$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time}}$$

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

Its direction is same as that of displacement which is from initial to final position.

Average speed $\geq$ |Average velocity|

$$\text{Average acceleration} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$$

Its direction is in the direction of change in velocity.

1. When a body is moving along a straight line with a constant speed, then

(*i*) Distance travelled = speed $\times$ time

(*ii*) Distance travelled = average speed $\times$ time

2. When a body travels half the distance at speed v_1, and the other half distance at speed v_2, then the average speed is given by the relation :

$$\text{Average speed} = \frac{2v_1v_2}{v_1+v_2}$$

Ex.1: A motor-cyclist goes from one station to the other at a speed of a 40 km/hr, but comes back at a speed of 60 km/hr. Find the average speed of the journey.

Sol.: $\text{Average speed} = \frac{2\times 40\times 60}{40+60} = \frac{4800}{100} = 48 \text{ km/hr}$

3. When a body travels at a speed of v_1 for the first half of the time and at a speed v_2 the for the second half of the time of its motion, then the average speed during total time of motion is

$$\text{Average speed} = \frac{v_1+v_2}{2}$$

Ex.2: A truck travels at a speed of 40 km/hr during first half of the total journey. After some unloading, during second half of the journey, the truck travels at a speed of 50 km/hr during first half time and at a speed of 70 km/hr during second half time (due to further unloading). Calculate the average speed of the truck during its entire journey.

Sol.: *During second half journey*, because the speed is 50 km/hr during first half of time and 70 km/hr during second half of time.

$\therefore$ Average speed during second half of journey $= \frac{v_1+v_2}{2} = \frac{50+70}{2} = 60 \text{ km/hr}$

$\therefore$ The speed is 40 km/hr during first half of journey and 60 km/hr during second half of journey.

$\therefore$ Average speed during entire journey $= \frac{2v_1v_2}{v_1+v_2} = \frac{2\times 40\times 60}{40+60} = 48 \text{ km/hr}$ ..... **Ans.**

4. When a body is moving with initial velocity u and with constant acceleration a such that in time t its velocity becomes v and its displacement is s, then

(*i*) $v = u + at$ ...... (*i*)
(*ii*) $s = ut + \frac{1}{2} at^2$ ...... (*ii*)
(*iii*) $v^2 = u^2 + 2as$ ...... (*iii*)

Also, in the above case, displacement covered in *n*th second (one particular second) is

$$S_{nth} = u + \frac{a}{2}(2n-1) \qquad \text{...... (iv)}$$

Proof for relation (*iv*) above :

S_{nth} = Distance travelled in *n*th second

= [Displacement covered in *n* seconds] – [Displacement covered in (*n* – 1) seconds]

$= [un + \frac{1}{2}an^2] - [u(n-1) + \frac{1}{2} a (n-1)^2]$ ...... [using (*ii*), $t = n$ and $t = n - 1$]

$= un + \frac{1}{2} an^2 - [un - u + \frac{1}{2} a (n^2 - 2n + 1)]$

$= un + {}^1/_2\, an^2 - un + u - {}^1/_2\, an^2 + {}^1/_2\, a \times 2n - {}^1/_2\, a$

$= u + {}^1/_2\, a \times 2n - {}^1/_2\, a$

$= u + \frac{a}{2}(2n - 1)$

Ex.3: A body covers 19 m in fourth second of its motion and 27 m in sixth second of its motion. Find (*i*) distance travelled in 10 s of its motion and (*ii*) distance travelled in 10th second of its motion.

Sol.: Using the relation $S_{nth} = u + \frac{a}{2}(2n - 1)$ and putting values for the two cases, we get

$19 = u + \frac{a}{2}(2 \times 4 - 1) \Rightarrow 19 = u + \frac{7a}{2}$ ...... (*i*)

$27 = u + \frac{a}{2}(2 \times 6 - 1) \Rightarrow 27 = u + \frac{11a}{2}$ ..... (*ii*)

By subtraction, we get $8 = \frac{4a}{2} \Rightarrow a = 4\ ms^{-2}$ ..... (*iii*)

From (*i*) $19 = u + \frac{7}{2} \times 4 \Rightarrow u = 5\ ms^{-1}$ ..... (*iv*)

