Chapter 12

12

Ratio and Proportion

IMPORTANT DEFINITIONS

1. Ratio

The ratio of two quantities of the same kind and in the same units is a fraction that shows how many times the one quantity is of the other.

Thus, the ratio of two quantities a and b ($(b \neq 0)$ is $a \div b$ or $\frac{a}{b}$ and is denoted by $a : b$

In the ratio a : b, the quantities (numbers) a and b are called the terms of the ratio. The former a is called the first term or antecedent and the later 'b' is known as the second term or consequent.

Note 1: In a ratio, he compare two quantities. The comparison becomes meaningless if the quantities being compared are not of the same kind i.e. they are not measured in the same units.

For example: It is just meaningless to compare 20 bags with 20 cross. Therefore, to find the ratio of two quantities, they must be expressed in the same units.

Note 2: Since the ratio of two quantities of the same kind determines how many times one quantity is contained by the other. So the ratio of any two quantities of the same kind is an abstract quality.

Note 3: The order of the term in a : b is very important. The ratio 3 : 2 is different from 2 : 3.

2. Proportion

An equality of two ratios is called a proportion.

Sign of Proportion: The Sign of Proportion is denoted as ":"

For example, the ratios 1:4 and 4:16 are equal which means both the ratios are in proportion. It can be written as 1:4::4:16.

Consider the two ratios 6 : 18 and 8 : 24 we find that

For example: 6 : 18 = 1 : 3 and 8 : 24 = 1 : 3

$\therefore$ 6 : 18 and 8 : 24

Thus, 6 : 18 = 8 : 24 is a proportion.

Note: Four numbers $a, b, c\ d$ are said to be a proportion, if the ratio of the first two is equal to the ratio of the last two i.e. $a : b = c : d$.

(i) **Continued Proportion:** Three numbers, *a, b, c, d* are said to be in continued proportion *a, b, b, c* are in proportion.
Thus if a, b, c are in continued proportion, then
a, b, b c are is proportion is $a : b : : b : c$
$\Rightarrow$ Product of extreme terms = product of mean terms
$\Rightarrow a \times c = b \times b$
$\Rightarrow ac = bb$
$\Rightarrow b^2 = ac$

(ii) **Mean Proportion:** If *a, b, c* are in continued proportion then *b* is called the mean proportional between *a* and *c* clearly, if *b* is the mean proportional between *a* and *c* then $b^2 = ac$.

Example 1: Express the ratio 45:108 in its simplest from?

Solution: In order to express the given ratio in it simplest form we divide its first and second term by their HCF.

We have, $45 = 3 \times 3 \times 5$ and $108 = 2 \times 2 \times 3 \times 3 \times 3$

So, HCF of 45 and 108 is $3 \times 3 = 9$

$$\therefore 45:108 = \frac{45}{108} = \frac{45 \div 9}{108 \div 9} = \frac{5}{12} = 5:12$$

Example 2: Divide `1,500 among A, B, C in the ratio 3 : 5 :2

Solution: We have,
Sum of the terms of the ratio = 3 + 5 + 2 = 10

A's share = $` \left( \frac{3}{10} \times 1,500 \right) = ` 450$

B's share = $` \left( \frac{5}{10} \times 1,500 \right) = ` 750$

C's share = $` \left( \frac{2}{10} \times 1,500 \right) = ` 300$

Example 3: If $(4x+5):(3x+11) = 13:17$, find the value of *x*.

Solution: We have,

$$(4x+5):(3x+11) = 13:17$$

$$\Rightarrow \quad \frac{4x+5}{3x+11} = \frac{13}{17}$$

$$17(4x+5) = 13(3x+11)$$

$$68x + 85 = 39x + 143$$

$$68x - 39x = 143 - 85$$

$$29x = 58$$

$$x = \frac{58}{29} = 2$$

Example 4: A bag contains `187 in the form of 1 rupee, 50 paise and 10 paise coin in the ratio 3 : 4 : 5. Find the number of each type of coins?

Solution: Let the number of 1 rupee, 50 paise and 10 paise coins be 3x, 4x and 5x respectively, Then,

$$3x + 4x \times \frac{50}{100} + 5x \times \frac{10}{100} = 187$$

$$3x + 2x + \frac{x}{2} = 187$$

$$\frac{11}{2}x = 187 \Rightarrow x = 34$$

$$\therefore 3x = 3 \times 34 = 102,\ 4x = 4 \times 34 = 136 \text{ and } 5x = 5 \times 34 = 170$$

Have, the number of 1 rupee, 50 paise and 10 paise coins are 102, 136 and 170 respectively.

