# Chapter-4

# ELECTRO-MAGNETIC INDUCTION & E.M. WAVES

1. Magnetic flux means magnetic lines of force. Its units are webers(Wb) or Tesla-metre$^2$ (T m$^2$). Its dimensions $ML^2T^{-2}A^{-1}$.
2. If the nomal to a plane points in the direction of magentic field, the magnetic flux is taken as positive; and if the normal points in the opposite direction, the magnetic flux is taken as negative.
3. Magnetic flux ($\phi$) associated with an area $A$ in a magnetic field of induction $B$ is given by the relation :

$$\phi = \vec{B} \cdot \vec{A} = BA \cos\theta \text{ if } \vec{B} \text{ is uniform over area } \vec{A}, \quad .....(i)$$

where $\theta$ is the angle between directin of $B$ and normal to $A$

$$\phi = \int \vec{B}.d\vec{A}, \text{ if } \vec{B} \text{ is not uniform} \quad .....(ii)$$

4. Whenever magnetic flux changes through a coil, then at the ends of the coil, e.m.f., is produced. This e.m.f. is called induced e.m.f. and the phenomenon is called electro-magnetic induction.
5. The induced e.m.f. is given by Faraday's laws of electro-magnetic induction :
   (i) Whenever magnetic flux changes through the coil, induced e.m.f. is produced.
   (ii) The induced e.m.f. produced is directly proportional to rate of change of magnetic flux, i.e.,

$$E \propto \frac{d\phi}{dt} \quad \Rightarrow \quad E \propto \frac{d}{dt}(\vec{B} \cdot \vec{A}) \quad .....(i)$$

   (iii) The induced e.m.f. produced is proportional to the number of turns in the coil i.e. $E \propto N$ .....(ii)

$\therefore$ Combining the above relations, we get

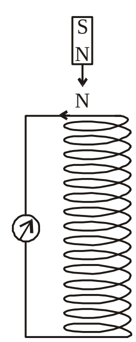
$$E \propto N\frac{d\phi}{dt} \quad \text{or} \quad E \propto \frac{d}{dt}(\vec{B} \cdot \vec{A})$$

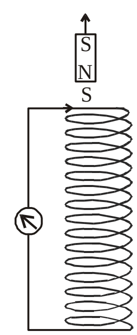
$$\Rightarrow \quad E = -N\frac{d\phi}{dt} \quad \text{or} \quad E = -N\frac{d}{dt}(BA\cos\theta) \quad .....(iii)$$

where –ve sign is in accordance with Lenz's law.

**Note :** Constant of proportionality in the above equation is 1. This has been obtained by adjusting the S.I. units of magnetic flux.

6. The induced e.m.f. is produced in accordance with the law of conservation of energy.




(1) When $N$-pole of the magnet is introduced into the coil, then the current is anti-clockwise, when seen from above.

(2) When $N$-pole of the magnet is withdrawn out of the coil, then current is clockwise, when seen from above.

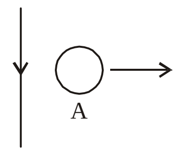
   (a) When $N$-pole of the magent is introduced into the coil, $N$-polarity is induced at the upper end. Work is required to be done, against the force of repulsion, to introduce the magnet. It is this work which appears as induced e.m.f.

(b) When $N$-pole of the magnet is withdrawn or taken out from the coil, direction of induced polarity changes (i.e. $S$-polarity is induced at the upper end). Work is again required to be done, against the force of attraction, to withdraw or take out the magnet. It is again this work which appears as induced e.m.f.

## Lenz's Law

1. The direction of induced current is such that it opposes the cause due to which the induced e.m.f. is produced .
2. For finding direction of induced e.m.f., note the following :
   (a) is magnetic flux changing, and
   (b) if so, is magnetic flux increasing or decreasing, and then
   (c) what is the direction of increase or decrease of magnetic flux; and then apply Lenz's law.

**Ex.1:** Coil $A$ is placed on a table near a long wire carrying current, as shown. When coil $A$ is suddenly removed away from the wire, what is direction of induced current in the coil ?




**Sol.:** (i) As the coil $A$ is moved away, it moves from a region of more magnetic flux to the region of less magnetic flux. So, magnetic flux through the coil is changing.
(ii) Also, the magnetic flux through the coil is decreasing.
(iii) Magnetic lines of force on the side of coil are perpendicular upward.
So, through the coil, magnetic flux decreases in the upward direction when the coil $A$ is moved away. Hence, direction of induced current will oppose the decrease of flux upward or it will increase the flux upward.
So, direction of induced current in the coil is in anticlockwise direction. .....**Ans.**

3. Lenz's law is based on law of conservation of energy.

## Methods of Production of induced E.M.F.

1. $E = -N\frac{d}{dt}(BA\cos\theta)$

$\therefore$ (a) Induced e.m.f. can be produced by varying $B$, keeping $A$ and $\theta$ constant.
(b) Induced e.m.f. can be produced by varying $A$, keeping $B$ and $\theta$ constant.
(c) Induced e.m.f. can be produced by varying $\theta$, keeping $A$ and $B$ constant.

2. *E.M.F. by varying B only* :

$$E = -NA\cos\theta\,\frac{dB}{dt} \Rightarrow E = -NA\cos\theta\,\frac{\Delta B}{\Delta t}$$

$$\Rightarrow E = -NA\cos\theta\,\frac{B_2 - B_1}{t}$$

**Ex.2:** A coil having 100 turns and area of cross-section 0.20 $m^2$ is placed normally in a magnetic field. If the magnetic field changes from 0.20 Wb $m^{-2}$ to 0.60 Wb $m^{-2}$ at a uniform rate over a period of 0.01s, calculate the induced e.m.f. in the coil.

**Sol.:** $N = 100$ $\quad A = 0.20\ m^2$ $\quad t = 0.01$ s

$\theta = 0^\circ \Rightarrow \cos\theta = 1$ $\quad B_1 = 0.20$ Wb $m^{-2}$ $\quad B_2 = 0.60$ Wb $m^{-2}$

$$E = -\text{N.A.}\cos\theta\,\frac{B_2 - B_1}{t}$$

$$= -100 \times 0.2 \times 1\,\frac{0.60 - 0.20}{0.01} = -800\text{ V}$$

$\Rightarrow$ Magnitude of induced e.m.f. = 800 V .....**Ans.**

3. *E.M.F. by varying A only* : There are two cases :

(1) (i) *When a rod of length l moves with velocity v in a uniform magnetic field of induction B,* then at its ends induced e.m.f. is produced given by the relation :

$$E = B\,l\,v\cos\theta \qquad .....(i)$$

where θ is the angle between (*i*) direction of *B* and (*ii*) normal to the area generated by moving the rod.

