Chapter-8

PROPERTIES OF SOLIDS, LIQUIDS & GASES

SOLIDS

Due to large molecular forces, solids have fixed volume and fixed shape. Solids are held together due to one of the following bindings, in order of decreasing strength of binding.

(*i*) Ionic binding (*ii*) Covalent binding (*iii*) Metallic binding (*iv*) Van der Waals binding

ELASTICITY

1. It is the property due to which a body regains its original length, volume or shape, when a deforming force, acting on it, is removed.

   Mathematically, $\text{Elasticity} = \dfrac{\text{Stress}}{\text{Strain}}$

2. Stress is the restoring force per unit area acting on the body and is due to molecular forces. Its unit is Nm^{-2} and dimensions are $ML^{-1}\,T^{-2}$

3. (a) $\text{Strain} = \dfrac{\text{Change in dimensions}}{\text{Original dimensions}}$, being pure ratio, it has no units.

   (b) It is of following three types

   (*i*) $\text{Longitudinal strain} = \dfrac{\text{Change in length}}{\text{Original length}} = \dfrac{\Delta L}{L}$

   (*ii*) $\text{Volumetric strain} = \dfrac{\text{Change in volume}}{\text{Original volume}} = \dfrac{\Delta V}{V}$

   (*iii*) Shearing strain is the angle through which a vertical plane shifts when a tangential deforming force is applied to it.

Young's Modulus of Elasticity

1. $Y = \dfrac{F/A}{\Delta L/L} = \dfrac{F}{A}\cdot\dfrac{L}{\Delta L} \quad \Rightarrow \quad Y = \dfrac{F}{\pi r^2}\cdot\dfrac{L}{\Delta L}$ ..... (*i*)

2. *F*/*A* is tensile stress i.e., stress acting perpendicular to area of cross-section of the wire.

3. $F = \dfrac{YA}{L}\cdot\Delta L$ ......from (*i*) $\Rightarrow F \propto \Delta L$ (extension)

   $\Rightarrow$ Within elastic limit, extension produced in a wire is directly proportional to the stretching force acting on it (Hooke's law)

4. Potential energy possessed by a stretched wire :

   (*i*) $\text{P.E.} = \dfrac{1}{2}\text{F}.\,\Delta\text{L} = \dfrac{1}{2}\text{Fx} = \dfrac{1}{2} \times \text{stretching force} \times \text{extension}$

   (*ii*) $\text{P.E.} = \dfrac{1}{2}kx^2$, where k = force constant $(\because\ F = kx)$

   (*iii*) $\text{P.E.} = \dfrac{1}{2}\dfrac{F^2}{k}$

5. Potential energy per unit volume (U), possessed by a stretched wire :

   (*i*) $U = \dfrac{1}{2} \times \text{stress} \times \text{strain}$ $\left[\because Y = \dfrac{\text{stress}}{\text{strain}}\right]$

   (*ii*) $U = \dfrac{1}{2} \times Y \times (\text{strain})^2$

(*iii*) $U = \frac{1}{2} \times \frac{(\text{stress})^2}{Y}$

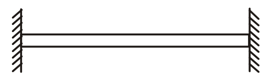
6. From $F = \frac{YA}{L} . \Delta L$ and $F = kx = k\Delta L$, by comparison we get

$$k = \frac{YA}{L} = \text{force constant for a wire}$$

7. Larger the value of Y, more elastic is the substance. For this reason, steel is more elastic than rubber.
8. When a rod is held horizontal within fixed supports, such that when it is heated, compressive force of supports does not allow it to expand (or when it is cooled, then the supports stretch the rod outwards), then in equilibrium,

$$\frac{FL}{AY} = \Delta L = L\alpha \,.\, t \quad \Rightarrow \quad \frac{F}{A} = \text{compressive stress} = Y\alpha t$$

$$\Rightarrow \quad F = \text{compressive force} = Y\alpha tA$$



where Y = Young's modulus of elasticity of rod
α = coefficient of linear expansion of rod
A = area of cross-section of rod
t = rise in temperature

Ex.1: A wire of length L and cross-sectional area A is made of a material of Young's modulus Y. If the wire is stretched by an amount x, find the work done.

Sol.: As per relation of Young's modulus of elasticity

$$Y = \frac{F}{A}.\frac{L}{\Delta L} \qquad \Rightarrow \qquad F = \frac{YA}{L} . \Delta L \qquad .....(i)$$

But within elastic limit of stretching, $F = K \times$ extension .....(ii)

Comparing (i) and (ii) K = force constant of wire = $\frac{YA}{L}$ .....(iii)

Now, work done is stretching a wire = $\frac{1}{2} K\, (\text{extension})^2$

$$= \frac{1}{2} . \frac{YA}{L} x^2 \qquad .....\textbf{Ans.}$$

Ex.2: Two wires of materials of force constants K_1 and K_2 have equal P.E. due to extensions in their lengths. Find the ratio of
(i) stretching forces acting on them
(ii) extensions produced in the wires

Sol.: (i) P.E. = $\frac{1}{2} \frac{(\text{Stretching force})^2}{\text{Force constant}} \quad \Rightarrow \quad \frac{F_1^2}{K_1} = \frac{F_2^2}{K_2}$

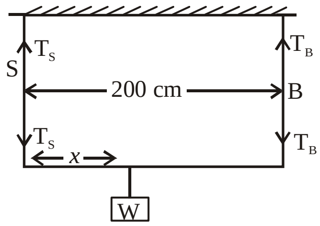
$\therefore \quad F_1 : F_2 \equiv \sqrt{K_1} : \sqrt{K_2}$ .....**Ans.**

(ii) P.E. = $^1/_2 \times$ force constant $\times$ (extension)^2

$\Rightarrow \quad K_1 x_1^2 = K_2\, x_2^2 \quad \Rightarrow \quad x_1 : x_2 \equiv \sqrt{K_2} : \sqrt{K_1}$ .....**Ans.**

Ex.3: A light rod, 200 cm long, is suspended from the ceiling, horizontally by means of two vertical wires of equal lengths, tied to its ends. One of the wires is made of steel and is of cross-section 0.1 cm^2 and the other wire is made of brass of cross-section 0.2 cm^2. Y for steel and brass are 20×10^{11} dyne/cm^2 and 10×10^{11} dyne/cm^2, respectively. Along the rod, at which distance from the steel wire, a weight must be hung to produce
(i) equal stresses in both the wires, and
(ii) equal strains in both the wires

Sol.: Let weight W be hung at distance x from steel wire. Let T_S and T_B be the tensions in steel wire and brass wire, respectively.



