OBJ. LEVEL – I

1. From dimensional homogeneity

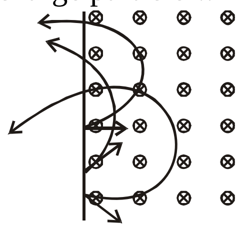
$at^2 = \text{Weber}$

$\therefore \quad a = \text{Web}/\text{second}^2 = \frac{V.s}{s^2} = \frac{\text{Volt}}{\text{second}}$

Ans. (b)

2. Ans. (c)

3. As from figure it is clear that no charge particle will ever complete the rotation.



Ans. (a)

4. As $\varepsilon = -\frac{d\phi}{dt}, \phi = 0$ does not mean that $\varepsilon = 0$ or vice-versa

If $\varepsilon \neq 0$, ϕ may or may not be zero

Ans. (c)

5. As time varying magnetic field can produce induce electric field which set up current in the ring. But a current carrying coil placed in uniform magnetic field will not experience any force. Also magnetic moment and area vectors are parallel to each other, so that torque about any axis is zero. Because of current element in magnetic field, the ring has a tension force along its length.

Ans. (d)

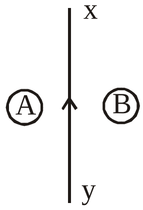
6. As $\vec{F} = q\,\vec{V} \times \vec{B}$, if $\vec{V}$ is parallel to $\vec{B}$, there will be no force.

Also in (b) and (c) case $\vec{V} \times \vec{B}$ is $\perp_r$ to $\vec{dl}$, so the direction of force is $\perp_r$ to $\vec{dl}$ and it is not possible to drift electron in that direction results no induced emf

Ans. (d)

7. As current in *xy* wire increases, so flux across both the loops increases, so induced $\vec{B}$ must oppose external magnetic field.

As also $\vec{B}$ due to wire in loop B is into and in loop *A* is outward so induced $\vec{B}$ must be outward in loop *B* and inward in loop *A*. So the direction of induced current in *B* is anticlockwise and in *A*, it is clockwise.



Ans. (a)

8. From Lenz's law, emf will induced in the ring in such a way that it always oppose the change in flux i.e., in both the motion the ring's polarity will develop in such a way that it always oppose the change in flux across ring and so magnet will get repelled while approaching and attracted while receding.

So, the result of both the motion, the magnet's acceleration will be less than g.

Ans. (c)

9. In L-R circuit, current decreases with time as

$$I = I_0 e^{-tR/L}$$

at $t = \frac{2L}{R}$ $\therefore$ $I = \frac{I_0}{e^2} = 0.135\ I_0$

Ans. (b)

10. $$e = \frac{50 \times 5 \times 10^4}{\pi} \times 10^{-2} \times \frac{1}{0.1} \times 4\pi \times 10^{-7} = 0.1\,V$$

11. $$e = L.\frac{di}{dt} = \frac{\phi}{I}.\frac{di}{dt} = \frac{0.8}{2} \times 0.4 = 0.16V$$

Ans. (d)

12. $$E = \frac{\pi r^2}{2\pi r}.\frac{dB}{dt} = \frac{r}{2}.\frac{dB}{dt}$$

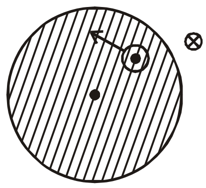
$$T = \int_0^Q dq.E.r = \frac{r^2}{2}.\frac{dB}{dt}.Q$$

Ans. (c)

13. As $Q = \int I.dt = \int \frac{e}{R}.dt = \int \frac{1}{R}\left(\frac{d\phi}{dt}\right)dt = \frac{\Delta\phi}{R}$

Ans. (d)

15. As disc is conducting, so any free electron which has velocity ωr tangential will experience a force $(\vec{F} = q\vec{V} \times \vec{B})$ away from centre and the centre is positively charged and periphery gets negatively charge.



So, potential difference between C and rim is $\int_0^r B.dr.\omega r$

i.e., $\frac{1}{2} B r^2 \omega$

Ans. (c)

SOLUTION OBJ. LEVEL - II

1. From, $\vec{F} = q\vec{V} \times \vec{B},$ magnetic force will act on electron towards A and because of this A gets positively charged and D has negative charge
Ans. (d)

2. In position II, there is no change in flux and so emf induced in this case is zero.
Ans. (a)

3. Motional emf for each wire = $Blv = 1 \times 4 \times 10^{-2} \times 5 \times 10^{-2} = 2 \times 10^{-3}$ Volt

In parallel, $\varepsilon_{eg} = \dfrac{\varepsilon_1/r_1 + \varepsilon_2/r_2}{1/r_1 + 1/r_2} = \varepsilon = 2\times 10^{-3} V$ [As $r_1 = r_2 = 2\ \Omega$]

$\therefore \quad r_{eg} = \dfrac{2\times 2}{2+2} = 1\Omega$

$\therefore \quad I = \dfrac{2\times 10^{-3}}{20} = 0.1\text{mA}$

Ans. (b)

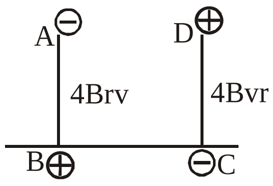
4. As flux across ring is constant so there is no induced emf across the ring and hence no current. Also as CE is parallel to $\vec{V}$, there is no potential difference across C and E.
Ans. (c)

