Lesson-8

CONGRUENCE GEOMETRY

CONGRUENCE GEOMETRICAL

Two geometric figure are said to be congruent if they are exactly of same shape and size or we simply say if two figures are congruent in a plane then if we move one figure on to the other then both figures are coincide. It represents by ≅ symbol. Now we have some questions on our mind.

1. When two line segments are congruent?

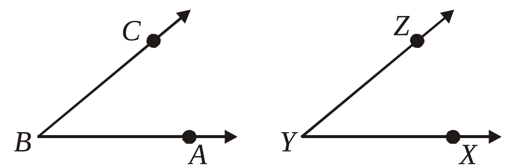
Two segments are congruent if they are of same length

$AB \cong CD$ iff $AB = CD$.

2. When two angles are congruent?

Two angles are congruent iff they have the same measure.

Now, $\angle ABC \cong \angle XYZ$ iff $\angle ABC = \angle XYZ$.

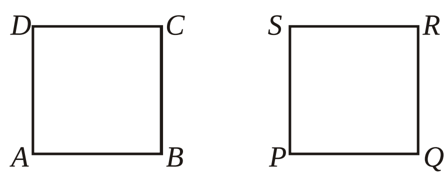


3. When two squares are congruent?

Two squares are congruent iff they have same length

Square $ABCD \cong$ square $PQRS$

iff $AB = BC = DC = AD = PQ = RQ = RS = SP$.

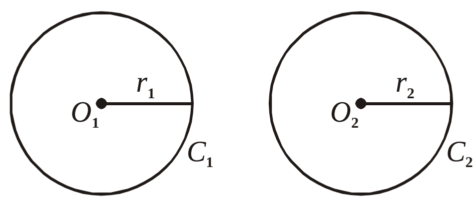


4. When two circles are congruent?

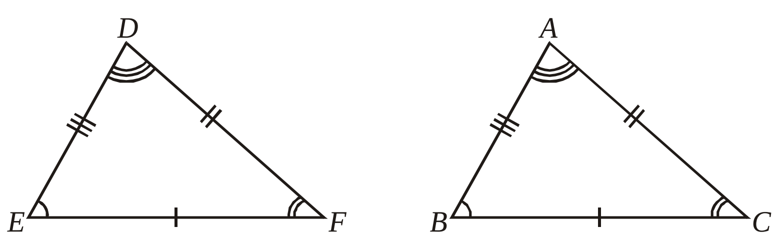
Two circles are congruent iff they have same radius

Circle $C_1 \cong$ circle C_2 iff $r_1 = r_2$.

Now the next question is when triangles are congruent?



Two triangles are congruent if and only if three sides and three angles of the one are congruent to the corresponding sides and the angles of the other. This follows immediately from the congruence of three segments and three angles. Thus if two triangle ABC and DEF are congruent, then



$AB = DE$, $AC = DF$, $BC = EF$, $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$ and we write $\Delta ABC \cong \Delta DEF$.

$\Delta ABC \cong \Delta DEF$ is read as triangle ABC is congruent to triangle DEF.

If $\Delta ABC \cong \Delta DEF$; A corresponds to D, B is corresponds to E and C corresponds to F.

If we try to superimpose ΔABC over ΔDEF, we need to put ΔABC over ΔDEF in such a way that A falls on D, B on E and C on F. While using the symbol of congruence it is necessary to keep the letters in the right order on both sides of it. We write the letters according to the correspondence e.g., $\Delta ABC \cong \Delta DEF$ or $\Delta BAC \cong \Delta EDF$ etc.

From the manner the two triangles are named, six equalities between the corresponding parts of the two congruent triangles can be written. We shall be using the abbreviation **'*c.p.c.t.*' to indicate corresponding parts of congruent triangles**.

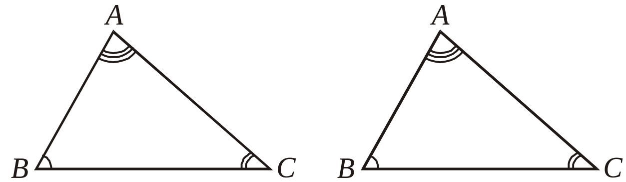
Some Important Facts on Congruence Relation in Triangle:

1. Every triangle ΔABC is congruent to itself.

$\angle A = \angle A$, $\angle B = \angle B$ and $\angle C = \angle C$

$AB = AB$, $BC = BC$ and $CA = CA$

$\Rightarrow$ $\Delta ABC \cong \Delta ABC$.



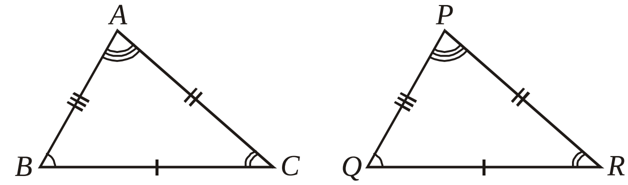
2. If $\Delta ABC \cong \Delta PQR$ then $\Delta PQR \cong \Delta ABC$

Since, $\Delta ABC \cong \Delta PQR$ then

$\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$

$AB = PQ$, $BC = QR$ and $AC = PR$

$\Rightarrow$ $\Delta PQR \cong \Delta ABC$.