(i) $\because$ Stresses in both the wires are equal

$$\therefore \quad \frac{T_S}{A_S} = \frac{T_B}{A_B} \quad \Rightarrow \quad \frac{T_S}{T_B} = \frac{A_S}{A_B} = \frac{0.1\,\text{cm}^2}{0.2\,\text{cm}^2}$$

$$\Rightarrow \quad T_B = 2T_S \qquad \text{.....(i)}$$

Also $T_S + T_B = W \quad \Rightarrow \quad T_S + 2T_S = W \quad \therefore \quad T_S = \frac{W}{3},\ T_B = \frac{2W}{3}$

Taking moments about W

$$T_S\, x = T_B\,(200 - x) \quad \Rightarrow \quad \frac{W}{3}.x = \frac{2W}{3}.(200 - x)$$

$\Rightarrow \quad x = 400 - 2x \quad \Rightarrow \quad x = 400/3$ cm .....**Ans.**

(ii) $\because$ Strains are same in both the wires

$$\therefore \quad \frac{T_S}{A_S Y_S} = \frac{T_B}{A_B Y_B} \qquad \left[\because \text{strain} = \frac{\text{stress}}{Y}\right]$$

$$\Rightarrow \quad \frac{T_S}{T_B} = \frac{A_S Y_S}{A_B Y_B} = \frac{(0.1\,\text{cm}^2)\,(20\times10^{11}\,\text{dyne/cm}^2)}{(0.2\,\text{cm}^2)\,(10\times10^{11}\,\text{dyne/cm}^2)} = \frac{1}{1} \quad \Rightarrow \quad T_S = T_B$$

Against taking moments about W

$T_S\, x = T_B\,(200 - x) \quad \Rightarrow \quad x = 200 - x \quad \Rightarrow \quad x = 100$ cm .....**Ans.**

Bulk Modulus of Elasticity

1. $B = \dfrac{F/A}{\Delta V/V} = -\dfrac{p}{\Delta V/V}$

   where p = hydrostatic pressure

   = pressure acting symmetrically from all sides
2. Reciprocal of bulk moduls of elasticity of a substance is called compressibility of the substance.
3. (a) Unit of compressibility = m^2N^{-1}

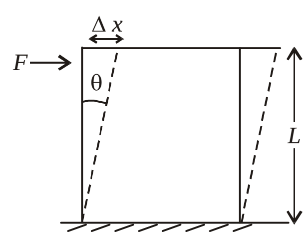
   (b) Dimensions of compressibility = $M^{-1}\,LT^2$

Shearing or Shear Modulus of Elasticity

1. This is only in case of solids
2. $n = \dfrac{F/A}{\theta} = \dfrac{F}{A.\theta}$ .... (i)

   where θ = angle through which vertical plane shifts

   F = tangential force



   For solids, θ is very small $\Rightarrow$ $\theta = \tan\theta = \dfrac{\Delta x}{L}$

   $\therefore$ The above relation (i) can also be written as :

   $n = \dfrac{F}{A}\cdot\dfrac{L}{\Delta x}$, where L = height of vertical plane
3. Shearing modulus of elasticity is also called rigidity of the substance.

Poisson's Ratio (σ)

1. $\sigma = \dfrac{\text{lateral comperssion strain}}{\text{longitudinal stretching strain}} = \dfrac{\Delta D/D}{\Delta L/L}$, where D = diameter of wire

2. Limiting values of σ are –1 and 0.5 i.e., the value of σ lies between –1 and 0.5.
3. If volume of wire remains same, then Poisson's ratio of the material of the wire is + 0.5.

Ex.4: If the volume of a wire remains same, when stretched by a tensile force, calculate Poisson's ratio of the material of the wire.

Sol.: Poisson's ratio = $\frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\Delta D / D}{\Delta L / L}$ .....(i)

where D is the diameter of the wire and ΔD is the change in the diameter; L is the length of the wire and ΔL is the change in the length.

Due to stretching force, let length L of the wire increase by ΔL. As volume remains same, so the diameter must decrease. Let ΔD be the decrease in diameter D of the wire.

$$\therefore \quad \frac{\pi D^2}{4} \times L = \frac{\pi[D - \Delta D]^2}{4} \times [L + \Delta L]$$

$$\Rightarrow \quad D^2L = [D^2 - 2D.\Delta D + (\Delta D)^2]\,[L + \Delta L]$$

$$= [D^2 - 2D.\Delta D]\,[L + \Delta L]$$ $\because$ $(\Delta D)^2$ is negligible as compared to D^2

$$D^2L = D^2L + D^2.\Delta L - 2LD.\Delta D - 2D.\Delta D.\Delta L$$

$$\Rightarrow \quad 2LD.\Delta D = D^2\Delta L$$ $\because$ $2D.\Delta D.\Delta L$ is negligible as compared to $D^2.L$

$$\Rightarrow \quad 2L\Delta D = D\,\Delta L$$

$$\Rightarrow \quad \frac{\Delta D}{D} = \frac{1}{2}.\frac{\Delta L}{L} \quad \Rightarrow \quad \frac{\Delta D / D}{\Delta L / L} = \frac{1}{2}$$ .....(ii)

$\therefore$ Poisson's ratio is 0.5, if volume of the wire remains constant. .....**Ans.**

Relations between Y, B(or K), n and σ

1. $\frac{9}{Y} = \frac{1}{K} + \frac{3}{n}$
2. $Y = 2n\,(1 + \sigma)$
3. $Y = 3K(1 - 2\sigma)$
4. $\sigma = \frac{3K - 2n}{2n + 6K}$

where Y = Young's modulus of elasticity
K = Bulk modulus of elasticity
n = Shearing modulus of elasticity
σ = Poisson's ratio

Some Other Relations of Elasticity

1. When a cylinder or a cylindrical rod of radius r and of length l, fixed at its upper end, is twisted from its lower end such that angle of twist at the lower end is θ, then