5.
$$de = (\vec{V}\times\vec{B}).\vec{dl} = \{(V_x\hat{i} + v_y\hat{j})\times B\hat{k}\}.(l_1\hat{i} + l_2\hat{j})$$
$$= (-V_xB\hat{j} + V_yB\hat{i}).(l_1\hat{i} + l_2\hat{j})$$
$$= V_yBl_1 - V_xl_2B$$
$$= (V_yl_1 - V_x\ell_2).B$$
Ans. (c)

6. emf $= \oint \vec{E}.\vec{dl} = \left|-\dfrac{d\phi}{dt}\right| = \pi r^2 \dfrac{dB}{dt}$

Ans. (d)

7. For bigger ring highest point is at high potential w.r.t. bottom point and for smaller ring highest point is at low potential w.r.t. bottom point. Thus



$V_D - V_C = 4Bv.r$ & $V_B - V_A = 4Brv$ As $V_B = V_C$

$\therefore \quad V_D - V_A = 8\,Bvr$

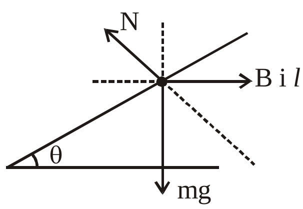
Ans. (d)

8.
$$e = (\vec{V}\times\vec{B}).\vec{dl} = \frac{1}{2}Blv$$
Ans. (a)

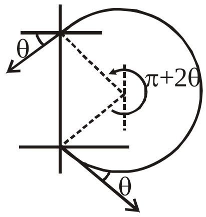
9. $mg\sin\theta = B\,i\,l\cos\theta$

$\therefore \quad B\,i\,l = mg\tan\theta$

Ans. (b)



10. Time spent $= \frac{T}{2\pi}(\pi + 2\theta)$



11. The duration of induced emf is the duration in which flux is changed
i.e., 10^{-3} s.

12. At steady state current in the coil $= \frac{\varepsilon}{R}$

As flux is constant, $\frac{\varepsilon}{R}L = I.\frac{L}{n}$

$\therefore \quad I = n.\frac{\varepsilon}{R}$

Ans. (c)

13. At steady state i.e. for a long time after switch S is closed current in the branch of inductor is $\frac{\varepsilon}{r}$ and after opening the switch at t = 0, energy stored in the inductor must be same and have same current 'ε/r' will flow across inductor thus p.d. across it in $\varepsilon/r \times r = \varepsilon$ at $t = 0$

Ans. (d)

14. As $\quad q = \int d\phi = Mi/R$

Ans. (c)

15. As $\quad e = Blv = 2 \times 0.1 \times 0.5 = 0.1\,V$

Keeping one resistance at rest, $i = \frac{e}{5+10} = \frac{1}{150}\;A$

Ans. (d).

SOLUTION SUB. LEVEL - I(C.B.S.E.)

1. According to Lenz's law, as flux changes across any loop, induced emf will set-up in such a way that it opposes the change in flux. i.e.,

If $\frac{d\phi}{dt}$ is +ve, induced $\vec{B}$ is opposite to $\vec{B}_{ext}$

and if $\frac{d\phi}{dt}$ is –ve, induced $\vec{B}$ will support $\vec{B}_{ext}$

In figure (i) $\frac{d\phi}{dt}$ is +ve, so $\vec{B}_i$ will oppose $\vec{B}_{ext}$ i.e., for dot $\vec{B}_i$ current must be anticlockwise

But for figure (ii) and (iii) as $\frac{d\phi}{dt}$ is –ve, so $\vec{B}_i$ will support $\vec{B}_{ext}$ i.e. for $\vec{B}_i$ to be $\otimes$ current must be clockwise.

2. From Lenz's law
- (a) As magnet approaches coil, south pole will develop in the near and of the coil and thus a current p to q set-up across the coil
- (b) along q P, along xy
- (c) along x y z
- (d) along z y x
- (e) along x y
- (f) no induced current

3. (a) As area of the loop increases, flux also increases so induced $\vec{B}$ will oppose $\vec{B}_{ext}$. Hence to make $\vec{B}_i$ onward current induced in the coil is anticlockwise.

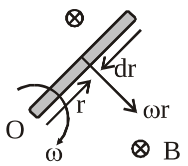
(b) Above explanation, current is anticlockwise.

4. $$e = \frac{d\phi}{dt} = \frac{8\times10^{-4}}{0.5} = 1.6\times10^{-3}V$$

5. As flux across the loop, $\phi = (\mu_0 nI)\ A$

$\therefore$ $$e = n.\mu_0 A.\frac{dI}{dt} = \frac{15}{10^{-2}}\times4\pi\times10^{-7}\times2\times10^{-4}\times\frac{2}{0.1}$$
$$= 7.5\times10^{-6}\ \text{Volt}$$

6. $$e = \int de = \int_0^l B.\omega r\ dr$$
$$e = \frac{1}{2}B\omega.l^2$$
$$= \frac{1}{2}\times0.5\times400\times1^2$$
$$= 100\ \text{V}.$$



7. Let θ be the angle area vector of the coil makes with magnetic field, the flux linked across the coil at that instant, $\phi = NBA\cos\theta$

$\therefore$ emf induced $= \frac{d\phi}{dt} = NBA.\sin\theta.\frac{d\theta}{dt}$
$$= NB\omega.\pi r^2\sin\theta$$

$\therefore$ $$e_{\max} = NB\omega\pi r^2 = 20\times3\times10^{-2}\times50\times\pi\times(8\times10^{-2})^2$$
$$= 0.603\ \text{V}$$

$\therefore$ $$e_{avg} = \int_0^{2\pi} e.d\theta = 0$$

Heat dissipated $= i^2R = \left(\frac{e}{R}\right)^2 .R = \frac{e^2}{R} = 0.036W$

Source of this power is the mechanical source which is rotating the coil.

