

Fractions and Decimals

FRACTION: A fraction is a number representing a part of a whole. The whole may be a single object or a group of objects.

Example: $\frac{2}{5}$ is a fraction with numerator 2 and denominator 5.

PROPER FRACTION: A fraction whose numerator is less than the denominator, is called a proper fraction.

Example: $\frac{7}{9}$, $\frac{3}{11}$, $\frac{2}{5}$ is a proper fractions.

IMPROPER FRACTION: A fraction whose numerator is more than or equal to the denominator, is called an improper fraction.

Example: $\frac{195}{111}$ is a improper fractions.

MIXED FRACTION: A combination of a whole number and a proper fraction is called a mixed fraction.

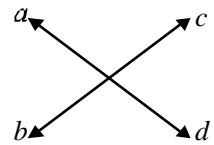
EQUIVALENT FRACTIONS: A given fraction and various fractions obtained by multiplying (or dividing) its numerator and denominator by the same non-zero number, are called equivalent fractions.

Example: $\frac{3 \times 2}{4 \times 2} = \frac{6}{8}$, $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$, $\frac{3 \times 4}{4 \times 4} = \frac{12}{16}$ etc., are equivalent fractions equivalent to the fraction $\frac{3}{4}$.

If $\frac{a}{b}$ and $\frac{c}{d}$ are two equivalent fractions, then

$$\boxed{a \times d = b \times c}$$

i.e., $\frac{a}{b} = \frac{c}{d} \Leftrightarrow a \times d = b \times c$



LIKE FRACTIONS: Fractions having the same denominators are called like fractions.

Example: $\frac{2}{15}$, $\frac{7}{15}$, $\frac{11}{15}$ etc., are like fractions.

UNLIKE FRACTIONS: Fractions with different denominators are called unlike fractions.

Example: $\frac{2}{13}$, $\frac{7}{24}$, $\frac{9}{125}$ etc., are unlike fractions.

FRACTION IN LOWEST TERMS: *A fraction is in its lowest terms if its numerator and denominator have no common factor other than 1.*

COMPARING FRACTIONS

In order to compare fractions, we may use the following steps:

- Step I** Find the LCM of the denominators of the given fractions.
- Step II** Convert each fraction to its equivalent fraction with denominator equal to the LCM obtained in step I.
- Step III** Arrange the fractions in ascending or descending order by arranging numerators in ascending or descending order.

CONVERSION OF UNLIKE FRACTIONS TO LIKE FRACTIONS

In order to convert unlike fractions to like fractions, we follow the following steps:

- Step I** Find the LCM of the denominators of the given fractions.
- Step II** Convert each of the given fractions into an equivalent fraction having denominator equal to the LCM obtained in step I.

ADDITION AND SUBTRACTION OF FRACTIONS

In order to add and subtract unlike fractions, we follow the following steps:

- Step I** Obtain the fractions and their denominators.
- Step II** Find the LCM of the denominators.
- Step III** Convert each fraction into an equivalent fraction having its denominator equal to the LCM obtained in step II.
- Step IV** Add or subtract like fractions obtained in step III.

Example: $2\frac{2}{3} + 3\frac{1}{2}$

$$= \frac{2 \times 3 + 2}{3} + \frac{3 \times 2 + 1}{2}$$

$$= \frac{8}{3} + \frac{7}{2}$$

$$= \frac{8 \times 2}{3 \times 2} + \frac{7 \times 3}{2 \times 3} \quad \left[\because \text{LCM of 3 and 2 is 6. So, convert each fraction} \right]$$

$$\quad \left[\text{to an equivalent fraction with denominator 6} \right]$$

$$= \frac{16}{6} + \frac{21}{6} = \frac{16 + 21}{6} = \frac{37}{6}$$

Step II Write the decimals in columns with their decimal points directly below each other so that tenths come under tenths, hundredths come under hundredths and so on.

Step III Add or subtract as we add or subtract whole numbers.

Step IV Place the decimal point, in the answer, directly below the other decimal points.