(ii) In vector form, $\vec{E} = \vec{B}\,.\,(\vec{l} \times \vec{v}) \quad \Rightarrow \quad \vec{B}.(l\,v) \quad \text{if} \quad \vec{l} \perp \vec{v}$

(iii) Therefore, if the rod moves parallel to the magentic field, no induced e.m.f. is produced at its ends.

(iv) If the rod moves in a direction perpendicular to the direction of magnetic field, then θ = 0º and induced e.m.f. produced is maximum i.e. $E = Blv$

(v) For finding the polarity of induced e.m.f. on the ends of the rod, apply Fleming's L.H. rule on the positive charge carriers (in the rod) when moved in the uniform magnetic field.

(2) (i) *When a rod of length l, fixed at one end, is rotated with angular velocity* ω *in a uniform magnetic field of induction B*, then at its ends induced e.m.f. is produced, given by the relation :

$$E = \tfrac{1}{2} Bl^2\omega\cos\theta \qquad .....(ii)$$

where θ = the angle between (i) direction of *B* and (ii) normal to the area generated by the rotation of rod.

(*ii*) If rod of length *l* is rotated about an axis perpendicular to its length and passing through its centre, then

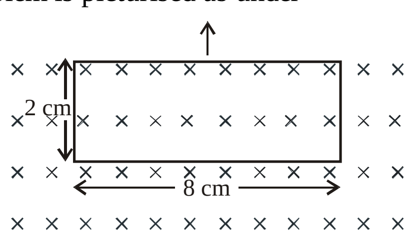
(a) e.m.f. across its ends = 0

(b) e.m.f. across its centre and ends $= \tfrac{1}{2} B(l/2)^2\,\omega\cos\theta$ .....(*iii*)

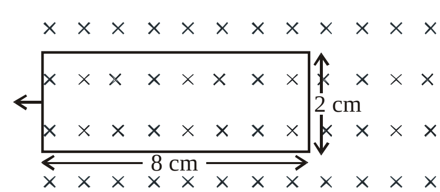
$$= \frac{Bl^2\omega\cos\theta}{8} \qquad .....(iv)$$

**Ex.3:** A rectangular loop of sides 8 cm and 2 cm, with a small cut, is moving out of a region of uniform magnetic field of magnitude 0.3 T, directed normal to the loop. What is the voltage developed across the cut, if the velocity of the loop is 1 cm $s^{-1}$ in a direction normal to (i) the longer side, and (ii) the shorter side. For how long does the induced voltage last in each case ?

**Sol.:** The problem is picturised as under




(i) velocity is normal to longer side.




(ii) velocity is normal to shorter side.

(i) *When the loop moves normal to the longer side :*

Induced emf, $E_1 = B\,l_{\text{longer}}\,v = 0.3 \times 0.08 \times 0.01$

$= 0.24 \times 10^{-3}$ V = 0.24 mV .....(i)**Ans.**

$t_1$ = time for which induced emf lasts

$$= \frac{\text{breadth}}{\text{speed}} = \frac{0.02}{0.01} = 2 \text{ sec.} \qquad .....(ii)\textbf{Ans.}$$

(ii) *When the loop moves normal to the shorter side :*

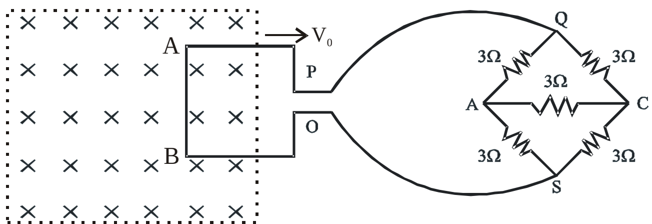
Induced emf, $E_2 = B\, l_{shorter}\, v = 0.3 \times 0.02 \times 0.1$

$= 0.06 \times 10^{-3}$ V = 0.06 mV .....(iii)**Ans.**

$t_2$ = time for which induced emf lasts

$$= \frac{\text{breadth}}{\text{speed}} = \frac{0.08}{0.01} = 8 \text{ sec.}$$ .....(iv)**Ans.**

**Ex.4:** A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity $v_0$ in a uniform magnetic field of induction, $B = 2$ Wb m$^{-2}$, as shown in the figure. The magnetic lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to a network of resistors, each of value 3 Ω. The resistances of the loop wires *OS* and *PQ* are negligible. (a) What should be the speed of the loop so as to have a steady current of 1 mA in the loop ? (b) Give the direction of the current in the loop.




**Sol.:** The network of resistances forms a balanced Wheatstone bridge $\therefore$ $R_{QS} = 3\ \Omega$

$\therefore$ Resistance of whole circuit = 3 + 1 = 4 Ω

$B = 2$ T $\quad l = 0.1$ m $\quad I = 10^{-3}$ A

$$\therefore \quad I = \frac{\text{induced emf}}{\text{resistance of cct.}} = \frac{B l v_0}{4}$$ .....(i)

$$\Rightarrow \quad v_0 = \frac{4 \times I}{B l} = \frac{4 \times 10^{-3}}{2 \times 0.1} = 2 \times 10^{-2}\,\text{m s}^{-1} = 2\ \text{cm s}^{-1}$$ .....**Ans.**

Imagine positive charge-carriers in the side *AB* of the square wire loop. With the motion of *AB* to the right, charge-carriers are also made to move to the right. Now, applying Flaming's L.H. Rule, positive charge carriers will move towards *A* and so electrons will move towards *B*. Hence, and *P* becomes positive and *O* becomes nagative. So, conventional current flows from *P* towards *O*, through the network of resistances.

4. *E.M.F. by varying* θ *only* :

(*i*) When a coil of area *A* and of *N* turns of wire is rotated in a uniform magnetic field of induction *B*, then at its ends induced e.m.f. is produced, given by the relation :

$$E = -NBA \frac{d}{dt}(\cos \omega t) = NBA\, \omega \sin \omega t \qquad (\because \text{ for rotation, } \theta = \omega t)$$

$$\Rightarrow \quad E = E_0 \sin \omega t \quad \text{where } E_0 = BA\omega N = BA(2\pi\nu)\, N = BA\left(\frac{2\pi}{T}\right)N$$

where ν = the frequency of rotation of coil

$T$ = time period of rotation of coil

(*ii*) *AC* current, through-out the world, is produced on this principle.