8. emf induced $= \frac{1}{2}B.\omega l^2 = \frac{1}{2}\times 0.4\times\frac{120\times 2\pi}{60}\times(0.5)^2\times 10^{-4}$

$= 6.28\times 10^{-5}\ V$

9.
$L_1 = \mu_0\pi r^2.n_1^2 l$

$L_2 = \mu_0\pi r^2 n_2^2 l$

$\therefore$ $M_{12} = n_2 l(\mu_0 n_1.\pi r^2.l)$

$M_{12} = \sqrt{L_1L_2}$

10. Magnetic field inside circular loop

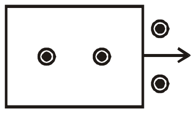
$B = \frac{\mu_0 I}{2b}$

$\phi = \vec{B}.\vec{A} = \frac{\mu_0 I}{2b}.\pi a^2$

$= \frac{\mu_0\pi a^2}{2b}.I$

$M = \frac{\phi}{I} = \frac{1}{2}\frac{\pi\mu_0.a^2}{b}$

11.



$\frac{\partial B}{\partial x} = -10^{-3}T/cm$

$\frac{\partial B}{dt} = -10^{-3}T/s$

$e = -\frac{\partial(a^{2.}.B)}{\partial x}.\frac{\partial x}{\partial t} - \frac{\partial(a^2B)}{\partial t} = -a^2\left[\frac{\partial B}{\partial x}.V + \frac{\partial B}{\partial t}\right]$

$i = \frac{e}{R} = -\frac{a^2}{R}\left[v\frac{\partial B}{\partial x} + \frac{\partial B}{\partial t}\right]$

$= \frac{(12\times 10^{-2})^2}{4.5\times 10^{-3}}\left[8\times 10^{-2}\times\frac{10^{-3}}{10^{-2}} + 10^{-3}\right]$

$= \frac{9\times 10^{-3}\times 144\times 10^{-4}}{4.5\times 10^{-3}} = 2.88\times 10^{-6}\ A$

12.
$\phi_1 = NAB$ $\qquad \phi_2 = 0$

$\therefore$ $Q = \frac{1}{R}\int d\phi = \frac{NAB}{R}$

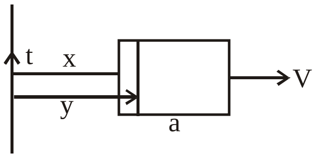
$\therefore$ $B = \frac{QR}{NAd} = \frac{0.5\times 7.5\times 10^{-3}}{25\times 2\times 10^{-4}} = 0.75T$

13.
$L = \mu_0 n^2 Al$

$= 4\pi\times 10^{-7}\times\left(\frac{500}{0.3}\right)^2\times 25\times 10^{-4}\times 0.3 = 2.61\times 10^{-3}$

$e = \left|L.\frac{dI}{dt}\right| = \left|L.\frac{2.5}{10^{-3}}\right| = 6.5\ V$

14. (a) $\phi = \int d\phi = \int_x^{x+a} \frac{\mu_0 I}{2\pi y}.ady$



$\therefore \quad M = \frac{\phi}{I} = \frac{\mu_0 a}{2\pi}\ln\left(\frac{x+a}{x}\right)$

(b) $e = -\frac{d\phi}{dt} = -\frac{\mu_0 aI}{2\pi}\left(\frac{x}{x+a}\right)\left(0 - \frac{a}{x^2}\right).\frac{dx}{dt}$

$= \frac{\mu_0 a^2 I}{2\pi x(x+a)}V = \frac{2\times10^{-7}\times0.1^2\times50\times10}{0.2\times0.3} = 1.7\times10^{-5}\,V$

15. Flux across disc

$\phi = \pi a^2 B_0$

As B is switched off

$\therefore \quad d\phi = \pi a^2 B$

$\therefore \quad e = \frac{d\phi}{dt} = \frac{\pi a^2.B}{\Delta t}$

$\therefore$ Again $\oint \vec{E}.\vec{dl} = -\frac{d\phi}{dt}$

$E = \frac{1}{2\pi R}.\frac{\pi a^2 B}{\Delta t} = \frac{a^2}{2R}.\frac{B_0}{\Delta t}$

$\therefore \quad T = (\lambda.2\pi R)E.R = 2\lambda\pi R^2 \times \frac{a^2}{2R}.\frac{B_0}{\Delta t}$

$\frac{\Delta L}{\Delta t} = \frac{\lambda\pi R.a^2 B_0}{\Delta t}$

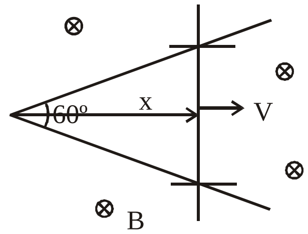
$\therefore \quad \Delta L = \lambda\pi R a^2 B_0$

$\therefore \quad I\omega = \lambda\pi R a^2 B_0$

$\therefore \quad \omega = \frac{\pi R a^2 \lambda B_0}{MR^2} = \frac{\pi a^2 \lambda B_0}{MR}$.