$\because$ (a) *s* = distance travelled in 10 s (using $s = ut + \frac{1}{2} at^2$)

$= 5 \times 10 + \frac{4}{2}(10)^2 = 50 + 200 = 250$ m

(b) s_{nth}= distance travelled in 10th *s* [using $s_{nth} = u + \frac{a}{2}(2n - 1)$]

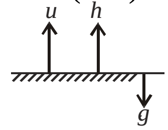
$= 5 + \frac{4}{2}(2 \times 10 - 1) = 5 + 38 = 43$ m

5. Fall of a Body under Gravity

When a body falls under gravity (due to the pull of earth), the acceleration (*a*) in the relations (*i*), (*ii*), (*iii*) and (*iv*) is replaced by '*g*. Take downward direction as positive and upward direction to be negative. However, remember following cases for assigning + and – sign to *g*, *u* and *s* (or *h*).

(*a*) *When a body is thrown up from the ground*

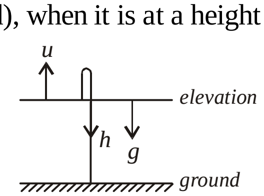
u and *h* are taken as negative but *g* is taken as positive.



(b) *When a body is thrown up from an elevation or a height* (like roof of a building) or when body is released from a rising balloon or a helicopter (with uniform speed), when it is at a height *h* above the ground and the body finally hits the ground.

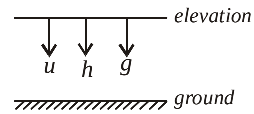
u is taken as negative

g and *h* are taken as positive



(c) *When a body is thrown down from an elevation or height h*, so that it hits the ground.

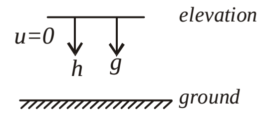
u, g and h – all are taken as positive



(d) *When a body is just released from a height h above the ground.*

then $u = 0$

g and h are taken as positive



Ex.4: A body is thrown up vertically with a velocity of 98 ms^{-1}. After what time will it be at a height of 411.6 m above the ground? Explain the significance for two values of times in the answer.

Sol.: $u = 98\ ms^{-1}$ $\quad h = 411.6$ m $\quad g = -9.8\ ms^{-2}$ $\quad t = ?$

Using the relation $h = ut + ½\, gt^2$, and putting proper values, we get

$$411.6 = 98t - \frac{1}{2} \times 9.8 \times t^2$$

$$\Rightarrow \quad 84 = 20t - t^2 \qquad \Rightarrow \quad t^2 - 20t + 84 = 0$$

$$\therefore \quad t = \frac{20 \pm \sqrt{400 - 4 \times 84}}{2} = \frac{20 \pm \sqrt{400 - 336}}{2} = \frac{20 \pm \sqrt{64}}{2} = 14 \text{ or } 6$$

$\Rightarrow$ The body is at a height of 411.6 *m* above the ground at 6 *s* as well as at 14 *s* from the time of its projection. The first time of 6 *s* is when the body is crossing the point in the upward direction and the second time of 14 *s* is when the body is crossing the same point while falling down (after rising further and then the falling back).

Ex.5: A helicopter is rising vertically up at a uniform speed of 10 ms^{-1}. When it is at a height of 75 m, a packet is just released from it. After what time will the packet hit the ground ? Take $g = 10\ ms^{-2}$.

Sol.: $u = -10\ ms^{-1}$ $\quad h = 75$ m $\quad g = 10\ ms^{-2}$ $\quad t = ?$

Using the relation $h = ut + ½gt^2$ and putting proper values, we get

$$75 = -10t + \frac{1}{2} \times 10t^2 \quad \Rightarrow \quad 5t^2 - 10t - 75 = 0 \quad \Rightarrow \quad t^2 - 2t - 15 = 0$$

$$\therefore \quad t = \frac{2 \pm \sqrt{4 + 4 \times 1 \times 15}}{2 \times 1} = \frac{2 \pm \sqrt{4 + 60}}{2} = \frac{2 \pm 8}{2}$$

$\Rightarrow \quad t = 5s$ (neglecting negative value of t)

Ex.6: A body is thrown up vertically. It crosses the same position after 5 s and 7s, after being thrown up. Calculate (i) the initial velocity with which it is thrown up and (ii) the maximum height attained by it.