Example 5: What must be added to the number 6, 10, 14 and 22 so that they are in proportion?

Solution: Let the required number be x then,

$6 + x, 10 + x, 14 + x, 22 + x$ are in proportion.

$$\Rightarrow (6 + x)(22 + x) = (10 + x)(141 + x)$$

$$\Rightarrow 132 + 6x + 22x + x^2 = 140 + 10x + 14x + x^2$$

$$\Rightarrow 132 + 20x = 140 + 24x$$

$$\Rightarrow 28x - 24x = 140 + 132$$

$$\Rightarrow 4x = 8$$

$$\Rightarrow x = \frac{8}{4} = 2$$

$$\Rightarrow x = 2$$

Example 6: If 3 : x : : 12 : 20, find the value of x?

Solution: We have,

$3 : x :: 12 : 20$

$\Rightarrow$ 3, x, 12, 20 are in proportion

$\Rightarrow$ Product of extremes = product of means

$\Rightarrow$ $3 \times 20 - x \times 12$

$\Rightarrow$ $60 = 12x$

$$\Rightarrow \frac{12x}{12} = \frac{60}{12}$$

$\Rightarrow$ $X = 5$

TIPS FOR COMPETITIVE LEVEL

1. Suppose it is given that A:B, B:C, C:D then A:D can be easily find out using the following relation:

$$\frac{A}{D}=\frac{A}{B}\times\frac{B}{C}\times\frac{C}{D}$$

A: C can also find out using the relation: $\frac{A}{C}=\frac{A}{B}\times\frac{B}{C}$

2. If a and b are two quantities then

i) Duplicate ratio of a:b = $a^2:b^2$

ii) Sub- Duplicate ratio of a:b = $\sqrt{a}:\sqrt{b}$

iii) Triplicate ratio of a:b = $a^3:b^3$

iv) Sub- Duplicate ratio of a:b = $\sqrt[3]{a}:\sqrt[3]{b}$

v) Inverse of reciprocal ratio of a:b = $\frac{1}{a}:\frac{1}{b}$

vi) Third proportional to a and b = $\frac{b^2}{a}$

Example 1: What do we mean by the ratio of two natural numbers?

Solution: It is their relationship with respect to relative size that we can express verbally in a sentence. Specifically, one number is a multiple of the other (so many times it), a part of it, or parts of it.

Example 2: What ratio has 15 to 5?

Solution:. 15 is three times 5.
That is the ratio -- the relationship -- of 15 to 5.

Example 3. What ratio has 5 to 15?

Solution: 5 is the third part of 15.

That is called the inverse ratio of 15 to 5

Example 4: Let's suppose you earn `200 a week. Your house rent is `40 weekly. What is the ratio of your rent to your income?

Solution: Make a ratio with the rent on top (numerator) and the weekly income on the bottom (denominator). Then reduce.

$$\frac{\text{Rent}}{\text{Income}}=\frac{40}{200}=\frac{1}{5}\ or\ 1:5$$

Example 5: Simplify the ratio 1/3:1/2.

Solution: Ratio is a comparison of numbers by division. Rewrite the above example as a division problem and solve.

1/3:1/2 = 1/3 ¸ 1/2 = 1/3 ´ 2/1 = 2/3 or 2:3

Example 6: On a workplace mathematics test of 20 questions, you missed 2 questions. What is the ratio of the number you answered correctly to the number you missed?

Step 1: Subtract the number of questions you missed from the total number of questions.
Total questions - 20
Number missed - 2
Number correct - 18

Step 2: Make a ratio with the number you answered correctly on top (numerator) and the number you missed on the bottom (denominator). Then reduce if necessary.

$$\frac{Number\ correct}{Number\ missed} = \frac{18}{2} = \frac{9}{1} = 9:1$$

Proportions

Example 1: 5 is to 15 as 8 is to 24. Is this a proportion?
This is a proportion because
5 : 15 = 5/15 = 1/3
8:24 =1/3

Example 2: Why is this a proportion? 16 is to 2 as 80 is to 10.

