d_3-d_1)}{(d_2+d_1)}$

6. A rectangular vessel when full of water takes 10 minutes to be emptied through an orifice in its bottom. How much time will it take to be emptied when half filled with water?

(1) 9 minute (2) 7 minute

(3) 5 minute (4) 3 minute

7. If A denotes the area of free surface of a liquid and h the depth of an orifice of area of cross-section a, below the liquid surface, then the velocity v of flow through the orifice is given by

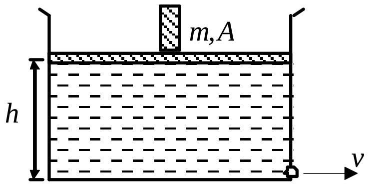
(1) $v=\sqrt{(2gh)}$

(2) $v=\sqrt{(2gh)}\sqrt{\left(\frac{A^2}{A^2-a^2}\right)}$

(3) $v=\sqrt{(2gh)}\sqrt{\left(\frac{A}{A-a}\right)}$

(4) $v=\sqrt{(2gh)}\sqrt{\left(\frac{A^2-a^2}{A^2}\right)}$

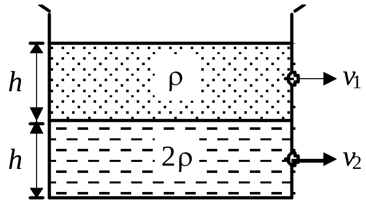
8. A cylindrical vessel contains a liquid of density ρ upto a height h. The liquid is closed by a piston of mass m and area of cross-section A. There is a small hole at the bottom of the vessel. The speed v with which the liquid comes out of the hole is



(1) $\sqrt{(2gh)}$ (2) $\sqrt{2\left(gh+\frac{mg}{\rho A}\right)}$

(3) $\sqrt{2\left(gh+\frac{mg}{A}\right)}$ (4) $\sqrt{2gh+\frac{mg}{A}}$

9. Equal volumes of two immiscible liquids of densities ρ and 2ρ are filled in a vessel as shown in the figure. Two small holes are punched at depth $h/2$ and $3h/2$ from the surface of lighter liquid. If v_1 and v_2 are the velocities of efflux at these two holes, then v_1/v_2 is



(1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{2}$

(3) $\frac{1}{4}$ (4) $\frac{1}{\sqrt{2}}$

10. A large tank filled with water to a height of h is said to be emptied through a small hole at the bottom. The ratio of time taken for the level of water to fall down from h to $h/2$ and from $h/2$ to zero is

(1) $\sqrt{2}$ (2) $\frac{1}{\sqrt{2}}$

(3) $\sqrt{2}-1$ (4) $\frac{1}{\sqrt{2}-1}$

ANSWERS (HYDRODYNAMICS)

LEVEL - I

1. (3)	5. (2)	9. (1)	13. (2)	17. (3)
2. (2)	6. (2)	10. (2)	14. (3)	18. (2)
3. (2)	7. (1)	11. (2)	15. (1)	19. (3)
4. (3)	8. (1)	12. (2)	16. (2)	20. (1)

LEVEL - II

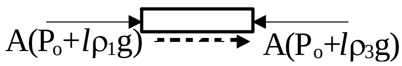
1. (2)	5. (4)	9. (3)	13. (1)	17. (1)
2. (3)	6. (3)	10. (3)	14. (2)	18. (4)
3. (3)	7. (3)	11. (3)	15. (3)	19. (1)
4. (3)	8. (3)	12. (1)	16. (3)	

LEVEL - III

1. (4)	3. (1)	5. (3)	7. (2)	9. (4)
2. (1)	4. (2)	6. (2)	8. (2)	10. (3)

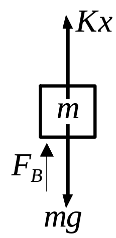
SOLUTIONS (LEVEL - III)

1. Since both the vertical limbs of U-tube are open to atmosphere, therefore, at surfaces of liquids the pressure is equal to atmospheric pressure. Since heights of the liquids in the two vertical limbs are equal and the liquids have different densities, pressure exerted by them will be different from each other. The liquids can remain in static equilibrium relative to the tube only, when the system is accelerating down with acceleration g. In that case liquids experience weightlessness.



Hence, at every point of tube, the pressure will become equal to atmospheric pressure. But if the tube is not accelerating down under gravity then heavier liquid will exert more pressure at bottom. Hence, at bottom of left vertical limb, pressure will greater than that at bottom of right vertical limb. It means on horizontal limb there will be a resultant horizontal force which will be towards right as shown in figure. Therefore, liquid in horizontal limb will have a rightward acceleration. In fact, this whole system must have a horizontally rightward acceleration. Obviously, only option (4) is correct.

2. The cube is in equilibrium under the following three forces:



(a) Spring force Kx, where x = elongation of the spring

(b) Gravitational force W = Weight of the cube $= mg$

(c) Buoyant force F_B (or upward thrust) imparted by the liquid on the cube given as $F_B = Vdg$

Where V = volume of immersed portion of the cube.