Sol.: (i) Let s be the height (displacement) of the point which the body crosses twice (once while going up and once while coming down). Let u be the initial velocity with which body is thrown up

$\therefore \quad s = u \times 5 - {}^1/_2 \times 9.8 \times 5^2$ .....(i) using $s = ut + {}^1/_2\, at^2$.

$s = u \times 7 - {}^1/_2 \times 9.8 \times 7^2$ .....(ii)

$\Rightarrow \quad 7u - {}^1/_2 \times 9.8 \times 49 = 5\,u - {}^1/_2 \times 9.8 \times 25$

$2u = 4.9\,(49 - 25) = 4.9 \times 24$

$u = 4.9 \times 12 = 58.8\ ms^{-1}$ ..... **Ans.**

(ii) Let H be the maximum height upto which it rises, where its velocity becomes zero.

$\therefore \quad u = 58.8\ ms^{-1}$ $\quad v = 0$ $\quad a = -g = -9.8\ ms^{-2}$

$\therefore \quad 0^2 - (58.8)^2 = 2(-9.8)H$ $\quad [v^2 - u^2 = 2\,a\,s]$

$$\Rightarrow \quad H = \frac{58.8 \times 58.8}{2 \times 9.8} = 176.4 \text{ m}$$ ..... **Ans.**

Ex.7: When a body is thrown up from a height with velocity u, it takes time t_1 to reach the ground. When the same body is thrown down from the same point with same velocity u, it takes time t_2 to reach the ground. Now, if the same body is just released from the same point show that, it will reach ground in time $\sqrt{t_1 t_2}$

Sol.: For first case $h = -ut_1 + {}^1/_2\, gt_1^2$ ..... (*i*)

For second case $h = ut_2 + {}^1/_2\, gt_2^2$ ..... (*ii*)

Multiplying (*i*) by t_2 and (*ii*) by t_1 and then adding, we get

$h\,(t_1 + t_2) = {}^1/_2 g t_1 t_2\,(t_1 + t_2)$

$\Rightarrow \quad h = {}^1/_2 g\, t_1\, t_2$ ..... (*iii*)

Now, for third case, $h = 0 + {}^1/_2 gt^2$ ..... (*iv*)

Comparing (*iii*) and (*iv*) we get $t = \sqrt{t_1\, t_2}$ .....**Ans.**

Ex.8 : A body while falling freely covers 44.1 m in last second of its motion. For how much time and from what height does body fall ?

Sol.: Let n^{th} be the last second of motion of the body

$44.1 = 4.9\,(2n - 1) \Rightarrow 2n - 1 = 9$ [using $s_{nth} = u + \frac{a}{2}(2n - 1)$]

$\therefore \quad n = 5$ sec.

$\Rightarrow$ The body falls for a total of 5 seconds .....**Ans.**

$h = {}^1/_2\, gt^2 = {}^1/_2 \times 9.8 \times 25 = 122.5$ m

$\Rightarrow$ The body falls from a height of 122.5 m .....**Ans.**

Ex.9: A stone of specific gravity 5 is just released from the surface of a pond 36 m deep. Find (i) the speed with which the stone will strike the bottom of the pond and (ii) the time after which the stone will hit the bottom of the pond. (Take $g = 10$ ms^{-2})

Sol.: As stone falls through water (density of which cannot be ignored as compared to density of stone) so,

$$a = g\left[1 - \frac{\rho_0}{\rho}\right]$$

Here, ρ_0 = density of medium (water) = 10^3 kg m^{-3}

ρ = density of body = sp.gr × 10^3 kg m^{-3} = 5 × 10^3 kg m^{-3}

(i) $\therefore \quad a = g\left[1 - \frac{\rho_0}{\rho}\right] = 10\left[1 - \frac{1}{5}\right] = 10 \times \frac{4}{5} = 8 \text{ m s}^{-2}$

$u = 0 \qquad s = 36 \text{ m} \qquad v = ?$

Using $v^2 - u^2 = 2\,as$, we get

$v^2 - 0^2 = 2 \times 8 \times 36 \quad \Rightarrow \quad v = 24 \text{ m s}^{-1}$ ..... **Ans.**