SOLUTION SUB. LEVEL - II

1. emf induced $= B.\frac{x}{\sqrt{3}} \times 2.V$



Total resistance $= \left(\frac{2}{\sqrt{3}}.x + 2\times\frac{2}{\sqrt{3}}.x\right)\lambda$

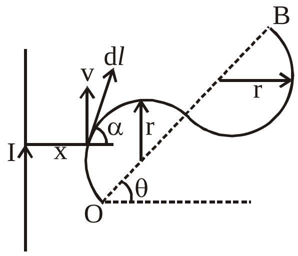
$= 2\sqrt{3}.x$

induced current $= \frac{\left(2/\sqrt{3}\right).Bvx}{(2\sqrt{3}.x)\lambda} = \frac{BV}{3\lambda}$

2. As

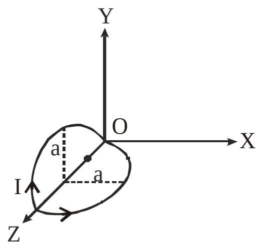
$e = \int B(x).V.dl\cos\alpha$

$= \int B(x).v.dx$



$$e = \int_{d-2r\sin\theta}^{d+2r\cos\theta} \frac{\mu_0 I}{2\pi x}.V.dx = \frac{\mu_0 I.V}{2\pi}\ln\frac{d+2r\cos\theta}{d-2r\cos\theta}$$

3. Flux associated with the loop



$\vec{A} = \frac{\pi a^2}{2}(+\hat{i}) + \frac{\pi a^2}{2}(\hat{j})$

$\vec{B} = \frac{B}{\sqrt{2}}\hat{i} + \frac{B}{\sqrt{2}}\hat{j}$

$\phi = \vec{B}.\vec{A}$

$= +\frac{\pi a^2}{2}\times\frac{B}{\sqrt{2}} + \frac{\pi a^2}{2}.\frac{B}{\sqrt{2}}$

$= +\frac{\pi a^2 B}{\sqrt{2}}$

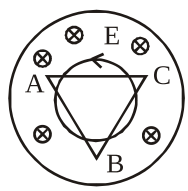
4. To lift the 'wire frame'

$\tau_g = \tau_B$

$mg\times\frac{a}{2} = a^2 i\times B$

$\therefore \quad i = \frac{mg}{2aB}$.

5.(a) As B increases with time, so induced electric field is set-up in such a way that it will produce magnetic field outward. Here $\vec{E}$ lines are concentric circle with anticlockwise sense.



(b) Again $\oint \vec{E}.\vec{dl} = -\frac{d\phi}{dt} = -\pi r^2 K$

$E = \frac{\pi r^2 K}{2\pi r} = \frac{r}{2}K$

$de = E.dx\cos[90-(60+\theta)]$

$= \frac{r}{2}K.dx.\sin(60+\theta)$

$$e=\int_0^a \frac{K.dx}{2}\times\frac{\sqrt{3}}{2}.\frac{a}{2}=\frac{\sqrt{3}K.a}{8}.a \qquad \left\{r\sin(60+\theta)=\frac{a}{2}\times\frac{\sqrt{3}}{2}\right\}$$

$$=\frac{\sqrt{3}Ka^2}{8}$$

(c) From symmetry, we can say that induced emf across BC is same as that of AB.

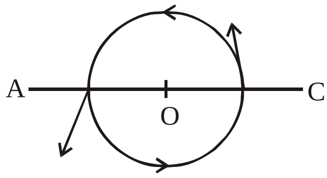
$\therefore \quad e_{BC}=\frac{\sqrt{3}}{8}a^2.K$

(d) For side CA, from symmetry about point *O*,

$V_C-V_0=V_A-V_O$

$\therefore \quad V_C-V_A=0$

$\therefore \quad V_{CA}=0$



(e) From faraday's law, emf induced across loop,

$$e_{loop}=\left|-\frac{d\phi}{dt}\right|=\left|-\frac{d}{dt}\left(-\frac{\sqrt{3}}{4}.a^2.B\right)\right|$$

$$=\frac{\sqrt{3}}{4}a^2.K$$

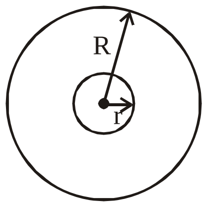
(f) Adding results of part (b), (c) and (d), we get

$$e_{AB}+e_{BC}+e_{CA}=\frac{\sqrt{3}}{8}a^2K+\frac{\sqrt{3}}{8}a^2K+0=\frac{\sqrt{3}}{4}.a^2K$$

The result is same as that of part (e)

6.(a) Let *I* be the current in bigger coil,

$$\vec{B}_{centre}=N_1\frac{\mu_0 I}{2R}$$



Flow across smaller coil $=N_2\pi r^2.B=\frac{\pi r^2\mu_0 I}{2R}\times N_1N_2$

$\therefore$ Mutual inductance, $M=\frac{\phi}{I}=\frac{\mu_0\pi r^2}{2R}.N_1N_2$

$$=\frac{4\pi\times10^{-7}\times\pi\times(2\times10^{-2})^2}{2\times20\times10^{-2}}\times20\times40=3.2\,\mu H$$

(b) emf induced in, B

$$e=M.\frac{dI}{dt}=3.2\times10^{-6}\times\frac{4}{2}=6.4\ \mu V$$

(c) $$dq=\int\frac{1}{R}d\phi=\frac{3.2\times10^{-6}\times4-0}{10}$$

$$=1.28\ \mu C$$

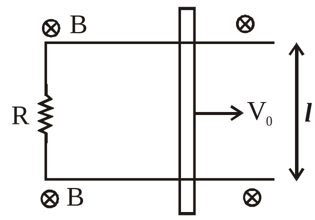
7.(a) Let x be the position of conductor from resistance R and *v* be the instantaneous velocity.