3. If $\Delta ABC \cong \Delta PQR$ & $\Delta PQR \cong \Delta XYZ$, then

$\Delta ABC \cong \Delta XYZ$

In the same manner we can show it

Different Conditions for Congruence of Triangles

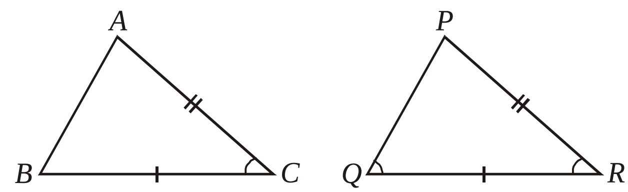
Now we can discuss different conditions so that two triangles are congruent.

Condition-1

"Two triangles are congruent if any two sides and the included angle of the triangle are equal to any two sides and the included angle of the other triangle."

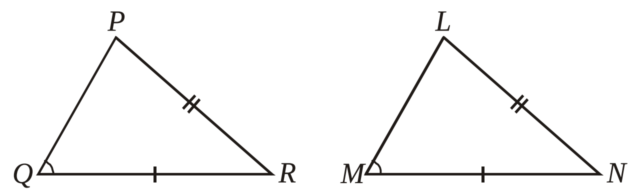
This case is generally known as side-angle-side or simply **SAS** case.

According to the theorem $\Delta ABC \cong \Delta PQR$ in shown figure, when the equal sides and angles are been marked by the same signs.



Is $AB = PQ$, in given figure? Name the other equal angles.

Note: In the given figure, two sides and an angle of ΔPQR are congruent to the two side and an angle of ΔLMN. The congruent parts are marked by the same sign. Are the two Δs congruent? NO why?

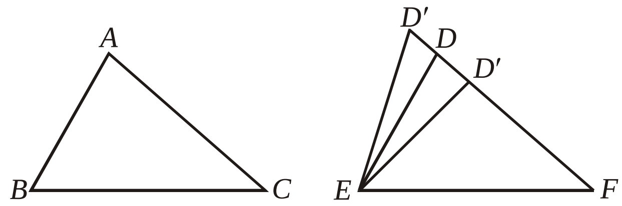


Because the word 'included' is very important in this case.

Condition-2

"Two triangles are congruent if any two angles and the included side of one triangle are equal to the corresponding angles and the included side of the other triangle".

This case is generally known as angle-side-angle or simply **ASA** case.



Given: Two Δs ABC and DEF such that

$\angle ABC = \angle DEF$,

$\angle ACB = \angle DFE$

and $BC = EF$

To prove: $\Delta ABC \cong \Delta DEF$

Proof.: **Case-I**

If $DF = AC$, then ΔABC will be congruent to ΔDEF by SAS and the theorem is proved.

Case-II

If $DF \neq AC$, then construct $D'F = AC$

$AC = D'F$ ...(Constant)

$BC = EF$ ...(Given)

& $\angle ACB = \angle D'FE$ ...(Given)

$\therefore$ $\Delta ABC \cong \Delta D'EF$ ...(SAS)

$\therefore$ $\angle ABC = \angle D'EF$ ...(c.p.c.t.)

But $\angle ABC = \angle DEF$ ...(Given)

$\therefore$ $\angle DEF = \angle D'EF$

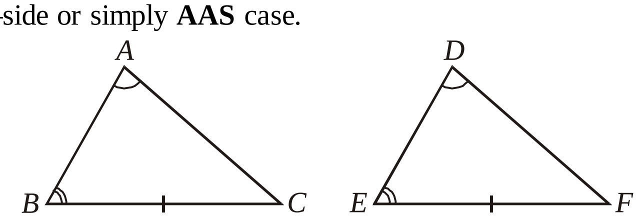
But D and D' lie on the same side of EF. It is possible only when D and D' coincide.

Hence $\Delta ABC \cong \Delta DEF$.

Condition-3

"Two triangles are congruent if any two angles and a non-included side of one triangle are equal to the corresponding angles and side of another triangle".

This case is generally known as angle-angle–side or simply **AAS** case.



Given: Two Δs ABC and DEF such that

$\angle A = \angle D$, $\angle B = \angle E$

and $BC = EF$

To prove: $\Delta ABC \cong \Delta DEF$

Proof.: $\because$ Sum of the angles of a triangle is 180°

$\therefore$ $\angle A + \angle B + \angle C = \angle D + \angle E + \angle F = 180°$

But $\angle A = \angle D$ and $\angle B = \angle E$ ...(Given)

$\therefore$ $\angle C = \angle F$ ...(i)

Now in ΔABC and ΔDEF,

$\angle B = \angle E$ ...(Given)

$\angle C = \angle F$ ...(Proved in (i))

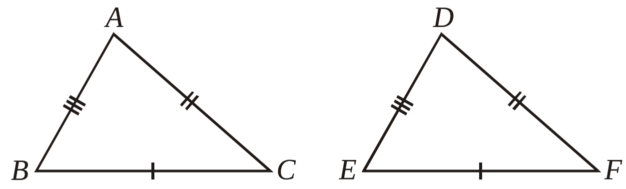
& $BC = EF$ ...(Given)

$\therefore$ $\Delta ABC \cong \Delta DEF$ ...(ASA)

Condition-4

"Two triangles are congruent if three sides of one triangle are equal to the corresponding three sides of the other triangle".