For complete immersion V = volume of cube

For equilibrium of the cube,

$$Kx + F_B = mg$$

$$\therefore \quad x = \frac{mg - F_B}{K} = \frac{mg - Vdg}{K}$$

where $V = m/D$

$$\therefore \quad x = \frac{mg}{K}\left[1 - \frac{d}{D}\right]$$

3. Considering the upward motion of a column of liquid of depth h

$$PA - P_oA - Ah\rho g = Ah\rho a$$

$$\therefore \quad P = P_o + h\rho g(g + a)$$

When the elevator is at rest

$$P_R = P_o + h\rho g$$

$$\therefore \quad \Delta P = P - P_R = h\rho a$$

Thus, the pressure at the bottom increases.

4. Let V = Volume of solid sphere

V_1 = Volume of the part of the sphere immersed in a liquid of density ρ_1

V_2 = Volume of the part of the sphere immersed in a liquid of density ρ_2

Hence, according to law of floatation

$$V\rho g = V_1\rho_1 g + V_2\rho_2 g \qquad (1)$$

and $V = V1 + V2$ (2)

Hence, $V_1\rho g + V_2\rho g = V_1\rho_1 g + V_2\rho_2 g$

or $V_1(\rho - \rho_1)g = V_2(\rho_2 - \rho)g$

or $\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}$

5. Let m be the mass and ρ the density of solid.

Volume = (m/ρ)

$$m_1 g = mg - \frac{m}{\rho}d_1 g \quad \text{... (1)}$$

$$m_2 g = mg - \frac{m}{\rho}d_2 g \quad \text{... (2)}$$

$$m_3 g = mg - \frac{m}{\rho}d_3 g \quad \text{... (3)}$$

Equation (2) – (1),

$$\rho = \frac{d_2 - d_1}{(m_1 - m_2)}m$$

Equation (2) – (3),

$$\rho = \frac{d_3 - d_2}{(m_2 - m_3)}m$$

$$\therefore \quad \frac{d_2 - d_1}{(m_1 - m_2)}m = \frac{d_3 - d_2}{(m_2 - m_3)}m$$

or

$$(d_2 - d_1)m_2 - (d_2 - d_1)m_3 = (m_1 - m_2)(d_3 - d_2)$$

$$m_3 = \frac{(d_2 - d_1)m_2 - (d_3 - d_2)(m_1 - m_2)}{(d_2 - d_1)}$$

$$= \frac{m_2(d_3 - d_1) - m_1(d_3 - d_2)}{(d_2 - d_1)}$$

6. $t_1 = 10 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H}\right]$

$$t_2 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{\frac{H}{2}}\right] = 10 \times \left[\frac{1}{\sqrt{2}}\right]$$

= 10 × 0.7 = 7 min

7. Applying Bernoulli's theorem, we have

$$\frac{P}{\rho} + \frac{1}{2}(v')^2 + gh = \frac{P}{\rho} + \frac{1}{2}v^2 + 0$$

where v' is the velocity of all surfaces of liquid and v the velocity of efflux.

Further, from continuity equation,

$$Av' = av \qquad \text{or} \qquad v' = \frac{av}{A}$$

$$\therefore \quad \frac{1}{2}\left(\frac{av}{A}\right)^2 + gh = \frac{1}{2}v^2$$

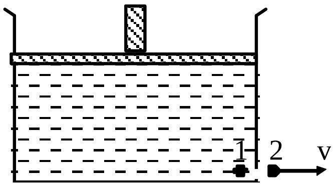
$$\Rightarrow \quad 2gh = v^2\left(1 - \frac{a^2}{A^2}\right)$$

$$v = \sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}}$$

8. Applying Bernoulli's theorem at points 1 and 2, Difference in pressure energy between 1 and 2 = difference in kinetic energy between 1 and 2.

or $\rho hg + \frac{mg}{A} = \frac{1}{2}\rho v^2$

or $v = \sqrt{2gh + \frac{2mg}{\rho A}} = \sqrt{2\left(gh + \frac{mg}{\rho A}\right)}$



9. $v_1 = \sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{gh}$ .... (1)

From Bernoulli's theorem

$$\rho gh + 2\rho g\left(\frac{h}{2}\right) = \frac{1}{2}(2\rho)v_2^2$$

$\therefore \quad v_2 = \sqrt{2gh}$ ... (2)

$\therefore \quad \frac{v_1}{v_2} = \frac{1}{\sqrt{2}}$

10. If R be the radius of tank and r that of the hole, then

$$-\pi R^2 \frac{dh}{dt} = \pi r^2 \sqrt{2gh}$$

or $-\frac{dh}{\sqrt{h}} = \frac{r^2}{R^2}\sqrt{2g}\, dt$

$\therefore$ Required ratio $\frac{t_1}{t_2} = \frac{\int_h^{h/2} \frac{dh}{\sqrt{h}}}{\int_{h/2}^{0} \frac{dh}{\sqrt{h}}}$

$$= \frac{\left[\sqrt{\frac{h}{2}} - \sqrt{h}\right]}{\left[0 - \sqrt{\frac{h}{2}}\right]} = \sqrt{2} - 1$$