(a) ϕ = angle of shear = $\frac{\theta r}{l}$

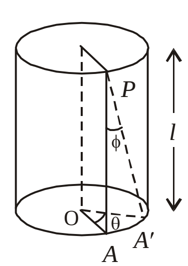
($\because$ $AA' = r\theta = l\phi$)

(b) T = Twisting couple acting on the cylinder = $\frac{\pi}{2}.\frac{n}{l}.r^4\theta$

where n = coefficient of shearing modulus,
= rigidity of the material of the cylinder

C = twisting couple per unit angular twist

$= \frac{\pi}{2}.\frac{n}{l}.r^4$

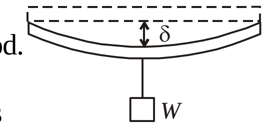


$\angle AOA' = \theta$

$\angle APA' = \varphi$

2. When a rectangular beam of length l, breadth b and depth d is loaded at its mid-point with a load W, then depression δ produced at the mid-point is given by the relation :

$$\delta = \frac{Wl^3}{4Ybd^3}$$, where Y = Young's modulus of material of rod.



3. If the beam is circular of length l and radius r, then the depression produced is

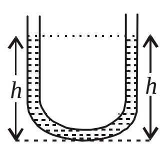
$$\delta = \frac{Wl^3}{12\pi r^4 Y}$$

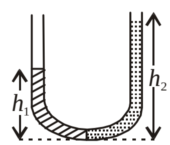
LIQUIDS

1. Liquids are one of the forms of matter in which molecular forces are not so strong as in case of solids. Due to this, these have fixed volume but not fixed shape. These have the shape of a container, as these require a container to be kept.
2. These exert pressure on the surfaces of the container in which these are kept.
3. At any point on the surface, pressure is normal to the surface.
4. $P = h\rho g$, where h = vertical height of a liquid above the point where pressure is required to be found
   ρ = density of the liquid
5. Inside the liquid, pressure at any liquid molecule acts from all directions. Therefore, resultant pressure on a liquid molecule, inside the liquid, is zero.
6. At the base of a U-tube, for liquids in equilibrium, pressures exterted by the liquids in the two limbs are equal or same.

   $h_1\rho g = h_2\rho g$

   $\Rightarrow h_1 = h_2$





   $h_1\rho_1 g = h_2\rho_2 g$

7. Atmospheric pressure = pressure of 76 cm of mercury column .... (*i*)
   $= 0.76 \times (13.6 \times 10^3) \times 9.8 \text{ Nm}^{-2}$
   $= 1.013 \times 10^5 \text{ Nm}^{-2}$ .... (*ii*)
8. (i) 1 Pa (1 pascal) is the pressure of 10^5 Nm^{-2}
   (ii) 1 Torr is the pressure of 1 mm of Hg
   (iii) Barometer was invented by Torricelli and the unit Torr is in his honour.
   (iv) Vacuum above the mercury level in the barometer is called Torricellian vacuum.
9. Pressure of 1 atmosphere at NTP can raise mercury upto a height of 76 cm or 0.76 m in a mercury barometer, but can raise water upto a height of 10.33 m.
10. Pressure at a point or at the base of a liquid column is independent of area of cross-section.

Archemede's Principle & Principle of Floatation

1. When a body is immersed (fully or partially) in a fluid (liquid or gaseous medium), it loses weight and the loss in weight is equal to the weight of fluid displaced.
2. The weight of fluid displaced is also called upward thrust or buoyant force. This is also equal to loss of weight of the body in that fluid.
3. When a body, lighter than a liquid, is put into the liquid, only a part of that body will be immersed. Volume of the body immersed will be such that the weight of the volume of the liquid displaced balances the weight of the body.
4. *For a floating body in a liquid*,
   Weight of floating body = weight of liquid displaced
   $\Rightarrow V_b \cdot \rho_b\, g = v_l \cdot \rho_l g$

5. Because a body loses weight in a liquid, so its effective weight in a liquid becomes less. Hence, effective value of acceleration due to gravity, due to which it is attracted towards the centre of the earth, deceases. The decreased value of acceleration due to gravity when a body of density ρ falls under gravity through a medium of density ρ_0, is given by the relation :

$$g' = g\left\{1 - \frac{\rho_0}{\rho}\right\}$$, where g' = new value of acceleration due to gravity

g = value, when medium is absent

For air ρ_0 is almost negligible $\Rightarrow$ $\rho_0 \approx 0$. So, take $g' = g$ while considering a body falling through air.

Density and Specific Gravity (or Relative Density)

1. Density of a body is its mass in a unit volume. Its units are g cm^{-3} or kg m^{-3} and its dimensions ar ML^{-3}
2. Density of water = 1 g cm^{-3} = 10^3 kg m^{-3} [$\because$ 1 g cm^{-3} = 10^3 kg m^{-3}]
3. Sp. gr = $\frac{\text{Density of body}}{\text{Density of water}}$, being a pure ratio it has no units.
4. Sp. gr of a solid body = $\frac{\text{Weight of the body in air}}{\text{Loss of weight of same body in water}}$
5. Sp. gr of a liquid = $\frac{\text{Loss of weight of a body in a liquid}}{\text{Loss of weight of same body in water}}$

Ex.5: A piece of solid weighs 120 g in air, 80 g in water and 60 g in a liquid. Calculate (i) density of solid and (ii) density of liquid.

Sol.: (i) Sp.gr. of solid = $\frac{\text{Weight of a body in air}}{\text{Loss of weight of same body in water}} = \frac{120}{120-80} = 3$

$\therefore$ Density of solid = sp.gr × density of water

= 3 g cm^{-3} or 3×10^3 kg m^{-3} .....(i) **Ans.**

(ii) Sp.gr of liquid = $\frac{\text{Loss of weight of a body in liquid}}{\text{Loss of weight of same body in water}} = \frac{120-60}{120-80} = \frac{3}{2}$

$\therefore$ Density of liquid = sp.gr × density of water

= 1.5 g cm^{-3} or 1.5×10^3 kg m^{-3} .....(ii) **Ans.**

Surface Tension

1. It is the force acting per unit length on an imaginary line in the surface of a liquid and this force acts perpendicular to this imaginary line and is also tangential to the plane of the liquid.
2. The phenomenon of surface tension is intimately linked with the surface film of the liquid, which consists of thickness of molecular range.
3. Surface energy is the amount of work done against the force of surface tension, in forming the liquid surface of a given area at a constant temperature

   $\therefore$ Surface energy = surface tension × area
4. Surface tension decreases with increase in temperature, given by the relation

   $\sigma_t = \sigma_0(1 - \alpha t)$, where σ_t = surface tension at t°C

   σ_0 = surface tension at 0°C

   α = temperature coefficient of surface tension

   $\therefore$ When temperature rises to a critical temperature, surface tension becomes zero.
5. Excess pressure (P), acting on a curved surface of radius of curvature R of a liquid of surface tension (σ) , is given by the relation.