**Ex.5:** A conducting rod of 1.0 m length moves with a frequency of 50 rev/s, with one end at the center and the other end at the circumference of a circular metallic ring of radius 1.0 m, about an axis passing through the centre of the coil and perpendicular to the plane of the coil. A constant magnetic field, parallel to axis of rotation, is present everywhere. What is the emf developed between the centre and the metallic ring ? ($B = 1.0$ Wb m$^{-2}$)

**Sol.:** $l = 1.0$ m $\quad B = 1.0$ Wb m$^{-2}$ $\quad \theta = 0^o \Rightarrow \cos\theta = 1$

$\omega = 2\pi \times \nu = 2\pi \times 50 = 100\,\pi$ rad s$^{-1}$ $\quad$ E = ?

$$E = \frac{1}{2} B\,\omega\, l^2 \cos\theta = \frac{1}{2} \times 1 \times (100\,\pi) \times 1^2 \times 1$$

$$= 50\,\pi = 50 \times 3.14 = 157 \text{ V}$$ .....**Ans.**

**Ex.6:** A rectangular coil of wire has dimensions 0.2 m × 0.1 m. The coil has 2000 turns and rotates, parallel to its length and perpendicular to a magnetic field of intensity 0.02 Wb m$^{-2}$. If the speed of its rotation is 4200 rpm, calculate

(i) the maximum value of induced emf in the coil, and

(ii) the instantaneous value of induced emf when the plane of the coil has rotated through an angle of 30º from its initial position.

**Sol.:** $A = 2 \times 10^{-2}$ m$^2$ $\quad N = 2000$ $\quad B = 2 \times 10^{-2}$ Wb m$^{-2}$

$$\omega = 2\pi\,\nu = 2 \times \frac{22}{7} \times \frac{4200}{60} = 440 \text{ rad s}^{-1}$$

(i) $E_0$ = max. induced emf = $BA\omega N$

$= (2 \times 10^{-2}) \times (2 \times 10^{-2}) \times 440 \times 2000 = 352$ V .....(i)**Ans.**

(ii) $E$ = instant value of induced emf = $BA\omega N \sin \omega t = BA\omega N \sin\theta$

$= E_0 \sin 30^o = 352 \times 0.5 = 176$ V .....(ii)**Ans.**

## Mutual Induction

1. It is the phenomenon of production of induced e.m.f. across the two ends of a coil due to change in current in the neighbouring coil.

2. Mathematically, $E = -M.\frac{dI}{dt}$, where the negative sign is in accordance with Lenz's law.

   and $M$ = mutual inductance between the two coils.

3. Units of $M$ (mutual inductance) are H(henry) or Ω.s (ohm-second);
   Dimensions of mutual inductance are [ML$^2$T$^{-2}$A$^{-2}$].

4. $M = \frac{\mu_0 A_s N_p N_s}{L_p}$ where $\frac{N_p}{L_p}$ = Number of turns per unit length of primary coil.

   $N_s$ = Number of turns in secondary coil.

   $A_s$ = Area of cross-section of the secondary coil.

5. Practically one coil is closely would over the other on a long solenoid, so that these have a common area of cross-section $A$. Number of turns per unit length of both coils are also same.
   Then, $M_{12} = M_{21} = M \Rightarrow$ It is immaterial which coil acts as the primary or the secondary coil.

**Ex.7:** If the current in the primary circuit of a pair of coils changes from 5 A to 1 A in 0.02 s, calculate the change of flux per turn in the secondary, if it has 200 turns. Mutual inductance between the two coils is 0.5 H.

**Sol.:** $E_{secondary} = -M\,.\,\frac{dI}{dt} = -M\,\frac{I_2 - I_1}{t}$ $\quad$ [$M = 0.5$ H, $\quad I_1 = 5$ A, $\quad I_2 = 1$ A, $\quad t = 0.2$ s]

$$= -0.5\,\frac{1-5}{0.02} = \frac{0.5 \times 4}{0.02} = 100 \text{ V}$$ .....(i)

But $E_{secondary} = -N \cdot \frac{d\phi}{dt} = -200 \times \frac{d\phi}{0.02}$

$\therefore$ $d\phi$ = change in flux per turn

$$= -E_{secondary} \times \frac{0.02}{200} = -\frac{100 \times 0.02}{200} = -0.01 \text{ Wb/turn}$$ .....**Ans.**

## Self-Induction

1. It is the phenomenon of production of induced e.m.f. in a coil when a changing current passes through it.
2. Mathematically, $E = -L \cdot \frac{dI}{dt}$, where negative sign is in accordance with Lenz's law, and $L$ = self-inductance of the coil.
3. Self-inductance and mutual-inductance have same units i.e., H(henry) or Ωs; and also same dimensions *i.e.* $[ML^2T^{-2}A^{-2}]$.
4. $L = \frac{\mu_0 N^2 A}{l}$, where $N$ = number of turns in the coil of linear length $l$

   $\Rightarrow L = \mu_0 n^2 A l$, where $n$ = number of turns per unit length of the coil.
5. When a current is passed through a coil, magnetic flux is linked with it. So, for induced e.m.f. due to variation of magnetic flux through a coil

   $$E = -\frac{d\phi}{dt} \quad .....(i)$$

   Also, for induced e.m.f. due to variation of current through the same coil

   $$E = -\frac{L\,dI}{dt} \quad .....(ii)$$

   $\therefore$ From (*i*) and (*ii*), we get

   $$\frac{L\,dI}{dt} = \frac{d\phi}{dt} \Rightarrow \phi = LI \quad .....(iii)$$

   Also, $\phi = LI = \frac{LdQ}{dt}$ .....(*iv*)

**Ex.8:** What is the self-inductance of iron-core-solenoid, 5.0 cm long and of mean radius 2 cm, if it has 500 turns ? Take $\pi^2 = 10$ and relative permittivity of iron = 1200.