Illustration 2: Simplify: $36.54 - 15.79 + 57.615 + 85.2$

Solution: We have

$$\begin{aligned}
 & 36.54 - 15.79 + 85.2 - 57.615 \\
 &= 36.650 - 15.790 + 85.200 - 57.615 \quad \left[\begin{array}{l} \text{Converting the given decimals into} \\ \text{like decimals} \end{array} \right] \\
 &= (36.650 + 85.200) - (15.790 + 57.615) \\
 &= 121.850 - 73.405 \\
 &= 48.445
 \end{aligned}$$

| | | |
|-----------------|-----------------|-----------------|
| 36.650 | 15.790 | 121.850 |
| <u>+ 85.200</u> | <u>+ 57.615</u> | <u>- 73.405</u> |
| 121.850 | 73.405 | 48.445 |

MULTIPLICATION OF A DECIMAL BY 10, 100, 1000, etc.

We follow the following rules to multiply a decimal by 10, 100, 1000, etc.

Step I On multiplying a decimal by 10, the decimal point is shifted to the right by one place.

Step II On multiplying a decimal by 100, the decimal point is shifted to the right by two places.

Step III On multiplying a decimal by 1000, the decimal point is shifted to the right by three places, and so on.

MULTIPLICATION OF A DECIMAL BY ANOTHER DECIMAL

In order to multiply a decimal by another decimal, we follow the following steps:

Step I Multiply the two decimals without decimal point just like whole numbers.

Step II Insert the decimal point in the product by counting as many places from the right to left as the sum of the number of decimal places of the given decimals.

Illustration 3: Find the product of 2.5 and 6.25?

Solution: We have,

$$\begin{array}{r}
 625 \\
 \times 25 \\
 \hline
 3125 \\
 \times 1250 \\
 \hline
 15625
 \end{array}$$

$$\therefore 625 \times 25 = 15625$$

Since the sum of the decimal places in the given decimals is $1 + 2 = 3$.

So, the product must contain 3 places of decimals.

$$\text{Hence, } 6.25 \times 2.5 = 15.625$$

DIVIDING A DECIMAL BY 10, 100, 1000 etc.

In order to divide a decimal by 10, 100, 1000 etc., we follow the following rules:

Rule I: When a decimal is divided by 10, the decimal point is shifted to the left by one place.

Rule 2: When a decimal is divided by 100, the decimal point is shifted to the left by two places.

Rule 3: When a decimal is divided by 1000, the decimal point is shifted to the left by three places.

DIVIDING A DECIMAL BY A DECIMAL

In order to divide a decimal by another decimal, we follow the following steps:

Step I Multiply the dividend and divisor by 10 or 100 or 1000 etc., to convert the divisor into a whole number.

Step II Divide the new dividend by the whole number obtained in step I. Following examples will illustrate the above procedure.

Illustration 4: Divide 0.0216 by 0.6

We have,

$$\frac{0.0216}{0.6} = \frac{0.0216 \times 10}{0.6 \times 10} = \frac{0.216}{6}$$

$$\begin{array}{r} 6 \overline{) 0.216} \quad (0.036 \\ \underline{0} \\ 21 \\ \underline{-18} \\ 36 \\ \underline{-36} \\ 0 \end{array}$$

$$\text{Hence, } 0.0216 \div 0.6 = 0.036$$

SOLVED PROBLEMS

Problem 1: Simplify: $4\frac{5}{6} - 2\frac{3}{8} + 3\frac{7}{12}$

Solution: We have,

$$4\frac{5}{6} - 2\frac{3}{8} + 3\frac{7}{12}$$

$$\begin{aligned}
 &= \frac{6 \times 4 + 5}{6} - \frac{2 \times 8 + 3}{8} + \frac{3 \times 12 + 7}{12} \\
 &= \frac{29}{6} - \frac{19}{8} + \frac{43}{12} \\
 &= \frac{29 \times 4}{6 \times 4} - \frac{19 \times 3}{8 \times 3} + \frac{43 \times 2}{12 \times 2} \quad [\because \text{LCM of } 6, 8, 12 \text{ is } 2 \times 3 \times 2 \times 2 = 24] \\
 &= \frac{116}{24} - \frac{57}{24} + \frac{86}{24} = \frac{116 - 57 + 86}{24} = \frac{202 - 57}{24} = \frac{145}{24}
 \end{aligned}$$