   $P = \frac{2\sigma}{R}$, P always acts towards concave side of the surface

6. For a soap bubble, $P = \frac{4\sigma}{R}$

7. For an air bubble inside a liquid, $P = \frac{2\sigma}{R}$

8. When two or more liquid drops combine to form a larger liquid drop, surface energy of the system decreases.

9. If N small drops, each of radius r, combine to form a bigger drop of radius R, then

   (*i*) $R = r \,.\, N^{1/3}$

   (*ii*) $\Delta E = N.\sigma\, 4\pi r^2 - \sigma\, 4\pi R^2 = \sigma 4\pi\, [Nr^2 - R^2]$ ...... (*i*)

   $= \sigma\, 4\pi\, R^2\, [N^{1/3} - 1]$ ...... (*ii*)

   $= \sigma\, 4\pi N r^2 \left[1 - \frac{1}{N^{1/3}}\right]$ ..... (*iii*)

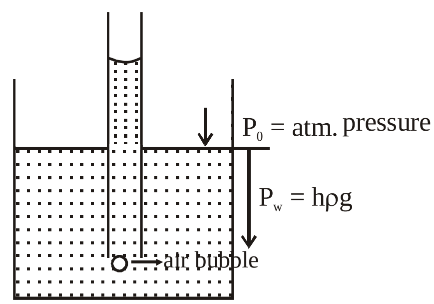
   $= 3.\sigma V\left[\frac{1}{r} - \frac{1}{R}\right]$ where $V = \frac{4}{3}\pi r^3.N$ ..... (*iv*)

10. (a) When small droplets combine to form a bigger drop the temperature of the system rises.

    (b) When a bigger drop is broken to form small droplets, the temperature of the system decreases.

Ex.6: The lower end of a capillary tube of diameter 2.0 mm is dipped 8.0 cm below the surface of water in a beaker. If surface tension of water is 73×10^{-3} Nm^{-1}, atmospheric pressure is 1.01×10^5 Pa and $g = 9.8$ m s^{-2}, calculate the pressure required in the tube to blow a bubble at its end in water.

Sol.: $\because$ Air bubble inside water has only one surface in contact water



$\therefore$ $P_{surf.\ tension}$ = excess pressure on its surface

$$= \frac{2\sigma}{R} = \frac{2 \times 73 \times 10^{-3}}{1 \times 10^{-3}} = 146 \text{ Nm}^{-2}$$

$$= 0.146 \times 10^3 \text{ Nm}^{-2}$$

Also air bubble is at a depth of 8 cm of water.

So, pressure at this depth is the sum of pressure of water and pressure of atmosphere. Now,

$P_{water} = h\rho g = (8 \times 10^{-2}) \times 10^3 \times 9.8 = 784 \text{ Nm}^{-2} = 0.784 \times 10^3 \text{ Nm}^{-2}$

$P_{atm} = 1.01 \times 10^5 \text{ Nm}^{-2} = 101 \times 10^3 \text{ Nm}^{-2}$

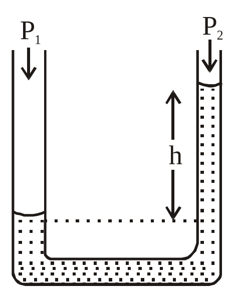
$\therefore$ Total pressure on air bubble $= P_{surf.tension} + P_{water} + P_{atm}$.

$= (0.146 + 0.784 + 101) \times 10^3$

$= 101.93 \times 10^3 \approx 1.02 \times 10^5 \text{ Nm}^{-2}$ .....**Ans.**

Ex.7: The limbs of a manometer consists of uniform capillary tubes of radii 14×10^{-4} m and 7.2×10^{-4} m. Find the correct pressure difference (P_1–P_2), if the level of liquid in the narrower tube stands 0.2 m above that in the broader tube. (Surface tension of water = 72×10^{-3} N m^{-1})

Sol.: If P_1 and P_2 are the pressure in the broader and the narrower tubes of radii r_1 and r_2, respectively, then the pressures just below the meniscuses in the respective tubes are



(i) $P_1 - \frac{2\sigma}{r_1}$ and (ii) $P_2 - \frac{2\sigma}{r_2}$

and this pressure difference of pressures below the meniscuses equals pressure difference of the liquid columns.

$$\therefore \qquad \left[P_1 - \frac{2\sigma}{r_1}\right] - \left[P_2 - \frac{2\sigma}{r_2}\right] = h\rho g$$

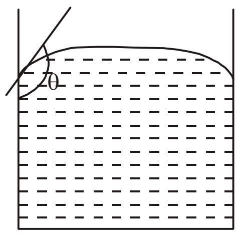
$$\Rightarrow \qquad P_1 - P_2 = h\rho g - \left[\frac{2\sigma}{r_2} - \frac{2\sigma}{r_1}\right] = h\rho g - 2\sigma\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

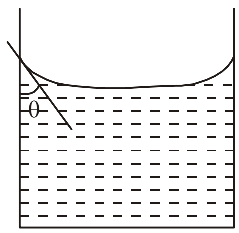
$$= 0.2 \times 10^3 \times 9.8 - 2 \times 72 \times 10^{-3} \times \left[\frac{1}{7.2\times10^{-4}} - \frac{1}{14\times10^{-4}}\right]$$

$$= 1.96 \times 10^3 - 1440 \left[\frac{14-7.2}{7.2\times14}\right] = 1960 - 1440 \times \frac{6.8}{7.2\times14}$$

$$= 1960 - 97.14 \approx 1863 \text{ Nm}^{-2}$$ .....**Ans.**

Angle of Contact





1. At the point of contact, angle of contact is the angle between
   (*i*) the tangent at the point of contact, and
   (*ii*) the surface of the container inside the liquid
2. If the cohesive force of of liquid molecules is greater than the adhesive force between the molecules of the liquid and the molecules of the container (or when the liquid does not wet the surface of the container), then the angle of contact is obtuse (greater than 90º). For example, angle of contact of mercury in glass is obtuse.
3. If the cohesive force of liquid molecules is smaller than the adhesive force between the molecules of the liquid and the molecules of the container (or when the liquid wets the surface of the container), then the angle of contact is acute (less than 90º). For example, angle of contact of water in glass is acute.
4. If the cohesive force of liquid molecules is equal to adhesive force between the molecules of liquid and the molecules of the container, then the angle of contact is zero. For example, angle of contact of pure water in silver utensils is zero.