**Sol.:** $l = 0.05$ m $\quad N = 500 \quad r = 2 \times 10^{-2}$ m

$\mu_r = 1200 \quad \mu_0 = 4\pi \times 10^{-7}$ Tm s$^{-1}$ $\quad L = ?$

$$L = \frac{\mu N^2 A}{l} = \frac{\mu_0 \mu_r N^2 A}{l} = \frac{(4\pi \times 10^{-7}) \times 1200 \times (500)^2 \times \pi(2 \times 10^{-2})^2}{0.05}$$

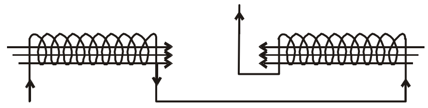
$$= 8 \times 12 \times 25 \times 4 \times \pi^2 \times 10^{-4} = 96 \times 10^{-1} \text{ H} = 9.6 \text{ H}$$ .....**Ans.**

## Relation between L & M

1. When two inductors, of self-inductances $L_1$ and $L_2$ and mutual inductance $M$, are held close to each other such that fields in them are in the same direction, then their resultant inductance $L$ is given by the relation.

   $L = L_1 + L_2 + 2M$
2. When two inductors of self-inductances $L_1$ and $L_2$ and mutual inductance $M$, are held close to each other such that fields in them are in opposite directions, then their resultant inductance $L$ is given by the relation

   $L = L_1 + L_2 - 2M$

3. When two inductors are held parallel to each other such that $M$ is neglible, then

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

4. When one coil is closely wound on the other, then $A$ and $l$ can be considered almost same for both the coils. In that case,

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \sqrt{\frac{\mu_0 N_1^2 A}{l} \times \frac{\mu_0 N_2^2 A}{l}} = \sqrt{L_1 L_2}$$

## Energy in an inductor

(1) $E = \frac{1}{2} LI^2$ where $L$ = self-inductance of inductor ; $I$ = current flowing through inductor

This energy in the coil, due to current through it, is in the form of magnetic flux. So, the energy is magnetic in nature.

(2) or $U$ = energy per unit volume of the inductor

$$U = \frac{1}{2} \frac{LI^2}{\text{volume of inductor}} \qquad L = \frac{\mu_0 AN^2}{l}$$

$$U = \frac{1}{2} \cdot \frac{\mu_0 AN^2}{l} \times \left(\frac{Bl}{\mu_0 N}\right)^2 \times \frac{1}{Al} \qquad B = \frac{\mu_0 IN}{l} \Rightarrow I = \frac{Bl}{\mu_0 N}$$

$$\Rightarrow \quad U = \frac{1}{2} \cdot \frac{B^2}{\mu_0} \text{ (its units are Jm}^{-3}\text{)}$$

## Eddy Currents

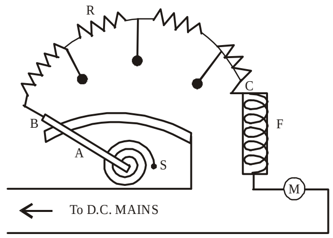
1. Whenever magnetic flux threading through a metallic plate or body changes, induced currents or eddy currents or Focault's currents are produced in it.
2. These currents cause unnecessary heating and wastage of power.
3. These are reduced by laminated soft iron cores.
4. Directions of eddy currents are given by Lenz's law.
5. Some of the applications of eddy currents are :
   (*i*) *Induction furnace*
   (*ii*) *Making galvanometer dead beat* : i.e., coil does not oscillate but stays in the position of deflection immediately.
   (*iii*) *Electric brakes* : A strong magentic field is applied to the drum attached to the axle of the wheel. Eddy currents, set up in the drum, exert a torque on the drum so as to stop it.
   (*iv*) *Speedometer* : A magnet placed inside the aluminium drum (pivoted and carefully held) rotates with the speed of the vehicle. Eddy currents, set up in the drum, oppose the motion of the magnet. A torque exerted on the drum, in opposite direction, deflects drum through an angle, proportional to speed of the vehicle.
   (*v*) *Induction motor* : A rotating magnetic field is produced by two single-phase alternating currents, having a phase difference of 90º. A metallic cylinder is placed in the magnetic field. The eddy currents set up in the cylinder tend to oppose the relative motion between the rotating magnetic field and the cylinder. As a result, the cylinder also starts rotating about its axis.
   (*vi*) *Inductothermy* : Eddy currents are used to heat localised tissues of the human body.

## Dyanamo/Generator

1. It is device which converts mechanical energy into electrical energy.
2. The mechanical energy is used to rotate an armature or a rectangular coil in a magnetic field. Then at the ends of this armature or rectangular coil, induced e.m.f. is produced. This induced e.m.f. is alternating in nature.
3. In a *DC* dynamo, a pair of split-rings is used and in *AC* dynamo, a pair of slip-rings is used, to draw current from the dynamo or a generator.

## Motor and Starter

1. Motor is device which converts electrical energy into mechanical energy.
2. A rotor or an armature placed in magnetic field of magnets, starts rotating when current is passed through it. Thus, electrical energy is converted into mechanical energy.
3. Initially, the speed of rotor is small and then increases due to the electrical torque $[I(\vec{A}\times\vec{B})N]$. So, initially, induced e.m.f. produced in the armature is small and increases with the increase in its speed.
4. Due to small induced opposing e.m.f., initially, large current flows through the motor. This large current may damage the motor, due to heat produced by large current. In order to decrease this initial large current, a starter is connected to the motor.




5. In small motors, no starter is used as these gain speed very rapidly and the quick induced e.m.f. reduces the current. But in heavy motors, a starter is a must.
6. Opposing induced e.m.f. in motors is called back e.m.f. ($e$).
7. If $E$ is the applied e.m.f., $e$ is the back e.m.f., then current through the motor is $\frac{E-r}{R}$, where $R$ is the resistance of the armature.
8. Efficiency of motor $= \frac{\text{Output power}}{\text{Input power}} = \frac{EI-I^2R(\text{heat dissipated})}{EI}$

$$= \frac{I(E-IR)}{IE} = \frac{E-IR}{E} = \frac{e}{E} \qquad [\because \quad I = \frac{E-e}{R} \Rightarrow E-IR = e]$$

$$= \frac{\text{back e.m.f.}}{\text{applied e.m.f.}}$$

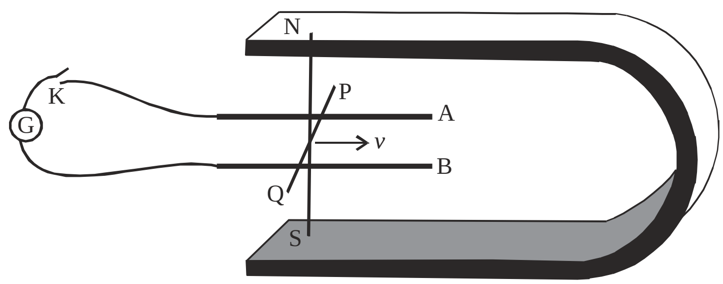
9. The efficiency of a motor is maximum when back e.m.f. is half the applied e.m.f.