This case is generally known as side-side-side or simply **SSS** case.



Verification:

We measure $\angle B$, $\angle C$, $\angle E$ and $\angle F$

We get $\angle B = \angle E$

and $\angle C = \angle F$

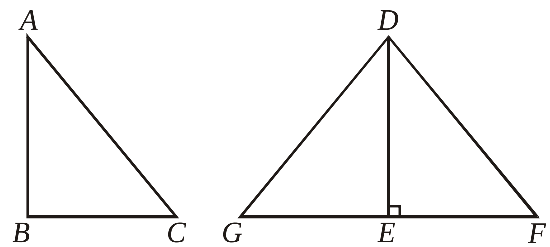
Also $BC = EF$ ...(Given)

$\therefore$ $\Delta ABC \cong \Delta DEF$ ...(ASA)

Condition-5

"Two right angled triangles are congruent if the hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle".

This case is generally known as right-hypotenuse-side or simply **RHS** case.



Given: Two right angles ABC and DEF right angled at B and E respectively in which

hyp. AC = hyp. DF

& side AB = side DE

To prove: $\Delta ABC \cong \Delta DEF$

Construction: Produce FE to G such that $EG = BC$. Join D to G forming ΔDEG.

Proof.: $\angle DEF + \angle DEG = 180°$ ...(DE stands on GF)

But $\angle DEF = 90°$ ...(Given)

$\therefore$ $\angle DEG = 90°$

Now in ΔABC and ΔDEG,

$AB = DE$ ...(Given)

$BC = EG$ ...(Costant)

& $\angle ABC = \angle DEG = 90°$ ...(Proved)

$\therefore$ $\Delta ABC \cong \Delta DEG$ ...(SAS)

$\therefore$ $\angle C = \angle G$ ...(c.p.c.t.) ...(i)

& $AC = DG$ ...(Given)

But $AC = DF$

Now in ΔDGF, $DG = DF$

$\therefore$ $\angle G = \angle F$ ...($\angle$s opp. equal sides) ...(ii)

From (i) & (ii), we get

$\therefore$ $\angle C = \angle F$

Now in ΔABC and ΔDEF,

$AB = DE$ ...(Given)

$\angle C = \angle F$ ...(Proved)

& $\angle ABC = \angle DEF = 90°$ ...(Given)

$\therefore$ $\Delta ABC \cong \Delta DEF$ ...(SAS)

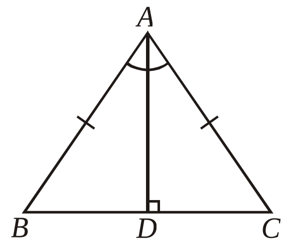
Some Applications of Congruence Relation

Theorem 1 The angles opposite to equal sides of a triangle are equal.

Given ΔABC in which $AB = AC$

To Prove $\angle B = \angle C$

Construction Draw AD, the bisector of $\angle A$ which meets BC at D.



Proof In ΔABC and ΔACD,

$AB = AC$ ...(Given)

$AD = AD$ ...(Common)

& $\angle BAD = \angle CAD$ ...(Constant)

$\therefore$ $\Delta ABD \cong \Delta ACD$ ...(SAS)

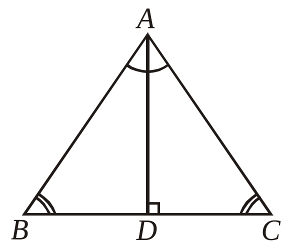
$\therefore$ $\angle B = \angle C$ ...(c.p.c.t.)

Theorem 2 The sides opposite to equal angles of a triangle are equal.

Given In ΔABC, $\angle B = \angle C$

To Prove $AB = AC$

Construction Draw AD, the bisector of $\angle A$ which meets BC at D.



Proof In ΔABD and ΔACD,

$\angle BAD = \angle CAD$ ...(Constant)

$\angle B = \angle C$ ...(Given)

& $AD = AD$ ...(Common)

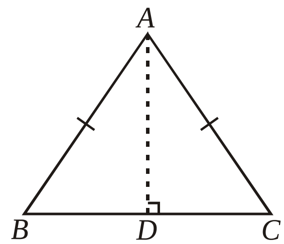
$\therefore$ $\Delta ABD \cong \Delta ACD$ ...(AAS)

$\therefore$ $AB = AC$ ...(c.p.c.t.)

Theorem 3 In an isosceles triangle altitude from the vertex bisects the base.

Given An isosceles triangle ABC such that $AB = AC$ and an altitude AD from A on side BC.

To Prove $BD = DC$



Proof In ΔADB and ΔADC

$\angle ADB = \angle ADC$ ...(Constant)

$AD = AD$ ...(Common)

$\angle B = \angle C$ ...(Given)

$\therefore$ $\Delta ADB \cong \Delta ADC$ ...(AAS)

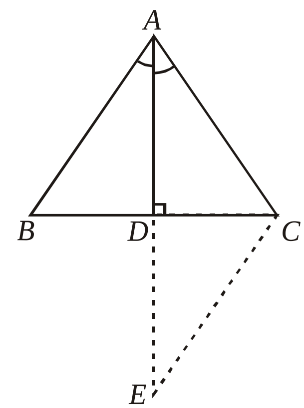
$\Rightarrow$ $BD = DC$ ...(c.p.c.t.)