| | | | |
|---|---|---|----|
| 2 | 6 | 8 | 12 |
| 3 | 3 | 4 | 6 |
| 2 | 1 | 4 | 2 |
| 2 | 1 | 2 | 1 |
| | 1 | 1 | 1 |

Problem 2: Divide: $\frac{5}{9}$ by $\frac{2}{3}$

Solution: We have,

$$\frac{5}{9} \div \frac{2}{3} = \frac{5}{9} \times \frac{3}{2} = \frac{5 \times 3}{9 \times 2} = \frac{5 \times 1}{3 \times 2} = \frac{5}{6}$$

THINGS TO REMEMBER

1. A fraction is a number representing a part of a whole.
2. A fraction can be expressed in the form $\frac{a}{b}$, where a, b are whole numbers and $b \neq 0$.
3. A fraction whose numerator is less than the denominator is called a proper fraction.
4. Product of two fraction = $\frac{\text{Product of their numerator}}{\text{Product of their denominators}}$
5. Two fractions are said to be reciprocal of each other, if their product is 1. The reciprocal of a non-zero fraction $\frac{a}{b}$ is equal to $\frac{b}{a}$.
6. The division of a fraction $\frac{a}{b}$ by a non-zero fraction $\frac{c}{d}$ is the product of $\frac{a}{b}$ with the reciprocal of $\frac{c}{d}$.

TIPS

1. **Comparison of fractions** can be done in the following ways:
 - (a) If $ad > bc$, then $\frac{a}{b} > \frac{c}{d}$
 - (b) If $ad < bc$, then $\frac{a}{b} < \frac{c}{d}$
 - (c) If $ad = bc$, then $\frac{a}{b} = \frac{c}{d}$
2. To **add** or **subtract** find equivalent fractions that have the same denominator.
3. To **multiply**: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$, then simplify.

4. Reciprocal of a fraction $\frac{a}{b}$ ($a \neq 0, b \neq 0$) = $\frac{b}{a}$.
5. To divide $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$.
6. If $\frac{a}{b}$ and $\frac{c}{d}$ ($a \in N, b \in N, c \in N, d \in N$) are two fractions, then $\frac{a+b}{c+d}$ is a fraction between $\frac{a}{b}$ and $\frac{c}{d}$.
7. A fraction $\frac{p}{q}$ is non-terminating repeating or recurring decimal if a digit a block of digits are repeated in the decimal part.
- $$\frac{2}{3} = 0.66\dots = 0.\overline{6}$$
- $$\frac{4}{7} = 0.571428571428 = 0.571428$$