emf induced = B *l*. *v*

$\therefore$ current, $i=\frac{Blv}{R}$

$\therefore \quad F=-ilB=\frac{B^2l^2}{R}.v$

$$a=-\frac{B^2l^2}{mR}.v$$



$$v.\frac{dv}{dx} = -\frac{B^2l^2}{mR}.v \quad \int_{V_0}^{v} dv = \int_0^x \frac{-B^2l^2}{mR}.dx$$

$$v - v_0 = -\frac{B^2l^2}{mR}.x$$

$$v = v_0 - \frac{B^2l^2}{mR}.x$$

Again, $a = -\frac{B^2l^2}{mR}.v$

$$\frac{dv}{dt} = -\frac{B^2l^2}{mR}.v$$

$$\int_{V_0}^{V} \frac{dv}{v} = -\frac{B^2l^2}{mR}\int_0^t dt$$

$$[\ln v]_{v_0}^{v} = -\frac{B^2l^2}{mR}.t$$

$$\ln\left({}^{v}\!/_{v_0} \right) = -\frac{B^2l^2}{mR}.t$$

$$v = v_0.\exp\left(-\frac{B^2l^2}{mR}.t \right)$$

8. Time taken by the loop π complete a solution $\frac{2\pi}{\omega}$. After time $t = \frac{T}{2\omega}$ the loop will be 11th to magnetic field. Let at any time *t* the loop is inside the field and making angle θ with the field

$$\omega t = \theta + \frac{5}{2} \qquad \Rightarrow \qquad \theta = \left(\omega t - \frac{\pi}{2} \right)$$

Angle between the area normal areal field $= \pi - \theta$

Flux through the loop $\phi = \vec{B}.\overrightarrow{ds}$

$$= B.\frac{\pi a^2}{2}\cos(\pi - \theta)$$

$$= -B\frac{\pi a^2}{2}\cos\theta$$

$$= -B\frac{\pi a^2}{2}\sin\omega t$$

$$\varepsilon = -\frac{d\phi}{dt} = \frac{B\pi\omega a^2}{2}\cos\omega t$$

$$I = \frac{\varepsilon}{R} = \frac{B\pi\omega a^2}{2R}\cos\omega t$$

$$I_{\max} = \frac{B\pi\omega a^2}{2R}$$

$$< P >= I^2 R$$

$$= (B\pi\omega a^2)^2$$

9. $\frac{dB}{dt} = 0.010$ T/s

emf induced across loop, $e = \frac{\pi r^2}{2}.\frac{dB}{dt} = \frac{\pi \times (10 \times 10^{-2})^2}{2} \times 0.01$

$\therefore$ current induced in the loop $= \frac{e}{R} = \frac{\pi}{2} \times 10^{-4} \times \frac{1}{4}$

$$= 3.9 \times 10^{-5} \text{ A}$$

10. L = 20 mH, R = 10 Ω, V = 5.0 V

As $I = \frac{V}{R}[1 - e^{-tR/L}]$

$\therefore$ emf induced, $e_L = \frac{L.dI}{dt} = L.\left\{0 - \frac{V}{R}e^{-tR/L} \times \frac{R}{L}\right\}$

$e_L = V.e^{-tR/L}$

Rate of charge of induced emf,

$$\frac{de_L}{dt} = \frac{V.R}{L}.e^{-tR/L}$$

(a) $\left.\frac{de_L}{dt}\right|_{t=0} = \frac{5\times10}{20\times10^{-3}} = 2.5\times10^3 \ \frac{V}{S}$

(b) $\left.\frac{de_L}{dt}\right|_{t=10ms} = \frac{5\times10}{20\times10^{-3}} \times e^{-\frac{10\times10^3\times10}{20\times10^{-3}}} = 17 \ \frac{V}{S}$

(c) $\left.\frac{de_L}{dt}\right|_{t=1.0s} = \frac{5\times10}{20\times10^{-3}} \times e^{-\frac{1\times10}{20\times10^{-3}}} = 0 \ \frac{V}{S}$.

SOLUTION SUB. LEVEL - III

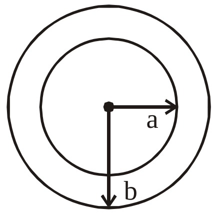
1. $e=(a^2-b^2)\dfrac{dB}{dt}=[0.2^2-0.1^2]\times B_0\omega$ cos ω t

$e=(0.2+0.1)\times 0.1\times 10\times 10^{-3}\times 100$ cos ω t

$i=\dfrac{0.3\times 0.1\times 10\times 10^{-3}\times 100}{50\times 10^{-3}\times 1.2}\cos\omega t$