(ii) Using $v = u + at \quad \Rightarrow \quad t = \frac{v-u}{a} = \frac{24-0}{8} = 3$ sec. ..... **Ans.**

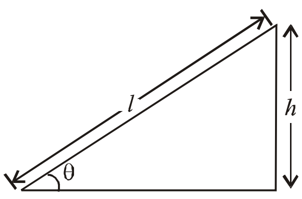
6. Motion along Inclined Smooth Surface

(*i*) When the body is moving down the plane: $a = g \sin\theta$

(*ii*) When the body is moving up the plane : $a = -g \sin\theta$

(*iii*) Velocity at the bottom = $\sqrt{2gh} = \sqrt{2gl\sin\theta}$

(*iv*) t, time taken to reach the bottom



$$= \left(\frac{2l}{g\sin\theta}\right)^{1/2} = \left(\frac{2h}{g\sin^2\theta}\right)^{1/2} = \frac{1}{\sin\theta}\left(\frac{2h}{g}\right)^{1/2}$$

(*v*) Keeping length of inclined plane same, if θ is increased from θ_1 to θ_2, then ratio of times taken to reach the bottom of inclined plane :

$$\frac{t_1}{t_2} = \left(\frac{\sin\theta_2}{\sin\theta_1}\right)^{1/2}$$

(*vi*) Keeping vertical height same if θ is increased from θ_1 to θ_2 (by changing l), then ratio of time taken to reach the bottom of inclined planes :

$$\frac{t_1}{t_2} = \frac{\sin\theta_2}{\sin\theta_1}$$

7. Some Related Examples

(*i*) When a body is released from a height above the ground then the ratio of distances travelled by the body in 1st, 2nd, 3rd, 4th,...... *n*th second is 1 : 3 : 5 .... : (2*n* – 1)

$\because \quad u = 0 \quad s_{nth} = \frac{g}{2}(2n-1)$

(*ii*) When a body is released from a height above the ground then the ratio of distances travelled by the body in 1 *s*, 2 *s*, 3 *s*, ... *n s* seconds is 1 : 4 : 9 .... : n^2

$\because \quad u = 0 \quad s = \frac{g}{2}.n^2$ $\qquad [s = ut + \frac{1}{2}gt^2, t = n]$

(*iii*) If a body moving with velocity *v* can be stopped by a force (or by the application of brakes) within a distance *s*, then the same body with velocity *nv* will be stopped by same force (or same brakes) within distance n^2 s.

$\because \quad 0^2 - v^2 = 2(-a)s$ . ... (*i*)

$0^2 - (nv)^2 = 2(-a)s'$ .... (*ii*) $\Rightarrow s' = n^2 s$

(*iv*) If we ignore the resistance of air, and a body is thrown vertically upwards, then with respect to any point on the line of its motion,

(a) time of ascent = time of descent

(b) the body crosses the point (while moving upwards or moving downwards) with same speed.

(*v*) If we don't ignore the resistance of air, and a body is thrown vertically upwards, then

(a) time of ascent < time of descent

(*vi*) When a body is falling freely (with no resistance), then distance travelled by body

in 1st second = 4.9 m

in 2nd second = 4.9 × 3 m

and in *n*th second = 4.9 (2*n* – 1) m

(*vii*) When a body of density ρ is falling under gravity through a medium of density ρ_0. (*e.g.* a stone falling through water, under gravity), then the acceleration (g') with which it falls down is given by the relation

$$g' = g\left(1 - \frac{\rho_0}{\rho}\right)$$

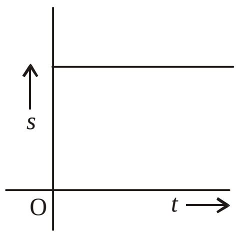
For air, ρ_0, is very small and hence negligible $\therefore\ g' = g$

8. Graphs & Motion of a body in a Straight Line

Displacement-time graphs

The slope of displacement-time graph, at any point, gives velocity at that point. The following graphs are possible in this case :

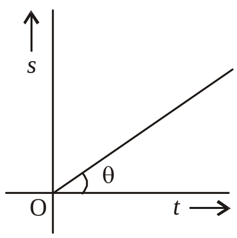
(a)



Here displacement is constant so the particle is at rest

Here, displacement increases with time. So the slope gives velocity. Further, the graph is a straight line. So, the velocity is uniform.