Theorem 4 If the bisector of the vertical angle of a triangle bisects the base of the triangle, then the triangle is isosceles.

Given A ΔABC in which AD is the bisector of $\angle A$ meeting BC in D such that $BD = DC$.

To Prove ΔABC is an isosceles triangle.

Construction Product AD to E such that $AD = DE$. Join EC.



Proof In ΔADB and ΔEDC

$BD = DC$ ...(Given)

$AD = DE$ ...(by construction)

& $\angle ADB = \angle EDC$

$\therefore$ $\Delta ADB \cong \Delta EDC$ ...(SAS)

$\Rightarrow$ $AB = EC$ ...(c.p.c.t.)

& $\angle BAD = \angle CED$ ...(i)

But $\angle BAD = \angle CAD$ ...(Given)

$\therefore$ $\angle CAD = \angle CED$

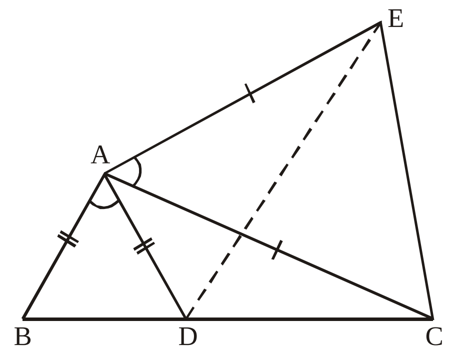
$\Rightarrow$ $AC = EC$ ...($\because$ Sides opposite to equal angles)

$\Rightarrow$ $AC = AB$ ...($\because$ from (i))

Hence, ΔABC is an isosceles triangle.

SOLVED EXAMPLES

Ex.1: In figure. $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Prove that $BC = DE$.



Sol.: Join DE.

We have, $\angle BAD = \angle EAC$

$\Rightarrow \quad \angle BAD + \angle DAC = \angle EAC + \angle DAC$

[Adding $\angle DAC$ to both sides]

$\Rightarrow \quad \angle BAC = \angle DAE$ .....(i)

Now, in triangles ABC and ADE, we have

$AB = AD$ [Given]

$\angle BAC = \angle DAE$ [from (i)]

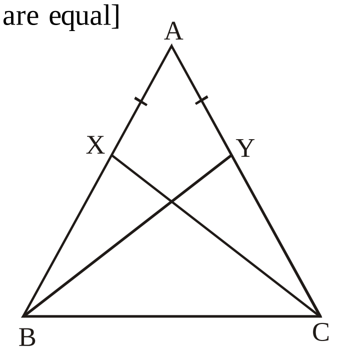
and $AC = AE$ [Given]

So, by SAS congruence criterion, we have

$\Delta ABC \cong \Delta ADE$

$\Rightarrow \quad BC = DE$ [$\because$ Corresponding parts of congruent triangles are equal]

Ex.2: In figure, X and Y are two points on equal sides AB and AC of a ΔABC such that $AX = AY$. Prove that $XC = YB$.



Sol.: In Δs AXC and AYB, we have

$AX = AY$ [Given]

$\angle A = \angle A$ [Common angle]

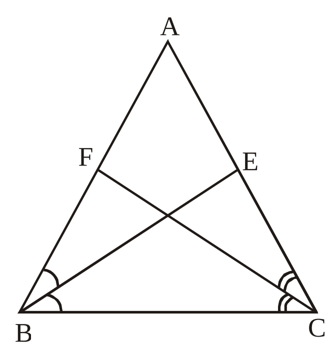
$AC = AB$ [Given]

so, by SAS criterion of congruence

$\Delta AXC \cong \Delta AYB$

$\Rightarrow \quad XC = YB$ [$\because$ Corresponding parts of congruent triangles are equal]

Ex.3: In figure, $AB = AC$, BE and CF are respectively the bisectors of $\angle B$ and $\angle C$. prove that $\Delta EBC \cong \Delta FCB$.



Sol.: In ΔABC, we have

$AB = AC$ [Given]

$\Rightarrow \quad \angle ACB = \angle ABC$

$\Rightarrow \quad \angle ECB = \angle FBC$ ...... (i) [$\because \angle ACB = \angle FCB$ and $\angle ABC = \angle FBC$]

Again, $\angle ACB = \angle ABC$

$\Rightarrow \quad \frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$

$\Rightarrow \quad \angle FCB = \angle EBC$ .....(ii) [$\because$ CF and BE are bisectors of $\angle ACB$ and $\angle ABC$ respectively]

Now, in Δs *EBC* and *FCB*, we have

$\angle ECB = \angle FBC$ [from (*i*)]

and, BC = BC [Common]

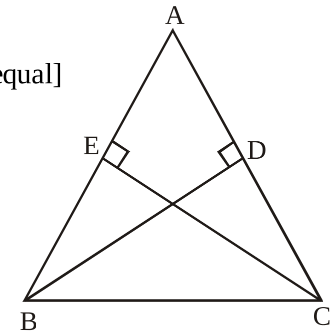
$\angle FCB = \angle EBC$

So, by ASA criterion of congruence

$\Delta EBC \cong \Delta FCB$.

Ex.4: If Δ *ABC* is an isosceles triangle with *AB* = *AC*. Prove that the perpendiculars from the vertices *B* and *C* to their opposite sides are equal.