∴ Amplitude of the current = 0.5 A

2. As magnetic filed varies with time, induced electric field is given by

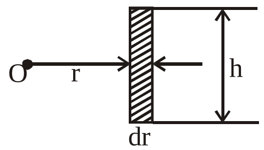


$\oint \vec{E}.\vec{dl}=-\dfrac{d\phi}{dt}=-\pi r^2\dfrac{dB}{dt}$

$E.2\pi r=-\pi r^2\dfrac{dB}{dt}$

or $|E|=\left|\dfrac{r}{2}\beta\right|$

Taking a differential element of are *h.dr* and applying Ohm's law, we get



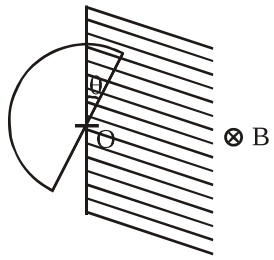
$J=\sigma E$

or $\dfrac{di}{dA}=\dfrac{1}{\rho}.\dfrac{r}{2}\beta$

$\int di=\dfrac{1}{\rho}\dfrac{\beta}{2}\int_a^b r.hdr$

∴ $i=\dfrac{\beta.h}{4\rho}(b^2-a^2)$

3. At any time 't'



$\theta=\dfrac{1}{2}\beta.t^2$

∴ Area of the sector, $=\dfrac{1}{2}a^2.\theta=\dfrac{a^2}{2}\times\dfrac{1}{2}\beta.t^2$

∴ Flux linked across semi-circle

$\phi=\dfrac{1}{2}a^2\theta B$

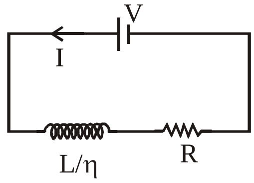
∴ $\dfrac{d\phi}{dt}=\dfrac{a^2B}{2}\times\dfrac{\beta}{2}\times 2t$

$e=\dfrac{a^2\beta.B}{2}t$

Taking anticlockwise as –ve and clockwise +ve, we have

∴ $e=(-1)^n.\dfrac{a^2\beta.B.t}{2}$

4. After a long time, the current in the circuit will be ε/R and so the flux associated with coil $=\dfrac{\varepsilon}{R}.L \quad =\dfrac{\varepsilon.L}{R}$



As inductance was decreased abruptly, the flux linked across the coil is constant i.e.,

$$\varepsilon.\frac{L}{R}=\frac{L}{\eta}.I_1 \qquad \text{i.e.,} \qquad I_1=\eta.\frac{\varepsilon}{R} \text{ at t = 0}$$

From Kirchhoff's voltage law,

$$\varepsilon-\frac{L}{\eta}.\frac{dI}{dt}-IR=0$$

$$\varepsilon-IR=\frac{L}{\eta}.\frac{dI}{dt}$$

or $$\int_{\frac{\eta\varepsilon}{R}}^{I}\frac{dI}{\varepsilon-IR}=\eta\int_0^t\frac{dt}{L}$$

or $$\left(\frac{1}{-R}\right)\left\{\ln[\varepsilon-IR]\right\}_{\frac{\eta\varepsilon}{R}}^{I}=\frac{t}{L}\eta$$

$$\ln\left[\frac{\varepsilon-IR}{\varepsilon-\eta\varepsilon}\right]=-\frac{t\eta R}{L}$$

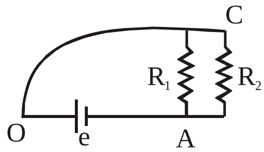
$$\frac{\varepsilon-IR}{\varepsilon(1-\eta)}=e^{-\left(\frac{\eta.Rt}{L}\right)}$$

$$\varepsilon-IR=\varepsilon(1-\eta)\;e^{-\eta Rt/L}$$

$$IR=\varepsilon\left[1-(1-\eta)e^{-\eta Rt/L}\right]$$

$$I=\frac{\varepsilon}{R}\left[1-(1-\eta)e^{-\eta Rt\;L}\right]$$

5. The circuit can be redrawn as



Here $R_1=\lambda.\ell\times\pi/2$

$$R_2=\lambda.\ell\times\frac{3\pi}{2}$$

$$\therefore \qquad R_{CA}=\frac{R_1R_2}{R_1+R_2}=\frac{(\lambda\ell)^2\,\pi/2\times3\pi/2}{(\lambda\ell)\times2\pi}=\frac{3}{8}\lambda\ell\pi$$

emf induced across OA, $e=\frac{1}{2}B\ell^2.\omega$

$\therefore$ Current in rode OA, $I=\frac{e}{R_{CA}}=\frac{1}{2}\frac{B\ell^2.\omega}{\frac{3}{8}\times\lambda\ell\pi}$

Also $R=\lambda\times2\pi\ell$

$$I=\frac{8}{3}\frac{B\ell^2\omega}{R}$$

6.(a) Let v be the speed of the frame at any instant.

emf induced = $B.d.v$

current in the wire, $i=\frac{B.d.v}{R}$

Force due to current on the frame

$$F_1=idB=\frac{B^2d^2}{R}.v$$

$\therefore$ Applying Newton's 2^{nd} law

$$a=\frac{F-F_1}{m}=\frac{F-\frac{B^2d^2}{R}.v}{m}$$

$$a = \frac{FR - B^2d^2.v}{mR}$$

(b) Again $\frac{dv}{dt} = \frac{FR - B^2d^2v}{mR}$

or $$\int_0^v \frac{dv}{FR - B^2d^2v} = \int_0^t \frac{dt}{mR}$$

$\therefore$ $$-\frac{1}{B^2d^2}\left|\ln[FR - B^2d^2.v\right|_0^v = \frac{t}{mR}$$

or $$\left|\ln\left\{\frac{FR - B^2d^2.v}{FR}\right\}\right|_0^V = -\frac{t}{mR}\times B^2d^2$$