Capillary Rise

1. It is due to surface tension.
2. If the angle of contact is acute, then the liquid rises in the capillary (*e.g.*, when a capillary is dipped in water).
3. If the angle of contact is obtuse, then the liquid is depressed in the capillary (e.g. when a capillary is dipped in mercury).
4. If h is the rise or depression in the capillary in liquid of surface tension σ, then

$$h\rho g = \frac{2\sigma}{R}$$ where ρ = density of the liquid

R = radius of curvature of the liquid inside the capillary

$$\Rightarrow \qquad hR = \frac{2\sigma}{\rho g}$$ = constant for a given capillary in a given liquid.

5. Capillary rise in terms of radius r of capillary is given by the relation :

$$h\rho g = \frac{2\sigma}{r} \times \cos\theta, \text{ where } \theta = \text{angle of contact}$$

$$\Rightarrow \quad hr = \frac{2\sigma\cos\theta}{\rho g} = \text{constant for a given liquid for two capillaries of different radii.}$$

6. From above equation, $h_1 r_1 = h_2 r_2$

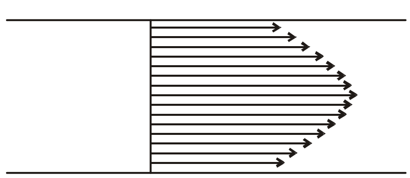
   $\Rightarrow$ Capillary rise is large in capillary of smaller radius and vice-versa.

Properties of Liquids in Motion

1. Motion of the liquids is of following forms :
   (a) Streamline motion
   (b) Steady state motion
   (c) Laminar motion
   (d) Turbulent motion
2. (a) For a liquid in motion, when liquid particles, preceding and succeeding a liquid particle, move along the same line, then this line is called streamline and then the motion of the liquid is called streamline motion.
   (b) Streamline motion takes place when the speed of liquid particles is very small.
   (c) No two streamlines cross in streamline motion of the liquid.
3. (a) For a liquid in motion, if all the liquid particles, crossing a point, cross it with same velocity, then the motion of the liquid is called steady-state motion.
   (b) Steady state motion takes place when the speed of liquid particles is very small.
   (c) Practically steady-state motion and streamline motion are same.
4. When motion of a liquid is fast, irregular with vortices, then motion of the liquid is called turbulent.
5. Reynold's number is used to distinguish between streamline motion and turbulent motion of a liquid.
6. If a liquid of density ρ and of coefficient of viscosity η is flowing through a pipe of radius r with a velocity v, then Reynold number N_R is given by the relation :

   $$N_R = \frac{vr\rho}{\eta} \quad \Rightarrow \quad v = \frac{\eta . N_R}{r\rho}$$

   (a) If $N_R < 1000$, then velocity of liquid is streamline
   (b) If $N_R > 1500$, then velocity of liquid is turbulent
   (c) If $1000 < N_R < 1500$, then nature of motion of the liquid keeps on changing from streamline to turbulent and vice-versa.
7. When different layers of a liquid flow with different velocities (when the liquid is flowing through a pipe) then the motion of the liquid is called laminar motion.
8. The velocity of liquid layers in contact with the boundary of the pipe is smaller and the velocity goes on increasing as the distance of layer increases from the surface.
9. The velocity of the layer exactly in the middle is maximum.



10. (a) The cause of laminar motion is that every layer below opposes the motion of a layer above it.
    (b) The opposing force per unit area between two layers is directly proportional to the velocity gradient between these two layers.

    i.e. $\frac{F}{A} \propto \frac{dv}{dx} \quad \Rightarrow \quad \frac{F}{A} = -\eta \frac{dv}{dx}$

(*i*) Negative sign is because opposing force acts in a direction opposite to the direction of flow of liquid.

(*ii*) η is a constant for a liquid and is called coefficient of viscosity of the liquid.

(*iii*) Dimensions of $\eta = ML^{-1}T^{-1}$

(*iv*) Units of $\eta = Nsm^{-2}$ ($Nm^{-2}.s$) or Pa.s ..... in *S.I.* system of units
= dyn cm^{-2}.s or Poise ..... in *C.G.S.* system of units

(*v*) Units of η in S.I. system = 10 × its unit in *CGS* system

Ex.8: (a) What is the largest average velocity of blood in an artery of radius 2×10^{-4} m, if the flow must remain laminar. Density of blood is 1.06×10^3 kg m^{-3} and viscosity of blood is 2.084×10^{-3} Pa s
(b) Also calculate the flow rate of blood through the artery.

Sol.: (a) $N_R = \dfrac{v\rho r}{\eta} \Rightarrow v = \dfrac{N_R.\eta}{\rho.r}$ .....(i)

Here, N_R = Reynold number (max. value) = 1000 $r = 2 \times 10^{-4}$ m
ρ = density of blood = 1.06×10^3 kg m^{-3} $\eta = 2.084 \times 10^{-3}$ Pa s

$$\therefore \quad v = \frac{1000 \times (2.084 \times 10^{-3})}{(1.06 \times 10^3) \times (2 \times 10^{-4})} = 9.8 \text{ m s}^{-1}$$ .....(a) **Ans.**

(b) Flow rate of blood = volume of blood crossing per sec across artery
= velocity × area of cross-section = $v \times (\pi r^2)$
= $9.8 \times 3.14\,(2 \times 10^{-4})^2 = 1.23 \times 10^{-6}$ $m^3 s^{-1}$ .....**Ans.**

Ex.9: A square plate of side 0.1 m moves, parallel to another plate, with a velocity of 0.1 m s^{-1}, both plates being immersed in water. If the viscous force between the plates is 2×10^{-3} N and the coefficient of viscosity of water is 10^{-3} decapoise, what is the distance between the plates ?