## Transformer

1. If electrical power is transmitted, along transmission lines, at higher voltage, then the loss of power due to Joule's heating is reduced.
2. The device to step-up or increase the voltage of electrical power is called step-up transformer and that for decreasing the votlage is called step-down transformer.
   For step up transformer $N_2 > N_1$ as $V_2 > V_1$ transformation ratio (K ) > 1.
3. The principle of transformer is based on :
   (i) production of magnetic flux due to applied varrying voltage across the primary coil, and
   (ii) production of induced e.m.f. across the secondary coil, due to varrying magnetic flux through the secondary coil.
4. For ideal transformer i.e., when output power = input power, $\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$, where $N_2$ is the number of turns in the secondary coil and $N_1$ is the number of turns is the primary coil. $\frac{N_2}{N_1}$ is also called transformer ratio.
5. Efficiency of a transformer, $\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{V_2I_2}{V_1I_1}$
6. Transformer is the most efficient machine developed so far. Its efficiency may be as high as 95% or even more.

7. Factors responsible for loss of energy in a transformer and the steps taken to minimise these losses of energy are as under :
   (i) *Copper loss* : Loss of energy due to Joule's heating in the coil. This is minimised by using copper wires for windings in primary and secondary coils.
   (ii) *Iron loss or Eddy Current loss* : Varying magnetic flux is associated with alternating current. This produces eddy currents in the core on which copper wires are wound. To reduce eddy currents, the core is made of thin laminated sheets placed one over the other, but insulated from one another.
   (iii) *Hysteresis loss* : This loss is minimised by using material of the core which has low hysteresis loss.
   (iv) *Loss due to magnetic flux leakage* : This loss is minimised by rounding off the corners of rectangular annular laminated sheets.

**Ex.9:** Figure shows a metal rod *PQ*, resting on the rails *AB* and positioned between the poles of a permanent magnet. The rails, the rod and the magnetic field are in three mutually perpendicular directions. *A* galvanometer *G* connects the rails through a switch *K*. Length of the rod is 15 cm, $B = 0.50$ T and the resistance of the closed loop containing the rod is 9.0 m Ω.




Answer the following questions :

(i) If *K* is open and the rod moves with a speed of 12 cm $s^{-1}$ in the direction shown, give the polarity and magnitude of induced emf.

(ii) Is there an excess charge built up at the ends of the rod when *K* is open? What, if *K* is closed?

(iii) With *K* open and the rod moving uniformly, there is no net force on the electrons in the rod *PQ*, even though these do experience magnetic force due to motion of the rod in the magnetic field. Explain.

(iv) What is the retarding force on the rod when *K* is closed.

(v) How much power is required by an external agent to keep the rod moving at the same speed of 12 cm $s^{-1}$, when *K* is closed? How much power is required when *K* is open ?

(vi) How much power is dissipated as heat in the closed circuit? What is the source of the power?

(vii) What is the induced emf in the moving rod when the permanent magnet is rotated to a vertical position so that the field becomes parallel to the rails?

**Sol.:** (i) Magnitude of $E$ $= B.(l \times v) = B \,.\, l\, v$ $[\because \; l \perp v]$

$= B\, l\, v$ $[\because$ area vector and $B$ are parallel$]$

$= 0.5 \times 0.15 \times 0.12 = 9 \times 10^{-3}$ V = 9 m V .....(i)**Ans.**

Imagine positively charged carriers, moved in the direction of motion of the rod, in the magnetic field and then apply Fleming's L.H. Rule. Accordingly, positive charge carriers move towards *P* and electrons towards *Q*. Thus, *P* is at positive potential and *Q* is at negative potential. ....**Ans.**

(ii) When key *K* is open, there is excess of electrons at end *Q* and deficiency of electrons at end *P*, due to induced emf.

When *K* is closed, this excess charge will flow in the closed circuit in the form of current, called

induced current $\left[=\frac{\text{induced emf.}}{\text{resistance of closed cct.}}\right]$

(iii) Initially, electrons start moving towards end *Q*; thus producing a p.d. across the two ends *P* and *Q* of the rod. After a short time (when the p.d., say, *V*, is so developed) force on electrons due to this p.d. balances magnetic force on electrons due to motion of rod. Then, net force on electrons in the rod becomes zero.

$$e\left[\frac{V}{l}\right] = -\,e\,(v \times B) \qquad \Rightarrow \quad V = l\,(v \times B)$$

$$\Rightarrow \qquad V = 0.15 \times 1.12 \times 0.5 = 9 \times 10^{-3}\ \text{V} = 9\ \text{mV}$$

i.e., this is the max. p.d. which can be developed across rod. After this, the electrons will not move across the ends of the rod, as resultant force on these will be zero.

(iv) Retarding force $= B\,I\,l = B\left[\frac{E_{\text{induced}}}{R_{\text{cct}}}\right] l$

$$= 0.5 \times \frac{9\ \text{mV}}{9\ \text{m}\Omega} \times 0.15 = 75\ \text{m N}$$ .....**Ans.**

(v) Power expanded = Force × velocity

$$= (75 \times 10^{-3}) \times (12 \times 10^{-2}) = 9 \times 10^{-3}\ \text{W} = 9\ \text{mW}$$ .....**Ans.**

(vi) Power dissipated as heat $= I^2R = \left[\frac{E_{\text{induced}}}{R_{\text{cct.}}}\right]^2 \times R$

$$= 1^2 \times 9 \times 10^{-3} = 9\ \text{mW}$$ .....**Ans.**

When the key K is closed power will be dissipated as heat and so the source must supply the power to keep the rod moving against the retarding force.

When the key is open, even then the external source must supply the power to maintain p.d. across the two ends of the rod.

(vii) When the magnetic field become parallel to the rod (after rotating the magnet), induced emf. will be zero.

($\because \quad E = B\,l\,v \sin 0^\circ = 0$)

**Ex.10:** A step-down transformer is used to reduce the main supply of 220 V to 11 V and its efficiency is 90%. If current in the primary is 5.0 A, what maximum current is delivered by the secondary?