$$\ln\left\{\frac{FR - B^2d^2v}{FR}\right\} = -\frac{t}{mR}B^2d^2$$

$$1 - \frac{B^2d^2}{FR}.v = e^{-\frac{B^2d^2}{mR}t}$$

$\therefore$ $$v = \frac{FR}{B^2d^2}\left[1 - e^{-\frac{B^2d^2}{mR}.t}\right]$$

for $v = v_0$ $\quad v_0 = \frac{FR}{B^2d^2}$

(c) Again, $v = v_0\left[1 - e^{-\frac{B^2d^2}{mR}.t}\right]$

$$v = v_0\left[1 - e^{-\frac{F.t}{mv_0}}\right]$$

7.(a) Induced emf across the connector

$$e = \int_a^b \frac{\mu_0 I}{2\pi r}.v.dr$$

$$e = \frac{\mu_0 I.v}{2\pi}\ln b/a$$

$\therefore$ induced current, $i = \frac{e}{R} = \frac{\mu_0 I.v}{2\pi R}\ln b/a$

(b) Force required to maintain the constant speed V,

$$dF = B.i\,dx$$

$$= \frac{\mu_0 I}{2\pi x}.i.dx$$

$$F = \frac{\mu_0 I}{2\pi}.\frac{\mu_0 IV}{2\pi R}\ln\frac{b}{a}\int_a^b \frac{dx}{x}$$

$$= \frac{\mu_0^2 I^2 V}{4\pi^2 R}\ln\left(b/a\right)\ln\left(b/a\right)$$

$$= \left(\frac{\mu_0 I.\ln\left(b/a\right)}{2\pi}\right)^2.\frac{V}{R}$$

8. As loop starts entering the region containing B field, flux across loop,

$$\phi = B\times\frac{1}{2}a^2.\theta = \frac{a^2B}{2}.\omega t$$

$\therefore \quad e = \frac{d\phi}{dt} = \frac{a^2 B\omega}{2}$

$\therefore$ Induced current, $i = \frac{e}{R} = \frac{a^2 B.\omega}{2R}$

Time in one revolution in which induced current will be present in the loop

$$= \frac{T}{4} = \frac{1}{4} \times \frac{2\pi}{\omega} = \frac{\pi}{2\omega}$$

heat generated in a revolution

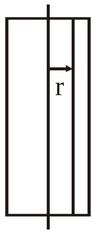
$$= \int_0^{\pi/2\omega} i^2 R.dt$$

$$= \left(\frac{a^2 B\omega}{2R}\right)^2 .R \times \frac{\pi}{2\omega}$$

$$3.14 \times 10^{-3} = \frac{a^4 .B^2 \omega^2}{4R} \times \frac{\pi}{2\omega}$$

$$\omega = \frac{10^{-3} \times 8 \times R}{a^4 \times B^2} = 100 \quad \text{rad/s}$$

9. $$\oint \vec{B}.\vec{dl} = \mu_0 \left(\frac{I}{\pi a^2}.\pi r^2\right)$$



$\therefore \quad B.2\pi r = \frac{\mu_0 .I.r^2}{a^2}$

$$B = \frac{\mu_0 .I.r}{2\pi a^2}$$

Again energy stored inside the rod

$$U = \left| \frac{1}{2\mu_0} .B^2 .2\pi rl.dr \right.$$

$$= \frac{\pi l}{\mu_0} \int_0^a \left(\frac{\mu_0 I.r}{2\pi a^2}\right)^2 .rdr$$

$$= \frac{\pi l.\mu_0^2 .I^2}{\mu_0 4\pi^2 a^4} \int_0^a r^3 dr$$

$$\frac{1}{2} LI^2 = \frac{\mu_0 l}{16\pi a^4} a^4 .I^2$$

$$L = \frac{\mu_0 l}{8\pi}$$

10. From equilibrium condition

$$a^2 iB \sin(90 - \theta) = mg \frac{a}{2} \sin\theta$$

$\therefore \quad i = \frac{mg \tan\theta}{2aB}$

SOLUTION SUB. IIT-JEE LEVEL

1.(a) Here torque must balance gravity.

Now,

$$\vec{\tau}_B = \vec{m} \times \vec{B} = m\hat{k} \times (3\hat{i} + 4\hat{k}) B_0 = 2mB_0 \hat{j}$$

Thus *m* is negative

$\therefore$ I should be clockwise

(b) $\vec{F} = I(\vec{l} \times \vec{B}) = I[(-b\hat{j}) \times (3\hat{i} + 4\hat{k})B_0]$

$= IB_0 b[3\hat{k} - 4\hat{i}]$

(c) $mg(a/2) = (abI)3B_0$

$\Rightarrow \quad I = \frac{mg}{6B_0 b}$

2. For the circuit shown taking Kirchoffs law or considering voltage

(a) $E - L\frac{dl}{dt} - lR = 0$

$\Rightarrow \quad \left|\frac{dl}{dt}\right| - \frac{Ldl}{dt} = lr$

(b) $\int d\varphi = \int_{X_0}^{2x} \frac{\mu_0 l_0}{2\pi x} dx - Li_1$

$= \frac{\mu_0 l_0}{2\pi} \ln 2 - Li_1$

Net charge which flows through resistance

$R = \frac{\text{Total charge in flux}}{\text{R}} = \frac{\mu_0 l_0 I}{2\pi} \ln 2 - Li_1$

(c) We know that

$l = l_l e^{-(t-T)^{RL}}$ …(i)

for $\quad T \le t \le 2T$

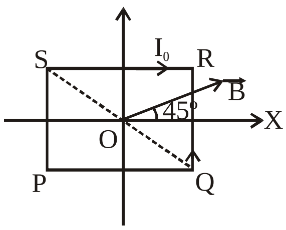
When $t = 2T \;\; l = l_1/4$

Sub. in 1

$\frac{l_l}{4} = l_l e^{-(2T-R)^{RL}}$

$\Rightarrow \quad \frac{L}{R} = \frac{T}{2\ln 2}$

3.(a) I_0 be the current flown through the wire $\vec{B}$ is the magnetic field.