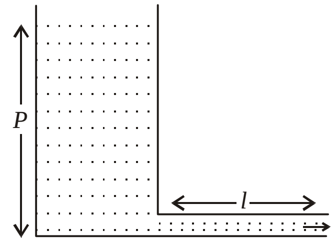
Sol.: $F = 2 \times 10^{-3}$ N $A = (0.1)^2 = 10^{-2}$ m^2
$\eta = 10^{-3}$ decapoise (C.G.S. unit) = 10^{-3} poiseuille (S.I. unit) (10 poise = 1 poiseuille)
dv = velocity of one plate w.r.t. the other plate = 0.1 ms^{-1}
dx = separation between the two plates = ?

$$\frac{F}{A} = -\eta \cdot \frac{dv}{dx} \quad \Rightarrow \quad dx = \frac{\eta . dv . A}{F}$$

$$= \frac{10^{-3} \times 0.1 \times 10^{-2}}{2 \times 10^{-3}} = 0.5 \times 10^{-3} \text{ m} = 0.5 \text{mm.}$$ .....**Ans.**

11. *Poiseuille's equation* : If a pressure *P* of a liquid column is maintained at one end of a horizontal tube of length *l* and radius *r*, such that a steady flow of liquid, of coefficient of viscosity η, is maintained through the pipe, then volume of liquid flowing per second through the pipe is given by the relation :

$$V = \frac{\pi}{8} \cdot \frac{P}{l} \cdot \frac{r^4}{\eta} \quad \Rightarrow \quad V \propto r^4$$



Ex.10: Glycerin flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is 4.0×10^{-3} kg s^{-1}, what is the pressure difference between the two ends of the tube? Density of glycerine = 1.3×10^3 kg m^{-3} and viscosity of glycerine = 0.83 Pa s.

Sol.: V = volume of glycerine flowing per sec = $\frac{\text{mass flowing per sec}}{\text{density}} = \frac{4\times10^{-3}}{1.3\times10^{3}}\ \text{m}^3\text{s}^{-1}$

$l = 1.5$ m $\quad r = 1 \times 10^{-2}$ m $\quad \eta = 0.83$ Pa s $\quad P = ?$

$$V = \frac{\pi}{8}\cdot\frac{P}{l}\cdot\frac{r^4}{\eta} \quad \Rightarrow \quad P = \frac{8}{\pi}\cdot\frac{Vl\eta}{r^4}$$

$$\therefore\ P = \frac{8\times4\times10^{-3}}{3.14\times1.3\times10^{3}}\times\frac{1.5\times0.83}{(1\times10^{-2})^4} = 9.76\times10^{2}\ \text{Pa}$$ .....**Ans.**

Stoke's Relation & Terminal Velocity

1. When a small spherical body of mass m and radius r is moving with velocity v through a viscous medium of coefficient of viscosity η, then it experiences an opposing viscous force F, given by the relation :

$$F = 6\pi\eta rv \qquad \text{...... (Stoke's relation)}$$

2. Clearly, viscous force F increases with the increase in velocity of the spherical ball through the medium i.e. $F \propto v$.
3. Terminal velocity v_0 is the maximum velocity which a body can attain while moving, through a viscous medium under some constant force like gravitation at force.
4. When a body of density ρ is falling under gravity through a viscous medium of coefficient of viscosity η and of density ρ_0, then at terminal velocity v_0 :

   Downward resultant weight = upward viscous force

   If r is radius of the spherical body falling through the medium, then

$$\frac{4}{3}\pi r^3 (\rho - \rho_0)\, g = 6\pi\eta r v_0 \qquad \text{..... }(i)$$

$$\Rightarrow \quad v_0 = \frac{2}{9}\,\frac{r^2(\rho-\rho_0)g}{\eta} \qquad \text{..... }(ii)$$

5. Terminal velocity, $v_0 \propto r^2$
6. Drops of rain water of almost same size, though falling from clouds at different heights, reach the ground with same terminal velocity. This is due to the viscosity of the atomspheric air.

Bernoulli's Theorem & its Applications

1. A liquid in motion has

   (i) kinetic energy, $\frac{1}{2}mv^2$,

   (ii) potential energy, mgh and

   (iii) pressure energy, PV or $\frac{Pm}{\rho}$, where ρ = density of liquid.

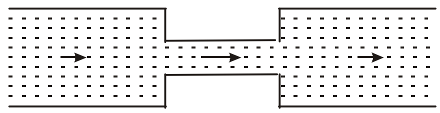
2. For a non-viscous and incompressible liquid with streamline motion, the sum of its $K.E.$ density, $P.E.$ density and pressure energy at any cross-section, is constant or same.

$$\Rightarrow \quad \frac{1}{2}\rho v^2 + \rho gh + P = \text{constant} \qquad \text{..... }(i)$$

   For any two cross sections :

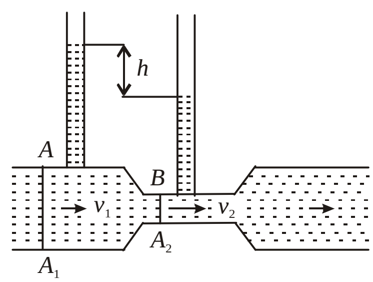
$$\frac{1}{2}\rho v_1^2 + \rho gh_1 + P_1 = \frac{1}{2}\rho v_2^2 + \rho gh_2 + P_2 \qquad \text{..... }(ii)$$

3. The Bernoulli's theorem is based on conservation of energy.
4. (a) For a continuous motion of a liquid, volume of the liquid crossing any cross-section is same *i.e.*
   $V_1 = V_2 \Rightarrow v_1A_1 = v_2A_2$
   where v_1 and v_2 are the velocities of the liquid at cross-sections A_1 and A_2, respectively.
   (b) From the equation $v_1A_1 = v_2A_2$, we see that where area of cross section is small, velocity of liquid is large and vice-versa.
5. For continuous motion of a liquid, where velocity is large pressure is small and vice-versa.