**Sol.:** Efficiency $= \frac{\text{Output power}}{\text{Input power}} = \frac{V_2 \times I_2}{V_1 \times I_1}$

$$\Rightarrow \quad \frac{90}{100} = \frac{11 \times I_2}{220 \times 5} \qquad \Rightarrow \quad I_2 = 90\ \text{A}$$ .....**Ans.**

## *E.M. Waves*

1. Ampere's circuital relation is :

$$\oint \vec{B}.d\vec{l} = \mu_0 I, \qquad \text{where } I = \text{current enclosed}$$

2. The above relation was found inconsistent by Clark Maxwell.
3. The modified Maxwell Ampere circuital relation is :

$$\oint \vec{B}.d\vec{l} = \mu_0[I_C + I_D] \qquad \text{where } I_C = \text{conduction current}$$

$$= \mu_0[I_C + \epsilon_0 \frac{d\psi}{dt}] \qquad I_D = \text{displacement current} = \epsilon_0 \frac{d\psi}{dt}$$

$$\frac{d\psi}{dt} = \text{rate of change of electric flux}$$

4. $\epsilon_0 \frac{d\psi}{dt}$ is displacement current and its magnitude is equal to conduction current $I_C$.

$$\Rightarrow I_C = I_D = \epsilon_0 \frac{d\psi}{dt}$$

5. The electric field vector $\vec{E}$ and magnetic field vector $\vec{B}$ of electromagnetic waves are related by the relation $\vec{E} = c\vec{B}$, where $c$ is velocity of light.
6. In e.m. waves, $\vec{E}$ and $\vec{B}$ are mutually perpendicular to each other, but are in same phase.
7. Velocity of light through vacuum is given by the relation :

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \qquad \text{where } \mu_0 = \text{permeability of vacuum} = 4\pi \times 10^{-7}\ \text{TmA}^{-1}$$

$$= \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{1}{4\pi \times 9 \times 10^9}}} \qquad \epsilon_0 = \text{permittivity of vacuum} = \frac{1}{4\pi \times 9 \times 10^9}\ \text{Fm}^{-1}$$

$$= \sqrt{9 \times 10^{16}} = 3 \times 10^8\ \text{ms}^{-1}$$

8. Velocity of light in any other medium :

$$c' = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_r\, \mu_0 . \epsilon_r\, \epsilon_0}} = \frac{1}{\sqrt{\mu_r . \epsilon_r}} \cdot \frac{1}{\sqrt{\mu_0\, \epsilon_0}} \qquad \left[\because \mu_r = \frac{\mu}{\mu_0} \text{ and } \epsilon_r = \frac{\epsilon}{\epsilon_0}\right]$$

$$= \frac{c}{\sqrt{\mu_r\, \epsilon_r}} = \frac{\text{velocity of light in vacuum}}{\sqrt{\mu_r\, \epsilon_r}}$$

9. (i) $V$ = induced e.m.f. $= -\frac{d\phi}{dt}$

(ii) Also, $V = -\int E.dl$

Combining above two relations, we get $\int E.dl = \frac{d\phi}{dt}$

The above equation also means that a *varying magnetic field produces electrical field*.

10. Also, $\oint B.dl = \mu_0 \epsilon_0 \frac{d\psi}{dt}$ .....from Maxwell's modified relation.

The above equation also means that a *varying electrical field produces magnetic field*.

11. From the above two relations, Maxwell in the year 1865 predicted the existence of e.m. waves. He argued that if we have a source of varying electrical field, this will automatically give rise to varying magnetic field, which in turn will give rise to varying electrical field again and so on. Thus e.m. waves start travelling.

12. Based on the above concept, Hertz in the year 1888, demonstrated the production of e.m. waves by using a source of varying electrical field. Due to high voltage between the plates, the air in the small gap between the plates gets ionised, thus providing a path for the discharge of plates. This system is able to produce very high frequency oscillations of charges on the plate and, hence, creating a source of varying electrical field.

13. Photons of e.m. waves are associated with

(i) energy, given by the relation, $E = h\nu = \frac{hc}{\lambda}$ .....(*i*)

(ii) momentum, given by the relation, $p = \frac{h}{\lambda} = \frac{E}{c}$ .....(*ii*)

(iii) rest mass which is zero .....(*iii*)

(iv) effective mass during motion $= \frac{E}{c^2}$ .....(*iv*)

14. Due to momentum, e.m. waves produce pressure, on the surface on which these fall.

(i) If e.m. waves are incident without reflection, then

$$P = \frac{\text{rate of change in momentum}}{\text{Area}} = \frac{\frac{d}{dt}(p)}{A} = \frac{\frac{d}{dt}\left[\frac{E}{c}\right]}{A}$$

$$= \frac{1}{c} \times \frac{\frac{d}{dt}[E]}{A} = \frac{1}{c} \times \text{ Rate of energy of light falling per second per unit area}$$

$$\Rightarrow P = \frac{I}{c}$$ where $I$ = intensity of light

= rate of energy of light falling per second per unit area

(ii) If light or e.m. waves are reflected back, change in momentum is doubled.

$$\therefore \quad P = \frac{2I}{c}$$

**Ex.11:** Light with an average energy flux of 18 W cm$^{-2}$ falls on a non-reflecting surface at normal incidence. If the surface has an area of 20 cm$^2$, find (i) the average pressure and (ii) the average force exerted by the energy flux on this surface.

**Sol.:** $I$ = intensity of energy flux = $18 \times 10^4$ W m$^{-2}$ $\qquad A = 20 \times 10^{-4}$ m$^2$

$\therefore$ For the non-reflecting surface

$$P \text{ (pressure)} = \frac{I}{c} = \frac{18 \times 10^4}{3 \times 10^8} = 6 \times 10^{-4} \text{ Nm}^{-2}$$ .....**Ans.**

$$F \text{ (force)} = P \times A = 6 \times 10^{-4} \times 20 \times 10^{-4} = 1.2 \times 10^{-6} \text{ N}$$ .....**Ans.**

15. Energy density in a region of electric field of intensity $\vec{E}$ :

(i) $U_E = \frac{1}{2} \epsilon_0 E^2$ (unit is Jm$^{-3}$)

(ii) $U_{E(\max)} = \frac{1}{2} \epsilon_0 E_0^2$ where $\vec{E}_0$ = max. intensity of electric field

(iii) $U_{E\,(\text{average})} = \frac{1}{2} \times \frac{1}{2}[\epsilon_0 E_0^2] = \frac{1}{4} \epsilon_0 E_0^2$

[$\because$ $E$ is alternating, so average value is taken as its rms value

$\therefore$ $U_{E(\text{average})} = \frac{1}{2} \epsilon_0 (E_{rms})^2 = \frac{1}{2} \epsilon_0 \left[\frac{1}{2} E_0^2\right] = \frac{1}{4} \epsilon_0 E_0^2$ ]

16. Energy density in a region of magnetic field of intensity $\vec{B}$ :

(i) $U_B = \frac{1}{2} \frac{B^2}{\mu_0}$ (unit is $Jm^{-3}$)

(ii) $U_{B(\max)} = \frac{1}{2} \frac{B_0^2}{\mu_0}$ where $\vec{B}_0$ = max. intensity of magnetic field

(iii) $U_{B\,(\text{average})} = \frac{1}{2} \times \left[\frac{1}{2} \frac{B_0^2}{\mu_0}\right] = \frac{1}{4} \cdot \frac{B_0^2}{\mu_0}$

17. Total energy density of e.m. waves is the sum of the energy density of electric field of intensity $\vec{E}$ and energy density of magnetic field of intensity $\vec{B}$.