$\vec{B} = B\cos 45° \hat{i} + B\sin 45° \hat{j}$

$\therefore \quad B = (Bi + Bj)\frac{1}{\sqrt{2}}$ …(i)

$\vec{\tau} = I_0(\vec{A} \times \vec{B})$

where $\vec{A} =$ Area $= L^2\hat{K}$

$\therefore \quad \vec{\tau} = I_0 L^2 \hat{K} \times (B\hat{i} + B\hat{j})\frac{1}{\sqrt{2}}$

$\vec{\tau} = \frac{BI_0L^2}{\sqrt{2}}(\hat{j} - \hat{i})$

$\therefore \quad |\vec{\tau}| = BI_0L^2$

(b) Since $|\vec{\tau}| = BI_0L^2$

and α = angular acceleration $= \frac{\tau}{I}$

$I_{PQ} = \frac{1}{2}\left[\frac{4}{3}ML^2\right]$

[From perpendicular axis theorem $I = I_x + I_y$]

$\therefore \quad \alpha = \frac{BI_0L^2}{2/3ML^2}$

$= \frac{3BI_0}{2M}$

Now $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha.t^2$

For given value of all the quantities we have

$\theta = \frac{3BI_0.(\Delta t)^2}{4M}$

4. Let m = mass of bar L = 0.2 kg

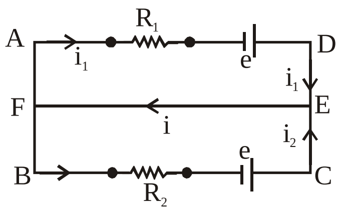
When x = length of the rail

V = terminal velocity of the bar $= \frac{dx}{dt}$

e = emf induced in the upper circuit

= emf induced in the lower circuit = B/V

i = current flowing through the rod



F_{mag} = magnetic force acting on the rod = I *l* B

According to the Lenz's law, the direction of current in the two circuits are shown in figure.

When terminal velocity is attained.

$F_{mag} = mg$

i/B = *mg*.

$\therefore \quad i = \frac{0.2 \times 9.8}{1 \times 0.6} = 3.267\,A$.

Now applying Kirchhoff's law to the two circuit, we get

For the circuit ACEFC, e = i_1R_1 …(i)

For the circuit BDEFD, e = i_2R_2 …(ii)

At the point E, $i = i_1 + i_2$ …(iii)

Multiplying both sides of equation (iii) by e, we get

$ei = ei_1 + ei_2$

$ei = i_1^2R_1 + i_2^2R_2$ By (i) and (ii)

Given that power dissipated in R_1 and R_2 are 0.76 W and 1.2 W respectively

$\Rightarrow \quad ei = 0.76 + 1.2 = 1.96$

$\Rightarrow \quad e = \frac{1.96}{3.267} = 0.6\,V$

Terminal velocity, $V = \frac{e}{Be} = \frac{0.6}{0.6 \times 1} = 1ms^{-1}$

Again power dissipated in R_1 = $ei_1 = 0.76\,w$

$\Rightarrow \quad i_1 = \frac{0.76}{e} = \frac{0.76}{0.6} = 1.267\,A$

power dissipated in R2 $= ei_2 = 1.2\,w$

$\Rightarrow \quad i_2 = \frac{1.2}{e} = \frac{1.2}{0.6} = 2A$

By equation (i), $R_1 = \frac{e}{i_1} = \frac{0.6}{1.267} = 0.474\Omega$

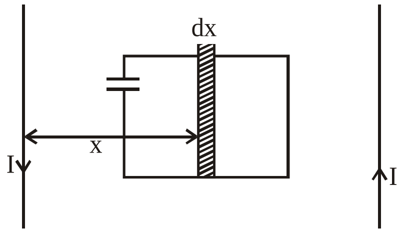
$\therefore \quad R_1 = 0.474\Omega$

By equation (ii), $R_2 = \frac{e}{i_2} = \frac{0.6}{2} = 0.3\Omega$

$\therefore \quad R_2 = 0.3\Omega$

5. Flux through the square loop

$$= \int_a^{2a} \frac{\mu_0}{4\pi} 2I \left| \frac{1}{x} + \frac{1}{3a - x} \right| adx = \frac{\mu_0}{4\pi} l\, a 4 \ln\, 2$$



Induced emf $e = -\frac{d\phi}{dt} = \frac{\mu_0}{\pi} a l_0 \omega\, \ln 2 \cos \omega t$

Charge on the capacitor

$$Q = Ce = -C\frac{\mu_0}{\pi} a l_0 \omega \ln\, 2 \cos \omega t$$

$$= -Q_0 \cos \omega t \;\; \text{(say)}$$

Current in the loop

$$= \frac{dQ}{dt} = \frac{\mu_0}{\pi} C I_0 \omega^2 a\, \ln\, 2 \sin \omega t$$

$$I_{\max} = \frac{\mu_0}{\pi} C I_0 a\, \omega^2 \ln 2$$