(b)



Slope = $\frac{dy}{dx}$ = tan θ

Here displacement is along *y*-axis, and time is along *x*-axis,

$\therefore\ \frac{dy}{dx}$ becomes $\frac{ds}{dt}$ and $\frac{ds}{dt}$ = velocity

⇒ slope of displacement-time graph gives velocity at any point.
In this case, because slope is same at all points, therefore the body is moving with uniform velocity.

Ex.10: The displacement-time graphs for bodies *A* and *B* make angles of 30º and 60º with time-axis. Find ratio of magnitude of velocities of *A* to *B*.

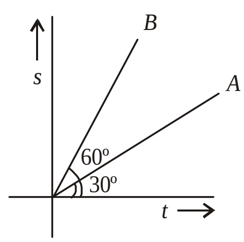
Sol.: v_A = velocity of *A* = slope of displacement-time graph of body *A*.

= tangent of angle which this graph (or line) makes with positive side of x-axis

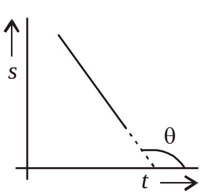
= tan 30º = $\frac{1}{\sqrt{3}}$

Similarly, v_B = tan 60º = $\sqrt{3}$

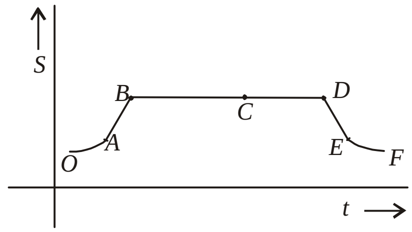
$\therefore\ \frac{v_A}{v_B} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$



(c)



Here displacement decreases with time. So, the slope of the line gives negative velocity; also the slope is negative *i.e*; tan θ is negative, because angle θ is obtuse.

(d) 

(*i*) During $O \rightarrow A$, velocity is positive and increasing ($\because$ slope is positive and increasing) so the body is accelerating

(*ii*) During $A \rightarrow B$, the velocity is constant ($\because$ slope is constant)

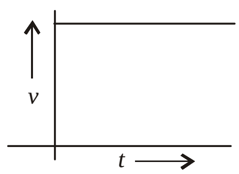
(*iii*) During $B \rightarrow C \rightarrow D$, body is at rest ($\because$ displacement does not change with time)

(*iv*) During $D \rightarrow E$, velocity is negative but uniform so body is non accelerated.

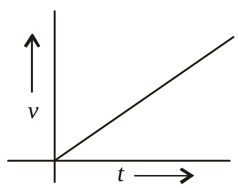
(*v*) During $E \rightarrow F$, velocity is negative but increasing so the body is accelerating

Velocity-time graphs

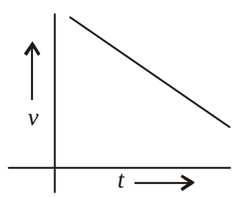
Slope of velocity-time graph, at any point, gives acceleration at that point. The graphs in this can be enterpreted as under :

(a) 

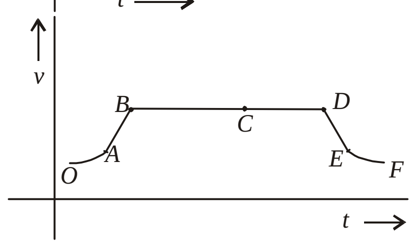
represents uniform velocity of a body.

(b) 

represents uniformly increasing velocity or uniform acceleration of a body

(c) 

represents uniformly decreasing velocity or uniform retardation of a body.

(d) 

(*i*) During $O \rightarrow A$, slope is +ve and increasing and hence body is having increasing acceleration.