6. Practically the relation "where velocity of the liquid is small, pressure on it is large and vice-versa" is applicable everywhere and is widely used in explaining many phenomenon based on Bernoulli's theorem.
7. Some of the phenomenon which can be explained on Bernoulli's theorem are :
   (a) Working of any sprayer or an atomiser
   (b) Streamline shape of wings of aeroplane to provide the 'lift' to it
   (c) A spinning cricket ball takes a curved path
   (d) A fast moving train attracts a person, standing close to railway track
   (e) Working of a venturimeter
8. Venturimeter is a device to measure rate of flow of fluid V :

$$V = \sqrt{2gh}\ \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}$$



where h = difference in levels of liquid in vertical tubes
(indicates pressure difference of liquids at cross-sections A and B)
A_1 = area of cross-section of pipe
A_2 = area of cross section of venturi-tube

Ex.11: An aeroplane is in level flight at constant speed and each of its two wings has an area of 25 m^2. If the speed of air is 180 km/h over the lower wing-surface and 234 km/h over the upper wing-surface, determine the mass of the aeroplane (take density of air = 1 kg m^{-3})

Sol.: Difference in pressure heads equals difference in velocity heads (Bernoulli theorm)

$\therefore$ Pressure difference from below upwards $\left[\frac{v^2}{2g} = \frac{P}{g\rho}\right]$

$$= (\text{density of air}) \times \frac{(v_{\text{upper surface}})^2 - (v_{\text{lower surface}})^2}{2}$$

$$= 1 \times \frac{65^2 - 50^2}{2} = 862.5\,\text{Pa}$$

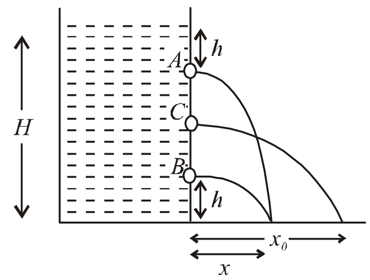
$\therefore$ Upward thrust = Upward pressure × Area of wings
= 862.5 × 50 = 43125 N

This upward thrust balances the weight of the aeroplane

$\therefore$ Mass of aeroplate = $\dfrac{43125}{9.8} \approx 4400$ kg .....**Ans.**

Torricelli's Theorem

Consider a tank in which liquid level of height H is maintained. Let there be a hole or an opening at A, at a depth h from the upper level of the liquid. Then,



1. Velocity of efflux of the liquid = $\sqrt{2gh}$

   (The velocity with which liquid comes out horizontally is same as if it has fallen freely through a vertical height h. This statement is known as Torricelli's theorem)
2. The horizontal range, $x = 2\sqrt{h(H-h)}$
3. For two taps, equidistant from the centre, horizontal range is same.
4. Horizontal range is maximum, if tap is at the centre of vertical height of liquid level.
5. If for two taps, at depths h_1 and h_2 from the upper surface, horizontal range is same, then $H = h_1 + h_2$
6. All the above relations are independent of density of the liquid inside the tank i.e., the relations are same for water, oil, etc.

GASES

1. Gases are one of the forms of matter in which molecular forces are negligible. Due to this, gases have no fixed shape and no fixed volume.
2. The volume of the gas is not only affected by temperature but also by pressure.
3. Assumptions of kinetic theory of gases :

   (*i*) The velocity of random motion is equally possible in all the directions.

   (*ii*) Collisions between the gas molecules themselves, and between the gas molecules and the molecules of the surface of the container are perfectly elastic.

   (*iii*) Time of contact during collisions of gas molecules is negligible and so the molecular forces are negligible during collisions.

   (*iv*) The actual volume of gas molecules is negligible as compared to the volume over which these are spread.
4. Using above assumptions of kinetic theory of gases, the following relations can be obtained :

   (*i*) $PV = \frac{1}{3}mnC^2$ where P = pressure of gas of volume V

   n = number of gas molecules, each of mass m

   C = *r.m.s.* velocity of gas molecules

   (*ii*) $P = \frac{1}{3}\rho C^2$ $\left(\because \frac{mn}{V} = \rho = \text{density of gas}\right)$

   (*iii*) $C = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3RT}{M}}$ [$\because$ for 1 mole, $PV = RT$, then M = molar mass]

   (*iv*) $PV = \frac{1}{3}mnC^2 = \frac{2}{3}n\left[\frac{1}{2}mC^2\right]$

   $\Rightarrow$ $PV = \frac{2}{3}n$ (energy of each molecule)

   (*v*) $RT = \frac{2}{3}E_{mole} \Rightarrow E_{mole} = \frac{3}{2}RT$ if $n = N$ for 1 mole (*vi*) $E_{molecule} = \frac{3}{2}\frac{R}{N}T = \frac{3}{2}kT$ [$k = \frac{R}{N}$ = Boltzmann constant = 1.38×10^{-23} JK^{-1}]

Kinetic Energy and Specific Heats of Gas Molecules (considered to be rigid)

1. Gas molecules have translational *K.E.* as well as rotational *K.E.*
2. The total kinetic energy of gas molecules (translational + rotational)
   = [Number of degrees of freedom] × [Energy associated with each degree of freedom]
   $\Rightarrow$ $E = n \times {}^1/_2\, RT$ for one mole ..... (*i*)
   $E = n \times {}^1/_2\, kT$ for one molecule ..... (*ii*)
3. n = number of degrees of freedom
   = 3 for monoatomic gases ..... (*iii*)
   = 5 for diatomic gases ..... (*iv*)
   = 6 for polyatomic gases ..... (*v*)
4. $C_p > C_v$ and $C_p - C_v = R$
5. (*i*) $\gamma = \frac{C_p}{C_v} = \frac{n+2}{n} = 1 + \frac{2}{n}$ where n = No. of degrees of freedom

   (*ii*) $n = \frac{2C_v}{C_p - C_v} = \frac{2}{\gamma - 1}$ where $\gamma = \frac{C_p}{C_v}$

When gas molecules are not rigid

1. When there is vibration within the molecule itself (this can happen in diatomic or polyatomic molecule), then kinetic energy associated with the gas molecule due to vibration at temperature T is given by the relation

   $E = f\,k\,T$ where k = Boltzmann constant
   f = number of vibrational modes
2. Therefore, *for diatomic gas molecule for one vibrational mode*