$\therefore$ $U = U_B + U_E = \frac{1}{2} \frac{B^2}{\mu_0} + \frac{1}{2} \epsilon_0 E^2$

Also $U_{\text{average}} = \frac{1}{4} \frac{B_0^2}{\mu_0} + \frac{1}{4} \epsilon_0 E_0^2$

18. $U_E = U_B$ at any instant, since

$U_B = \frac{1}{2} \frac{B^2}{\mu_0} = \frac{1}{2} \frac{E^2}{c^2 \mu_0} = \frac{1}{2} \frac{\mu_0 \epsilon_0 E^2}{\mu_0} = \frac{1}{2} \epsilon_0 E^2 = U_E$ $\left[\because c^2 = \frac{1}{\mu_0 \epsilon_0}, \vec{E} = c\vec{B}\right]$

19. $\vec{S}$ = Poynting vector

= energy vector of e.m. waves falling per second per unit area, held perpendicular to the path of propagation of waves. It is also called intensity of e.m. waves.

$\Rightarrow$ $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$ ($Js^{-1}\,m^{-2}$ or $Wm^{-2}$) $= c\,\epsilon_0 E^2 = \frac{c\,B^2}{\mu_0}$

$\therefore$ If $\vec{S}$ is along $+x$-axis, $\vec{E}$ will be along $+y$-axis and $\vec{B}$ will be along $+z$-axis.

20. If em waves are moving towards $x$-axis, then

(i) Electric energy vector is along $y$-axis (but $E_x = 0$ and $E_z = 0$)

$E_y = E_0 \sin[kx - \omega t]\,\hat{\mathbf{j}}$ where $k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

$= E_0 \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]\,\hat{\mathbf{j}}$

(ii) Magnetic energy vector is along z-axis (but $B_x = 0$ and $B_y = 0$)

$B_z = B_0 \sin[k\,x - \omega\,t]\,\hat{\mathbf{k}}$

$= B_0 \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]\,\hat{\mathbf{k}}$

**Ex.12:** At any instant, the electric field part of em waves in vacuum is

$E = 3.1 \cos[1.8\,y + 5.4 \times 10^6\,t]\ \hat{\mathbf{i}}$ , where $y$ is in m and $t$ is in sec.

(a) What is direction of propagation of the waves ?

(b) What is wavelength of the waves ?

(c) What is the frequency of the waves ?

(d) What is the amplitude of the magnetic field part of the waves ?

(e) Write an expression for magnetic field part of the waves, at the same instant.

**Sol.:** (a) The direction of propagation of waves is towards – $y$-axis. .....**Ans.**

(b) Given wave : $E_x = 3.1 \cos[1.8\,y + 5.4 \times 10^6\,t]\ \hat{\mathbf{i}}$ .....(i)

Standard wave : $E_x = E_0 \cos\left[\frac{2\pi}{\lambda}y + \frac{2\pi}{T}t\right]\hat{\mathbf{i}}$ .....(ii)

By comparison : $1.8 = \frac{2\pi}{\lambda} \quad \Rightarrow \quad \lambda = \frac{2 \times 3.14}{1.8}$ m = 3.49 m .....**Ans.**

(c) Again, by comparison : $5.4 \times 10^6 = \frac{2\pi}{T} = 2\,\pi\,\nu$

$\therefore \quad \nu = \frac{5.4 \times 10^6}{2 \times 3.14} \approx 8.6 \times 10^7$ Hz = 86 MHz. .....**Ans.**

(d) Amplitude of electric field, $E_0 = 3.1\ \mathrm{NC^{-1}}$

$\therefore$ Amplitude of magnetic field = $\frac{E_0}{c} = \frac{3.1}{3 \times 10^8} = 1.03 \times 10^{-8}$ T $\approx$ 10.3 nT .....**Ans.**

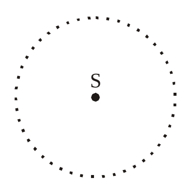
(e) Using, $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$ and knowing that the direction of wave is along $-\hat{j}$ and that of $E$ is along $\hat{i}$ , the direction $B$ is along $+\hat{k}$. Also value of $B_0$ has been calculated as above.

$\therefore$ Expression of magnetic field part of the wave is

$\Rightarrow \quad B = 1.03 \times 10^{-8} \cos(1.8\,y + 5.4 \times 10^6\,t)\ \mathrm{T}\ \hat{\mathbf{k}}$ .....**Ans.**

**Ex.13:** About 5% of the power of a 100 W bulb is converted into visible radiations. (a) What is the average intensity of visible radiations at a distance of 1 m from the bulb ? If the bulb is giving out monochromatic waves of wavelength 6000 Å, then how many photons will be reaching at a point, distant 1 m from the bulb ?

**Sol.:** (a) Energy per second emitted from the source $S$, as light = $100 \times \frac{5}{100} = 5$ W




$\therefore \quad I$ = intensity of light at a distance 1 m from source $S$

$= \frac{\text{Energy emitted per second}}{4\pi r^2}$

$= \frac{5}{4\pi(1)^2} = \frac{5}{4 \times 3.14} = 0.398\ \mathrm{Wm^{-2}}$ .....**Ans.**

(b) Let $n$ be the number of photons reaching at the point of above intensity

Energy of each photon $= \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{6000\times10^{-10}}$ J $= 3.315 \times 10^{-19}$ J.