Sol.: In Δ *ABC*, we have



$AB = AC$ [Given]

$\Rightarrow \angle B = \angle C$ .....(*i*) [∵ Angle opposite to equal sides are equal]

Now, in Δ*s BCE* and *BCD*, we have

$\angle B = \angle C$ [From (*i*)]

$\angle CEB = \angle BDC$ [Each equal to 90°]

and, $BC = BC$ [Common]

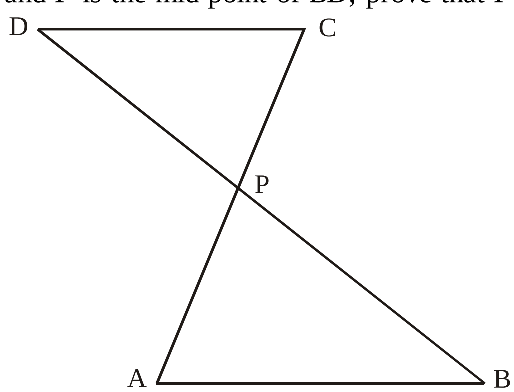
So, by AAS criterion of congruence, we have

$\Delta BCE \cong \Delta BCD$

$\Rightarrow BD = CE$ [∵ Corresponding parts of congruent triangles are equal]

Hence, *BD* = *CE*.

Ex.5: In figure, if *AB* ∥ *DC* and *P* is the mid-point of *BD*, prove that *P* is also the mid-point of *AC*.



Sol.: Since *AB* ∥ *DC* and transversal *AC* cuts them and *A* and *C* respectively. Therefore,

$\angle PAB = \angle PCD$ .....(*i*) [Alternate angles]

Similarly, *AB* ∥ *DC* and transversal *BD* cuts them at *B* and *D* respectively. Therefore,

$\angle ABP = \angle CDP$ .....(*ii*) [Alternate angles]

Since *AC* and *BD* intersect at *P*. Therefore,

$\angle APB = \angle CPD$ .....(*iii*) [Vertically opposite angles]

Thus, in triangles *APB* and *CPD*, we have

$\angle ABP = \angle CDP$ [From (*i*)]

$BP = DP$ [∵ *P* is the mid-point of *BD* (Given)]

and, $\angle APB = \angle CPD$ [From (*iii*)]

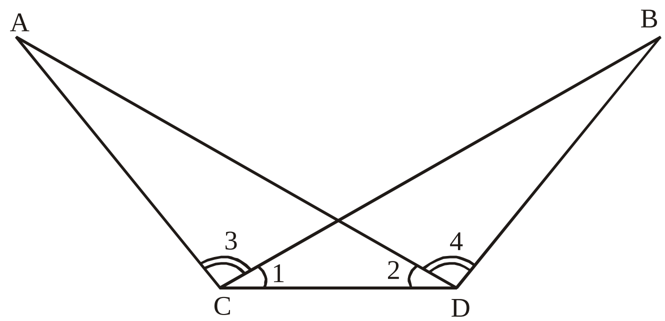
So, by ASA congruence criterion, we have

$\Delta APB \cong \Delta CPD$

$\Rightarrow$ $AP = PC$ [$\because$ Corresponding parts of congruent triangles are equal]

Hence, P is the mid-point of AC.

Ex.6: In figure, $\angle BCD = \angle ADC$ and $\angle ACB = \angle BDA$. Prove that $AD = BC$ and $\angle A = \angle B$.



Sol.: We have,

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

$\Rightarrow$ $\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow$ $\angle ACD = \angle BDC$ .....(*i*)

Thus, in triangles ACD and BDC, we have

$\angle ADC = \angle BCD$ [Given]

$CD = CD$ [Common]

and $\angle ACD = \angle BDC$ [From (*i*)]

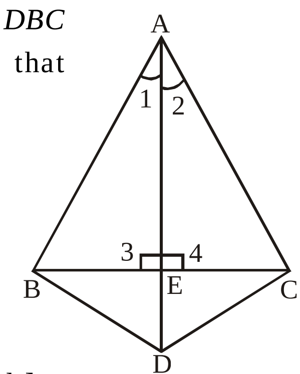
So, by ASA criterion of congruence, we have

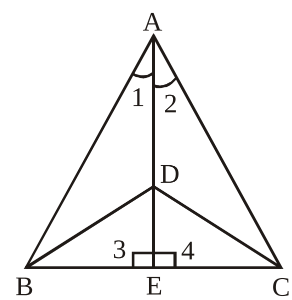
$\Delta ACD \cong \Delta BDC$

$\Rightarrow$ $AD = BC$ and $\angle A = \angle B$ [$\because$ c.p.c.t.]

Ex.7: If two isosceles triangles have a common base, the line joining their vertices bisects them at right angles.

Sol.: Given Two isosceles triangles ABC and DBC having the common base BC such that $AB = AC$ and $DB = DC$.





To Prove AD (or AD produced) bisects BC at right angle.