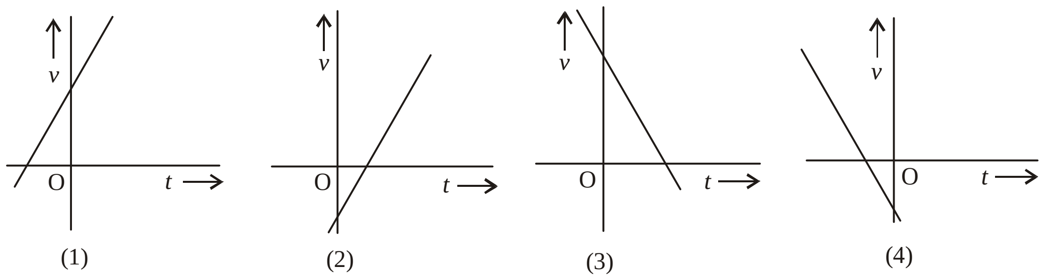
(*ii*) During $A \rightarrow B$, slope is +ve and uniform, so acceleration is uniform.

(*iii*) During $B \rightarrow C$, velocity is constant or there is no acceleration.

(*iv*) During $D \rightarrow E$, slope is uniform and negative, so acceleration is negative and uniform

(*v*) During $E \rightarrow F$, slope is –ve but increasing so the body is accelerating, but acceleration is not uniform.

Ex.11: A body has a velocity which varies with time and is given by the relation : $v = -2 + 4t$. Which one of the following graphs gives its velocity with time t ?



Sol.: Given equation $v = -2 + 4t$ ...... (*i*)

Standard equation $y = c + mx$ ...... (*ii*)

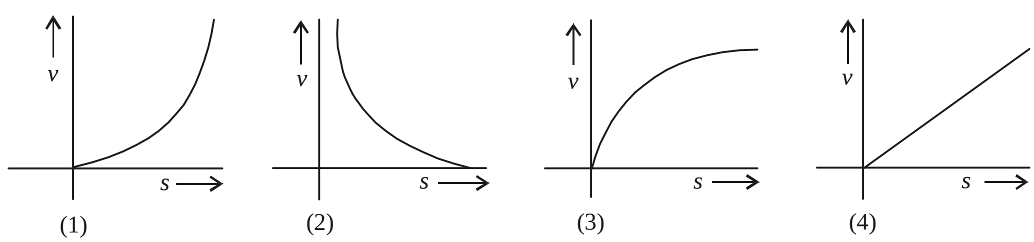
By comparison, $c = -2 \Rightarrow$ intercept $= -2$ (negative) ...... (*iii*)

$m = 4 \Rightarrow \tan\theta = 4$ ...... (*iv*)

(*iii*) means that line cuts *y*-axis at 2 units on –*y*-axis *i.e*; 2 units below the origin on *y*-axis. So (1) and (3) are not possible.

(*iv*) means tan θ is positive (+4), so the angle which the line makes with positive side of *x*-axis in less than 90º ($\because$ tangent of an angle is positive, when angle is in first quadrant or angle is less than 90º). So, (4) is not possible. Hence (2) is the answer.

Ex.12: A body is released from a height towards the ground. Which one of the following graphs represents the relation between (*i*) distance travelled and (*ii*) velocity gained ?



Sol.: $v^2 - 0^2 = 2gs \quad \Rightarrow \quad v^2 \propto s$

Clearly graph is parabola about *s* (linear variable).

$\therefore$ (3) represents the relation between *v* and *s*.

Velocity time-graph (general)

If we draw a graph between velocity along y-axis and time along x-axis, then the area under the curve on x-axis gives the displacement of the body. the area under the curve on *x*-axis gives the magnitude of a physical quantity, the units of which are the products of the variables along the two axes.

(*i*) Area of curve for graph between *F* along *y*-axis and *t* along *x*-axis gives impulse or change in momentum

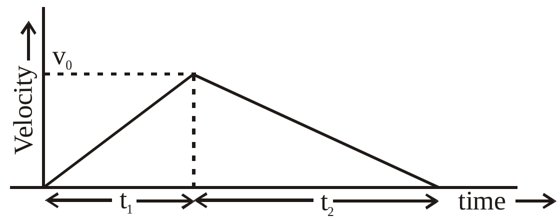
(*ii*) Area of graph between *P* along *y*-axis and *V* along *x*-axis, for a given gas enclosed in an enclosure, gives work done by the gas during expansion or work done on the gas during compression

Ex.13: A body, starting from rest accelerates along a straight path at the rate of α for some time. On attaining some speed, it is immediately retarded at the rate of β to bring it to rest. If the total time taken for the journey is t, find

(i) the maximum speed attained by the body and

(ii) total distance travelled by the body.