$\therefore \quad n \times 3.315 \times 10^{-19} = 0.398 \quad \Rightarrow \quad n = \frac{0.398}{3.315\times10^{-19}} = 1.2 \times 10^{18}\ s^{-1}$ .....**Ans.**

## Spectrum of E.M. Waves

1. *Radiowaves* (or radio-frequency waves) : Wavelength range = few km to 0.3 m. Frequency range ⊔ $10^5$ Hz to $10^9$ Hz. These waves are used in radio and television broadcasting. These are generated by oscillating circuits or electronic devices.
2. *Microwaves* (or UHF waves or ultra high frequency waves with respect to radiowaves) Wave-length range = 0.3 m to 1 mm. Frequency range = $10^9$ Hz to $3 \times 10^{11}$ Hz. These are used in radar and other communications. These are also used to study structure of atoms or molecules.
3. *Infra-red waves* (or heat waves) : Wave-length range = $10^{-3}$ m to $7.5 \times 10^{-7}$ m. Frequency range = $3 \times 10^{11}$ Hz to $4 \times 10^{14}$ Hz. These are produced by hot bodies or molecules. These have applications in industry, medicine, astronomy, etc.
4. *Visible light waves* **:** Wavelength range = $7.5 \times 10^{-7}$ to $4 \times 10^{-7}$ m ; ($\lambda_{red} = 7.5 \times 10^{-7}$ m, $\lambda_{violet} = 4 \times 10^{-7}$m, $\lambda_{average} = 6 \times 10^{-7}$ m). Frequency range = $4 \times 10^{14}$ to $7.5 \times 10^{14}$ Hz.
   (i) Average wavelength is for yellow light = $6 \times 10^{-7}$ m
   (ii) Sensitivity to eyes is maximum for green waves ($\lambda = 5.6 \times 10^{-7}$ m)
5. *Ultra-violet waves* : Wavelength range = $4 \times 10^{-7}$ m to $6 \times 10^{-10}$ m. Frequency range = $7.5 \times 10^{14}$ to $5 \times 10^{17}$Hz. These are produced by atoms and molecules in electrical discharges. These are also radiated by the sun. These ionise the medium through which these pass. Used for killing germs, sterilisation and some medical applications.
6. *X-rays* : Wavelength range = $10^{-9}$ to $6 \times 10^{-12}$ m. Frequency range = $3 \times 10^{17}$ to $5 \times 10^{19}$ Hz. These are produced when high speed electrons (cathode rays) are incident on a target of large atomic mass and high melting point (e.g. tungsten). These produce ionisation and can penetrate through bodies of smaller atomic masses but are stopped by bodies of larger atomic masses. These have large use in medical diagnosis, medical therapy (treatment of cancer), industry, agriculture, and research.
7. *Gamma rays* : These overlap the upper limit of *X*-rays. So, wavelength range = $10^{-10}$ m to $10^{-14}$ m or below. Frequency range = $3 \times 10^{18}$ to $3 \times 10^{22}$ Hz. These are produced from the nucleus of radioactive elements, after the emission of $\alpha$-particles and $\beta$-particles. These have large ionising power and large penetration power. These can be stopped by thickness of few milimetres of lead. These produce serious effects on living organisms.
8. *Cosmic rays* : Very high energy waves (of wavelength smaller than that of gamma rays and of frequency larger than that of gamma rays) from cosmos or outer-space. Large intensity of cosmic rays or even small intensity of cosmic rays for larger duration can produce destruction of delicate organs, even leading to death of animals and other living-beings. These can produce even genetic defects in animals and other living beings.

## Earth's Atmosphere :

It is divided into four regions of approximate heights from the surface of earth, as under :

1. *Troposphere*: extends upto 12 km from the surface of the earth; temperature falls as we go up, from 290 K to 220 K. Density of air goes on decreasing. This region contains $CO_2$, other gases, water-vapours and clouds which reflect back infra-red radiations and keep atmosphere warmer. This is called 'Green-house' effect.

2. *Stratosphere* : It lies between 12 km and 50 km from the surface of the earth. Temperature rises, as we go above, from 220 K to 280 K. The upper layer of stratosphere has ozone layer which entraps harmful ultraviolet waves from the sun. All radiations of wavelengths less than $3 \times 10^{-7}$ are absorbed by ozone layer.
3. *Mesosphere* : This layer extends from 50 km to 80 km from the surface of the earth. The lower layer of this region starts from ozone layer. Temperature in this region falls from 280 K to 180 K, as we go up.
4. *Ionosphere* : This layer extends from 80 km to 400 km from the surface of the earth. Temperature rises from 180 K to 700 K as we go up. Cosmic rays and ultra-violet rays coming from cosmos and the sun produce ionisation in this region. This region consists of large number of electrons and ionised particles. A layer of large density of electrons of thickness of few kilometres above 110 km from the surface of earth is named *Kennelly Heaviside layer*. Beyond this layer, electron density decreases upto a height of 250 km. Again electron density increases and this second layer of increased density of electrons is called *Appleton layer*.

## Propagation of Radio and TV Waves

1. *Amplitude Modulated Band* consists of e.m. waves of frequency less than 30 MHz (or of wavelength more than 10 m). These AM radio waves transmitted from an antenna from a radio-station can be received at another place on the surface of earth in following two possible ways.
   (i) *Ground waves/surface wave propagation* : AM e.m. waves of frequency upto 1.5 MHz (1500 kHz) or of wavelength more than 200 m can bend round the corners and so can reach upto large distances from the radio-station. These waves are called ground waves. However their intensity falls with distance. Further, ground-waves become weaker as frequency increases. These waves are also called medium waves.
   (ii) *Sky wave propagation* : AM e.m. waves of frequencies more than 1.5 MHz (or wavelength less than 200 m) can be reflected from the ionosphere and can reach large distances from the radio-station, via sky. These waves are also called short waves.
2. *Frequency Modulated Band* consists of e.m. waves of frequencies 80 MHz to 200 MHz.
3. Radiowaves of frequencies more than 40 MHz or not reflected back by ionosphere and on reflection at the surface, these have large attenuation (i.e. these get distorted). So their propagation is directly from transmitting station to places on earth, and the transmission is called *space wave propagation*.
4. *T.V. waves* : have e.m. waves of frequency range between 100 MHz to 220 MHz. These cannot be propagated either as ground waves or as sky waves. These are directly transmitted *as space wave* from high T.V. towers.
5. *Range of T.V. tower*:
   (i) Distance upto which TV radiations can reach, $d = \sqrt{2hR}$

   where $h$ = height of T.V. tower and $R$ = radius of earth

   (ii) Area served around T.V. tower = $\pi d^2 = \pi \times 2\ Rh$
   (iii) Population benefitted = Area served × population density.