Proof In Δs ABD and ACD, we have

$AB = AC$ [Given]

$BD = CD$ [Given]

$AD = AD$ [Common side]

So, by SSS criterion of congruence

$\Delta ABD \cong \Delta ACD$

$\Rightarrow$ $\angle 1 = \angle 2$ .....(*i*) [$\because$ c.p.c.t. are equal]

Now, in Δs ABE and ACE, we have

$AB = AC$ [Given]

$\angle 1 = \angle 2$ [From (*i*)]

and, $AE = AE$ [Common side]

So, by SAS crition of congruence,

$\Delta ABE \cong \Delta ACE$

$\Rightarrow \quad BE = CE$ [$\because$ c.p.c.t. are equal]

and, $\angle 3 = \angle 4$

But, $\angle 3 + \angle 4 = 180°$ [$\because$ Sum of the angles of a linear pair is 180°]

$\Rightarrow \quad 2\angle 3 = 180°$ [$\because \angle 3 = \angle 4$]

$\Rightarrow \quad \angle 3 = 90°$

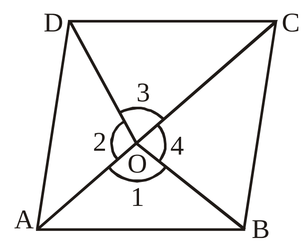
$\therefore \quad \angle 3 = \angle 4 = 90°$

Hence, *AD* bisects *BC* at right angles.

Ex.8: A point *O* is taken inside an equilateral four sided figure *ABCD* such that its distances from the angular points *D* and *B* are equal. Show that *AO* and *OC* are in one and the same straight line.

Sol.: **Given** A point *O* inside an equilateral quadrilateral four sided figure *ABCD* such that *BO* = *OD*.

To Prove *AO* and *OC* are in one and the same straight line



Proof In Δs *AOD* and *AOB*, we have

$AD = AB$ [Given]

$AO = AO$ [Common side]

$OD = OB$ [Given]

So, by SSS criterion of congruence,

$\Delta AOD \cong \Delta AOB$.

$\Rightarrow \quad \angle 1 = \angle 2 \quad .....(i)$ [$\because$ c.p.c.t. are equal]

Similarly, $\Delta DOC \cong \Delta BOC$

$\Rightarrow \quad \angle 3 = \angle 4 \quad .....(ii)$ [$\because$ c.p.c.t. are equal]

But $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 4$ right angles [Sum of the angles at a point is 4 right angles]

$\therefore \quad 2\angle 2 + 2\angle 3 = 4$ right angles [Using (*i*) and (*ii*)]

$\Rightarrow \quad \angle 2 + \angle 3 = 2$ right angles

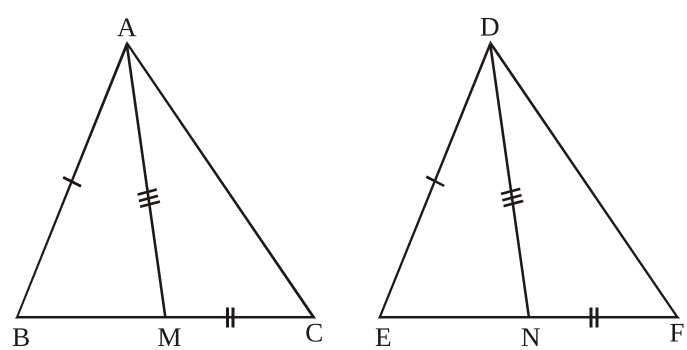
$\Rightarrow \quad \angle 2 + \angle 3 = 180°$

$\Rightarrow \quad \angle 2$ and $\angle 3$ form a linear pair.

$\Rightarrow$ *AO* and *OC* are in the same straight line

$\Rightarrow$ *AC* is a straight line.

Ex.9: In figure, two sides *AB* and *BC* and the median *AM* of ΔABC are respectively equal to sides *DE* and *EF* and the median *DN* of ΔDEF. Prove that $\Delta ABC \cong \Delta DEF$.



Sol.: Since AM and DN are medians of ΔABC and ΔDEF respectively. Therefore,

$BM = MC$ and $EN = NF$

$\Rightarrow$ $BM = \frac{1}{2} BC$ and $EN = \frac{1}{2} EF$.

But, $BC = EF$ [Given]

$\Rightarrow$ $\frac{1}{2} BC = \frac{1}{2} EF$

$\Rightarrow$ $BM = EN$ .....(*i*)

In triangles ABM and DEN, we have

$AB = DE$ [Given]

$BM = EN$ [From (*i*)]

$AM = DN$ [Given]

So, by SSS congruence criterion, we have

$\Delta ABM \cong \Delta DEN$

$\Rightarrow$ $\angle B = \angle E$ ...(*ii*)

Now, in triangles ABC and DEF, we have

$AB = DE$ [Given]

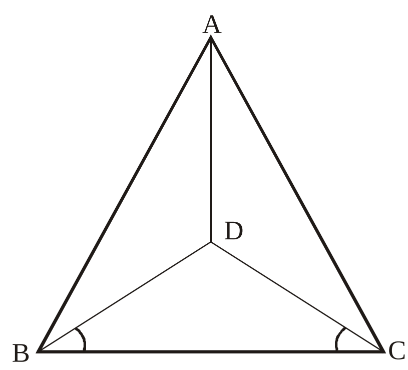
$BC = EF$ [Given]

and $\angle B = \angle E$ [From (*ii*)]

So, by SAS congruence criterion, we have

$\Delta ABC \cong \Delta DEF$

Ex.10: In Figure $AB = AC$, D is the point in the interior of ΔABC such that $\angle DBC = \angle DCB$. Prove that AD bisects $\angle BAC$ of ΔABC.