Solution: This is a proportion because 16: 2 =16/2= 8
80:10 = 80/10 =8

Example 3: Complete this proportion: 8 is to 32 as 9 is to ?

Solution: Let the missing number be x 8:32:: 9:x
USE: **Product of extremes = Product of means**

$$\Rightarrow 8 \times x = 32 \times 9$$

$$x = \frac{32 \times 9}{8}$$

$$x = \frac{288}{8} = 36$$

Example 4: Complete this proportion: 27 is to 3 as ? is to 5

Solution: Let the missing number be x 27:3:: x :5
USE: **Product of extremes = Product of means**
$\Rightarrow 27 \times 5 = 3 \times x$
$x = \frac{27 \times 5}{3}$
$= \frac{135}{3} = 45$

Example 5: In each item below, what ratio has *a* to *b*?

Solution: a) Since 1 is the sixth part of 6, then *a* is the sixth part of *b*.
b) Since 10 is ten times 1, then *a* is ten times *b*.
(*a* simply means the first term; *b* means the second.)
a) *a* is to *b* as 1 is to 6.
b) *a* is to *b* as 10 is to 1.

Example 6: Read this proportion, and complete it:
$\frac{8}{2} = \frac{20}{?}$

Solution: Let the missing number be x
$\frac{8}{2} = \frac{20}{x}$
$\Rightarrow 8 \times x = 20 \times 2$
$x = \frac{20 \times 2}{8} = \frac{40}{8} = 5$

Example 7: Complete this proportion
$\frac{7}{21} = \frac{4}{?}$:

Solution: Let the missing number be *x*
$\frac{7}{21} = \frac{4}{x} \Rightarrow 7 \times x = 21 \times 4$
$x = \frac{21 \times 4}{7} = \frac{84}{7} = 12$

Example 8: Complete this proportion:
$\frac{2}{3} = \frac{?}{12}$

Solution: Let the missing number be x
$\frac{2}{3} = \frac{x}{12} \Rightarrow 2 \times 12 = x \times 3$
$x = \frac{2 \times 12}{3} = \frac{24}{3} = 8$

PART – I: MISCELLANEOUS DOMAIN

1. Let's assume there are 150 employees at your facility and 105 employees belong to some type of recreation club. What is the ratio of the number of employees who do not belong to a recreation club to the total number of employees?

2. In a workplace mathematics class with 20 students, there are 12 women. What is the ratio of the number of women to the total number of students?

3. Simplify the ratio 1/4:2/3.

4. Out of a total weekly budget of $180.00, $30.00 is spent for food. What is the ratio of the amount spent for food to the amount not spent on food?

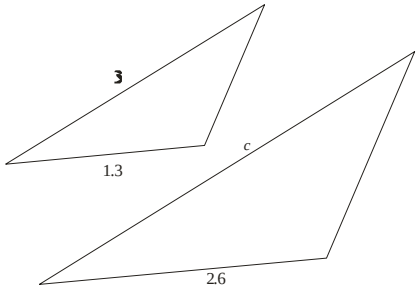
5. Divide Rs. 77 in the ratio 2:9

6. If a:b =3:7 and b:c = 6:7, find a:c

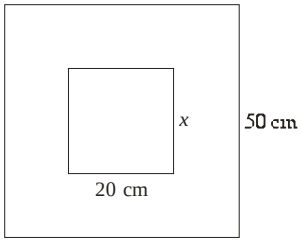
7. What must be added to each term of the ratio 7:12 so that the ratio becomes 3:4?

8. Find the mean proportional to 36 and 64.

9. A and B are similar triangles. What is side c? show work.



10. For the similar squares below, find x.



11. Find the third proportional to 10 and 20.

12. If $\frac{a}{2}=\frac{b}{5}=\frac{c}{2}$ then find the value of $\frac{a+b+c}{c}$

13. If m:n = 3:5, find the value of $\frac{4m+3n}{6m-2n}$

14. The sides of a triangle are in the ratio of $\frac{1}{3}:\frac{1}{4}:\frac{1}{8}$ If the perimeter is 102 cm, then find the length of the smallest side ?

15. The difference between the present ages of P and Q is 12 years and the ratio of their present ages is 2:5 respectively. What is P's present age?