Sol.: The motion of the body can be represented graphically as under. Let the body be accelerated for time t_1, till its maximum velocity becomes v_0, and then decelerated for time t_2 till it comes to rest again. So that $t = t_1 + t_2$ ..... (i)



(i) *During acceleration*

$u = 0,\qquad v = v_0,\qquad a = \alpha,\qquad t = t_1$

$$\therefore\; t_1 = \frac{v_0 - 0}{\alpha} = \frac{v_0}{\alpha} \qquad \text{..... (ii) using } v - u = at$$

During retardation

$u = v_0\qquad v = 0\qquad a = -\beta\qquad t = t_2$

$$t_2 = \frac{0 - v_0}{-\beta} = \frac{v_0}{\beta} \qquad \text{..... (ii) using } v - u = at$$

$$\therefore\; t = t_1 + t_2 = \frac{v_0}{\alpha} + \frac{v_0}{\beta} = v_0\frac{\alpha + \beta}{\alpha\beta} \quad\Rightarrow\quad v_0 = \frac{\alpha\beta}{\alpha + \beta}t \qquad \text{..... } \textbf{Ans.}$$

(ii) Area under the curve on time-axis gives the distance travelled by the body.

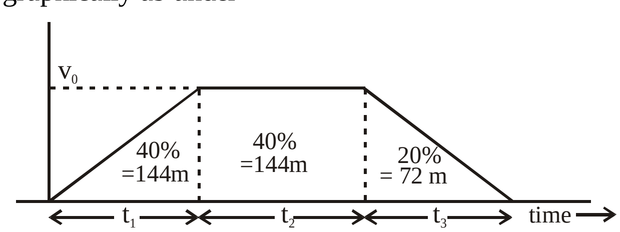
$\therefore\; s$ = distance travelled = area of triangle

$$s = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times t \times v_0$$

$$s = \frac{1}{2}t\left[\frac{\alpha\beta}{\alpha+\beta}t\right] \quad\Rightarrow\quad s = \frac{1}{2}\frac{\alpha\beta}{\alpha+\beta}t^2 \qquad \text{..... } \textbf{Ans.}$$

Ex.14: A body at rest is accelerated uniformly and covers 40% of the distance during acceleration. Then it covers another 40% of the distance with the speed attained. Brakes are applied to bring it to rest, covering the remaining distance. If total distance travelled is 360 m in 24 seconds, find (i) the maximum speed attained (ii) acceleration and (iii) retardation given to the body.

Sol.: The problem is shown graphically as under



Let v_0 be the maximum speed attained. Now, area under velocity-time graph on time-axis gives the distance travelled. From the graph

$$\tfrac{1}{2} v_0 t_1 : v_0 t_2 : \tfrac{1}{2} v_0 t_3 \equiv 144 : 144 : 72 \equiv 2 : 2 : 1$$

$$\Rightarrow \quad t_1 : 2t_2 : t_3 \equiv 2 : 2 : 1$$

$$\Rightarrow \quad t_1 : t_2 : t_3 \equiv 2 : 1 : 1$$

$$\therefore \quad t_1 = \frac{2}{4} \times 24 = 12 \text{ s} \qquad t_2 = \frac{1}{4} \times 24 = 6 \text{ s} \qquad t_3 = \frac{1}{4} \times 24 = 6 \text{ s.}$$

$\therefore$ (i) $v_0 \times t_2 = 144 \quad \Rightarrow \quad v_0 = \frac{144}{6} = 24 \text{ m s}^{-1}$ ..... **Ans.**

(ii) $\tfrac{1}{2} \alpha \, t_1^2 = s_1 = 144$

$\Rightarrow \quad \alpha = \frac{2 \times 144}{12 \times 12} = 2 \text{ m s}^{-2}$ ..... **Ans.**

(iii) $\tfrac{1}{2} \beta \, t_3^2 = s_3 = 72$

$\Rightarrow \quad \beta = \frac{2 \times 72}{6 \times 6} = 4 \text{ ms}^{-2}$ ..... **Ans.**