Sol.: In ΔBDC, we have

$\angle DBC = \angle DCB$

$\Rightarrow$ $DC = DB$ ...(*i*)

[$\because$ Sides opposite to equal angles of ΔDBC are equal]

Now, in ΔABD and ΔACD, we have

$AB = AC$ [Given]

$BD = CD$ [From (*i*)]

and, $AD = AD$ [Common side]

So, by SSS congruence criterion, we have

$\Delta ABD \cong \Delta ACD$

$\Rightarrow$ $\angle BAD = \angle CAD$ [$\because$ c.p.c.t.]

Hence, AD is the bisector of ΔBAC.

Ex.11: Two Δs ABC and DEF are such that $AB = DE$, $AC = DF$ and $\angle A = \angle D$. If $BC = 6$ cm, $\angle A = 60°$ and $\angle B = 40°$, find the measurement of EF and $\angle F$.

Sol.: $\because$ $AB = DE, AC = DF, \angle A = \angle D$ ...(Given)

$\therefore$ $\Delta ABC \cong \Delta DEF$

$\therefore$ $BC = EF, \angle B = \angle E$ and $\angle C = \angle F$

$\therefore$ $BC = 6$ cm $\Rightarrow$ $EF = 6$ cm

$\because$ $\angle A + \angle B + \angle C = 180°$ ...(Angles of ΔABC)

$\therefore$ $60° + 40° + \angle C = 180° \Rightarrow \angle C = 80°$

$\therefore$ $\angle C = \angle F \Rightarrow \angle F = 80°$

Hence $EF = 6$ cm and $\angle F = 80°$.

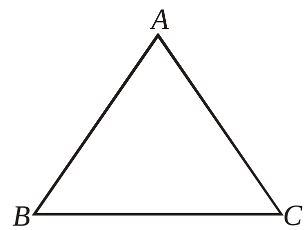
Ex.12: If ΔABC is an equilateral triangle, find the measurement of each angle.

Sol.: Let ΔABC

$\because$ $AB = AC = BC$ (ΔABC is equilateral)

$\therefore$ $AB = AC \Rightarrow \angle C = \angle B$ ...(i) (Angle opposite to equal sides)

Again $BC = AC \Rightarrow \angle A = \angle B$ ...(ii)



From (i) and (ii), we get

$\angle A = \angle B = \angle C$ ...(iii)

But $\angle A + \angle B + \angle C = 180°$ (Angle of ΔABC)

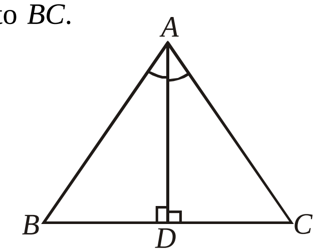
or $\angle A + \angle A + \angle A = 180°$

or $3\angle A = 180°$

$\angle A = 60°$

Hence from (iii), we get $\angle A = \angle B = \angle C = 60°$

Ex.13: Prove that ΔABC is isosceles if the bisector of $\angle A$ is perpendicular to BC.



Sol.: In ΔABC and ΔACD,

$\angle 1 = \angle 2$ (AD is the bisector of $\angle A$)

$\angle ADB = \angle ADC = 90°$ ($AD \perp BC$)

& $AD = AD$ (Common)

$\therefore$ $\Delta ABD \cong \Delta ACD$ (ASA)

$\therefore$ $AB = AC$ (c.p.c.t.)

Hence ΔABC is isosceles.

Ex.14: AB is a line segment, AX and BY, two equal line segments are drawn on opposite sides of line AB such that $AX \parallel BY$. If line segments AB and XY intersect each other at O, prove that

(i) $\Delta AOX \cong \Delta BOY$ and

(ii) O is the mid point of both AB and XY.

Sol.: (i) In $\Delta AOX \cong \Delta BOY$, $AX \parallel BY$ and XY intersects them

$\therefore$ $\angle 1 = \angle 2$ (Alt. $\angle s$)

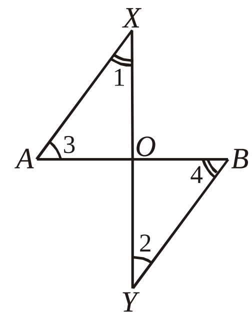
Similarly,

$\angle 3 = \angle 4$ (Alt. $\angle s$)

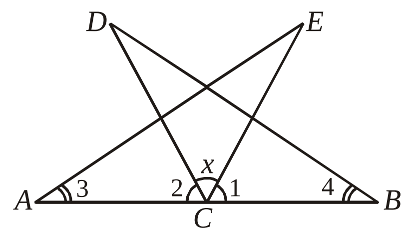
& $AX = BY$ (Given)

$\therefore$ $\Delta AOX \cong \Delta BOY$ (ASA)

(ii) Hence $AO = BO$ and $OX = OY$ (c.p.c.t.)



Ex.15: In figure, if $AC = BC$, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$, then prove that $\Delta DBC \cong \Delta EAC$.