16. Find x in the proportion 6:9:: 36:x?

17. The monthly salary of A, B and C are in the proportion 3:4:6. If C's monthly salary is Rs.2400 more than A's monthly salary, then find B's annual salary.

18. If $x=\frac{1}{y}$ and $y=\frac{1}{3}z$ then find x: y: z?

HIGHER ORDER THINKING SKILLS (HOTS)

19. Divide `1,250 between Aman and Amit in the ratio 2 : 3?

20. The ratio of copper and zinc in an alloy is 5 : 3. If the weight of the copper in the alloy is 30.5 gm, find the weight of zinc in the alloy?

21. What must be added to each term of the ratio 2 : 5 so that it may become equal to 5 : 6?

22. The sides of triangle are in the ratio 1 : 2 : 3. If the perimeter is 36 cm. Find its sides?

23. If $x : y = 3 : 5$ find the ratio $3x + 4x : 8x + 5y$.

24. The boys and the girls in a school are in the ratio is 7 : 4. If total strength of the school be 550, find the number of boys and girls.

25. The ratio of monthly income to the saving of a family is 7 : 2. If the saving be of `500. Find the income and expenditure.

26. Find the ratio of the price of pencil to that of ball pen. If pencils cost `16 per score and ball pens cost `8.40 per dozen?

27. The scale of a map is 1 : 3000000. What is the actual distances between the two towns, if they are 3 cm part on the map?

28. If three leaves by bread are consumed by 9 people, how many people will consume 9 leaves of bread?

29. A electric pole casts a shadow of length 20 meters at a time when a tree 6 metres high casts shadow of length 8 metres. Find the height of the pole.

30. Find the ratio of least prime number to the least composite number?

31. Find 12 : 3 : : x : 1 value of x?

32. In a cricket coaching camp, 1,200 children are trained one of which 900 are selected for various matches. Find the ratio of non-selective children to the total number of children?

33. If $3\frac{1}{3}$ of A = 1.5 of B = $\frac{1}{8}$ of C, then what is A : B : C?

34. A sum of money is divided into 2 parts such that 6 times of one part added to 15 times the other gives 8 times the whole. What is the ratio of one part to the other?

35. If $3x = 2k$ and $5y = 8k$ then find the ratio of $x : y$?

36. If $x : 2\frac{1}{3} :: 21 : 50$ then what is the value of x?

37. Instead of dividing `117 among P, Q, R in the ratio $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$, by mistake it was divided in the ratio 2 : 3: 4 and who gained in the transaction?

38. If $\frac{5a+3b}{2a-3b}=\frac{23}{5}$, the what is the value of $a : b$?

PART – II: MULTIPLE CHOICE QUESTIONS

1. If a, b, c are in proportion, then
   (a) $a^2 = bc$ (b) $b^2 = ac$ (c) $c^2 = ab$ (d) None of these

2. Choose the correct statement:
   (a) (5 : 8) > (3 : 4) (b) (5 : 8) < (3 : 4)
   (c) two ratios cannot be compared (d) None of these

3. If `760 is divided between A and B in the ratio 8 : 11, then B's share is
   (a) `440 (b) `320 (c) `430 (d) `330

4. The sides of a triangle are in the ratio 1 : 3 : 5 and its perimeter is 90 cm. The length of its largest side is
   (a) 40 cm (b) 50 cm (c) 36 cm (d) 54 cm

5. If bus covers 195 km in 3 hours and a train covers 300 km in 4 hours, then the ratio of their speed is
   (a) 13 : 15 (b) 15 : 13 (c) 13 :12 (d) 12 : 13

6. If cost of 5 bars of soap is `82.50, the cost of one dozen such bars is
   (a) `208 (b) `192 (c) `480 (d) `720

7. If 4 : 5 : : x : 35 then the value of x is
   (a) 42 (b) 32 (c) 28 (d) None of these

8. The 1^{st}, 2^{nd} and 4^{th} terms of a proportion are 12, 21 and 14 respectively. Its third term is
(a) 16 (b) 18 (c) 21 (d) 8

9. If a, b, c are in proportion, then
(a) $a^2 = bc$ (b) $b^2 = ac$ (c) $c^2 = ab$ (d) None of these

10. 10 boys can dig a pitch in 12 hours, how long will 8 boys take to do it?
(a) 9 hour 36 minutes (b) 15 hour (c) 6 hour 40 minutes (d) 13 hour 20 minutes

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