   E_{molecule} = (energy due to translational and rotational motion) + (energy due to vibrational motion within it)

   $$= \left(5 \times \frac{1}{2}kT\right) + 1 \times kT = \frac{7}{2}kT$$

   $$\therefore \quad C_v = \frac{7}{2}R \qquad C_p = \frac{9}{2}R \qquad \gamma = \frac{9}{7}$$
3. *For polyatomic gas molecule of f vibrational modes*

   E_{molecule} = (energy due to translational and vibrational motion) + (energy due to vibrational motion within it)

   $$= 6 \times \frac{1}{2}kT + f\,kT = (3 + f)\,kT$$

   $$\therefore \quad C_v = (3 + f)\,R \qquad C_p = (4 + f)\,R \quad \Rightarrow \quad \gamma = \frac{4 + f}{3 + f}$$

Other Gas Laws

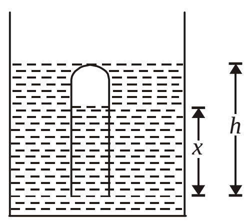
1. *Avogadro's law* : equal volumes of all gases at same temperature and pressure have equal number of

molecules.

2. *Boyle's law* : For a given mass of a gas, at constant temperature, the product of its pressure and volume is constant *i.e.* PV = constant $\Rightarrow$ $P_1V_1 = P_2V_2$
3. *Dalton's law of partial pressures* : the resultant pressure exerted by a mixture of the gases, in a given volume, is the sum of pressures exerted by each gas as if each gas occupies the same volume *i.e.*
$$P = P_1 + P_2 + P_3 + ...$$
4. *Graham's law of diffusion* : At a given temperature and pressure, the rate of diffusion of a gas is inversely proportional to the square root of its density i.e.,
$$r = \text{rate of different} \propto \frac{1}{\sqrt{\rho}} \qquad \Rightarrow \qquad \frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$
5. *Gas equation* : $PV = nRT$ where n = No. of moles = $\frac{\text{mass of gas}}{\text{molecular mass of gas}}$

   For 1 mole of the gas at S.T.P.
   V = 22.4 litres T = 273 K
   R = 8.31 JK^{-1} mol^{-1}
6. If C_{rms} is *r.m.s.* velocity of gas molecules and C_{sound} is the velocity of sound in that gas, then
$$\frac{C_{rms}}{C_{sound}} = \frac{\sqrt{3P/\rho}}{\sqrt{\gamma P/\rho}} = \sqrt{\frac{3}{\gamma}} \qquad \left(\text{where } \gamma = \frac{C_p}{C_v}\right)$$

Ex.12: A tube of length h, which is wide enough to make effects of surface tension negligible, is closed at one end. It is then lowered into a tank of mercury to a depth h, as shown in the figure, so that mercury rises to a height x into the tube. If the mercury barometer also stands at h, then find h in terms of x.



Sol.: *Before the tube of length h is lowered into water*, it has air at atmospheric pressure (equal to barometric height h). Let A be its area of cross-section , then volume of air in the tube is Ah.

$\therefore$ $P_1V_1 = h\,Ah = A\,h^2$ .....(i)

After the tube has been lower into the water

Pressure of air enclosed + pressure of water column of height x

= Atmospheric pressure + pressure of water column of height h

$\Rightarrow$ $P_2 + x = h + h$ $\Rightarrow$ $P_2 = 2h - x$

V_2 = volume of air = $A\,(h - x)$

$\therefore$ $P_2\,V_2 = (2h - x)A\,(h - x)$ .....(ii)

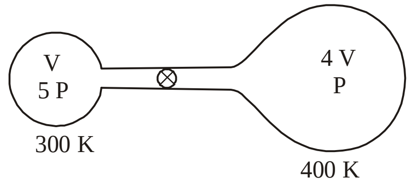
Applying Boyle's law, $P_1V_1 = P_2V_2$

$Ah^2 = (2h - x)A\,(h - x)$ $\Rightarrow$ $h^2 = (2h - x)\,(h - x)$ .....(iii)

$\Rightarrow$ $h^2 = 2h^2 - 3hx + x^2$ $\Rightarrow$ $h^2 - 3hx + x^2 = 0$

$$h = \frac{3x \pm \sqrt{9x^2 - 4x^2}}{2} = \frac{x\left(3 \pm \sqrt{5}\right)}{2}$$ .....**Ans.**

Ex.13: Two glass containers of internal volume V and 4 V are connected with a small tube, of negligible internal volume, which is provided with a stopper, as shown. The pressures of air in them are 5 P and P and their temperatures are 300 K and 400 K, respectively. If the respective temperatures of the containers are kept constant and the stopper is removed to connect the two containers then find the common pressure in both the containers.



Sol.: Let P' be the common pressure in both the tubes after the stopper is removed. Then, applying the gas equation

$$\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3} + \frac{P_4V_4}{T_4}$$

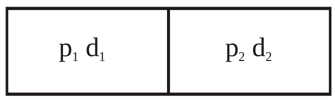
and putting the proper values we get

$$\frac{5P \times V}{300} + \frac{P \times 4V}{400} = \frac{P' \times V}{300} + \frac{P' \times 4V}{400}$$

$$\Rightarrow \quad \frac{5}{3}P + P = \frac{P'}{3} + P' \quad \Rightarrow \quad \frac{8P}{3} = \frac{4P'}{3} \quad \Rightarrow \quad P' = 2P$$ .....**Ans.**

Ex.14: Two chambers containing m_1 and m_2 grams of gases of densities d_1 and d_2 and at pressures p_1 and p_2, respectively, are put into communication. Find the pressure of the mixture.

Sol.:



Assuming temperature to be constant ($\because$ there is no mension of variation of temperature), Boyle's law can be applied. Let p be the common pressure.

$$\therefore \quad p[\text{total volume}] = p_1V_1 + p_2V_2 \qquad V_1 = \frac{m_1}{d_1},\ V_2 = \frac{m_2}{d_2}$$

$$\Rightarrow \quad p\left[\frac{m_1}{d_1} + \frac{m_2}{d_2}\right] = p_1\frac{m_1}{d_1} + p_2\frac{m_2}{d_2}$$

$$\Rightarrow \quad p[m_1 d_2 + m_2 d_1] = p_1 m_1 d_2 + p_2 m_2 d_1$$

$$\Rightarrow \quad p = \frac{p_1m_1d_2 + p_2m_2d_1}{m_1d_2 + m_2d_1}$$ .....**Ans.**