Sol.: $\angle 1 = \angle 2$ (Given)

Adding $\angle x$ to both sides, we get

$\angle 1 + \angle x = \angle 2 + \angle x$

or $\angle BCD = \angle ACE$ ...(i)

Now in ΔDBC and ΔEAC,

$\angle 4 = \angle 3$ (Given)

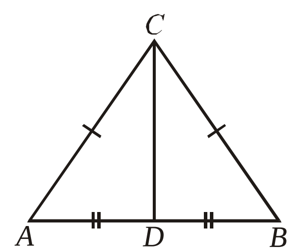
$\angle BCD = \angle ACE$ (By (i))

& $BC = AC$ (Given)

$\therefore$ $\Delta DBC \cong \Delta EAC$ (ASA)

Ex.16: In an isosceles ΔABC with $AC = BC$, D is the mid-point of AB. Prove that $CD \perp AB$.

Sol.: In ΔCAD and ΔCBD,



$AC = BC$ (Given)

$CD = CD$ (Common)

& $AD = BD$ (Given)

$\therefore$ $\Delta CAD \cong \Delta CBD$ (SSS)

$\therefore$ $\angle CDA = \angle CDB$ (c.p.c.t.)

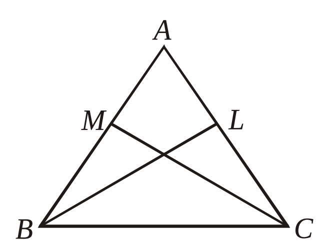
But $\angle CDA + \angle CDB = 180^\circ$ (Linear pair)

$\therefore$ $\angle CDA = \angle CDB = 90^\circ$

Hence $CD \perp AB$.

Ex.17: From the vertices B and C of ΔABC, perpendicular BL and CM are drawn to the opposite sides AC and AB respectively. If $BL = CM$, prove that ΔABC is isosceles.

Sol.: In right Δs BLC and CMB,



hyp. BC = hyp. CB (Common)

& $BL = CM$ (Given)

$\therefore$ $\Delta BLC \cong \Delta CMB$ (RHS)

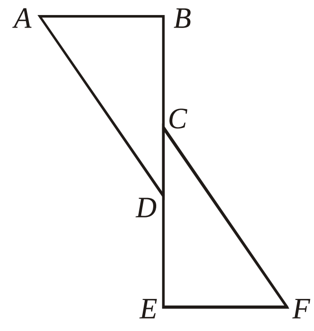
$\therefore$ $\angle BCL = \angle CBM$

$\therefore$ $AB = AC$

Hence ΔABC is isosceles.

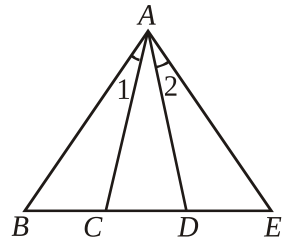
BASIC LEVEL ASSIGNMENT

1. Given $AB \perp BE$, $EF \perp BE$, $BC = DE$, $AD = CF$, then prove $\angle A = \angle F$.



2. If two sides AB and BC and the median AM of ΔABC are respectively equal to sides DE, EF and median DN of ΔDEF. Prove that $\Delta ABC \cong \Delta DEF$.

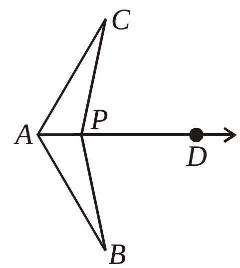
3. If $\angle 1 = \angle 2$, $BC = DE$, $\angle B = \angle E$, then prove that $\angle BDA = \angle ECA$.



4. ΔABC and ΔDBC are two isosceles triangles on the same base BC such that $AB = AC$ and $DB = DC$. Prove that $\angle ABD = \angle ACD$.

5. In a right angled triangle, one acute angle is double the other. Prove that the hypotenuse is double the smallest side.

6. In figure, $\angle CPD = \angle BPD$ and AD is bisector of $\angle BAC$. Prove that $\Delta CAP \cong \Delta BAP$ and hence $CP = BP$.



7. $PQRS$ is a quadrilateral and T and U are respectiely points on PS and RS such that $PQ = RQ$, $\angle PQT = \angle RQU$ and $\angle TQS = \angle UQS$. Prove that $QT = QU$.

8. If D is the mid point of the hypotenuse AC of a right angle triangle ABC. Prove that $BD = \frac{1}{2}AC$.

9. By the means of congruence relation. Prove that each angle of an equilateral triangle is of measure 60°.

10. Can you find the breath of the river without crossing it. If No/Yes? Explain your answer.

True and False

11. Sides opposite to equal angles of a triangle may be unequal.

12. The bisector of two equal angles of a triangle are equal.

13. Two altitudes corresponding to two equal sides of a triangle need not be equal.

14. If all three angles of two triangles are equal then these two triangle are congruent.

Fill in the blanks

15. In a ΔPQR if $\angle P = \angle R$ then $PQ =$ ....

16. Angle opposite to equal sides of a triangle are ....

17. If altitudes RX and QY of a triangle PQR are equal, then $PQ=$ ....

18. In an equilateral triangle all angles are ....

19. In an isosceles triangle altitude from vertex ....the base.

20. Any triangle ABC is congruent to ....

ANSWERS

Basic Level Assignment

11. False	**12.** True	**13.** False	**14.** False
15. QR	**16.** equal	**17.** QR	**18.** equal
19. bisects	**20.** itself		