PART – I: MISCELLANEOUS DOMAIN

- Solve: (a) $2 - \frac{3}{5}$ (b) $7\frac{1}{2} + 3\frac{1}{4} - 2\frac{1}{4}$ (c) $\frac{8}{3} - 1$ (d) $\frac{2}{9} + \frac{5}{27}$
- Sahil solved $\frac{2}{5}$ part of an exercise while Rahim solved $\frac{3}{7}$ of it. Who solved lesser part? By how much?
- A cyclist covers $4\frac{1}{2}$ km in one hour. How far does he go in $3\frac{1}{2}$ hours?
- Find: (a) $18 \div \frac{3}{5}$ (b) $12 \div 2\frac{3}{4}$ (c) $2\frac{2}{5} \div 1\frac{1}{10}$ (d) $\frac{7}{5} \div 14$ (e) $12\frac{3}{4} \div 17$ (f) $\frac{144}{5} \div \frac{12}{5}$
- Which is greater: 3.05 or 3.50?
- Express as rupees using decimals: (a) 9 paise (b) 8 rupees (c) 725 paise.
- Multiply: (a) $28 \times 2\frac{3}{7}$; (b) $3\frac{1}{4} \times 16$; (c) $\frac{2}{9} \div \frac{81}{3}$ (d) $7\frac{2}{3} \times 3\frac{3}{4}$; (e) $3\frac{1}{4} \times \frac{16}{5}$
- A rectangular table top is $2\frac{1}{2}$ m its long and $1\frac{3}{4}$ m wide. Find its perimeter?
- Calculate $\frac{2}{3} \left[1 + \frac{1}{2} \left(\frac{3}{4} - \frac{7}{2} \right) \right]$
- Write the place value of 5 in the following decimal numbers: (a) 2.35 (b) 85.32 (c) 29.205 (d) 128.250 (e) 10.2935.
- Find (a) 100.3×10.3 (b) 8.9×1000 (c) 0.5×0.008 (d) 0.2×10 (e) 233.33×3 (f) 18.25×0.25

12. The side of an equilateral triangle is 5.5 cm. Find its perimeter?
13. If a car covers a distance of 19.5 km in one litre of petrol. How much distance will it cover in 19.5 litres of petrol?
14. Find: (a) $125.086 \div 26$ (b) $8.397 \div 1000$ (c) $30.24 \div 0.36$ (d) $66.65 \div 0.215$
(e) $9.5 \div 1.9$ (f) $27.39 \div 0.1$ (g) $12.397 \div 10$.
15. Find the average of 8.4, 7.5, 12.9 and 10.6?
16. A train covers 311.50 km in 3.5 hours. What is the distance covered by it in 1 hour?
17. Production of wheat is $2\frac{1}{4}$ times that of rice, but the cost of rice is $1\frac{1}{4}$ times that of wheat. If a farmer produces wheat in place of rice, then what is his income in terms of the previous income?
18. Simplify $\frac{0.3 \times 0.3 - 0.2 \times 0.2}{(0.3 - 0.2)}$
19. Eight people are planning to share equally the cost of a rental car. If one person withdraws from the arrangement and the others share equally the entire cost of the car, then by how much is the share of each of the remaining persons increased in terms of the original share?
20. Mohan ate half a pizza on Monday. He ate half of what was left on Tuesday and so on. He followed this pattern for one week. How much of the pizza would he have eaten during the week?
21. A lamp post has half of its length in mud, $\frac{1}{3}$ of its length in water and $3\frac{1}{3}$ m above the water, but in the mud. Find the total length of the post?
22. Chandran gave one-fourth of his money to Suresh. Suresh in turn gave one-third of what he received to Jayesh. If the difference between the amount of Suresh and Jayesh is ₹100, how much did Chandran have?
23. Convert (i) $0.\overline{89}$ (ii) $4.\overline{173}$ (iii) $6.\overline{2797}$ to vulgar fraction.
24. Which of the following numbers 0.1, 0.11, $(0.11)^2$, $\sqrt{0.0001}$ is the greatest?
25. Simplify: $0.\overline{4} + 0.\overline{61} + 0.\overline{11} - 0.\overline{36}$.
26. In the expression: $2.5 + 0.05 - [1.6 - \{3.2 - (3.2 + 2.1 \div x)\}] = 0.65$, find the value of x ?
27. Simplify: $\frac{0.2 \times 0.2 + 0.2 \times 0.02}{0.044}$
28. Evaluate: $\frac{0.0203 \times 2.92}{0.0073 \times 14.5 \times 0.7}$
29. What is $0.\overline{09} \times 7.\overline{3}$ equal to?

VALUE BASED QUESTIONS

1. Find the value of $\frac{1}{1 + \frac{1}{3 - \frac{4}{2 + \frac{1}{3 - \frac{1}{2}}}}} + \frac{3}{3 - \frac{4}{3 + \frac{1}{1 - \frac{1}{3}}}}$
2. If $N = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \dots + \frac{1}{156}$, what is the value of N ?
3. What is the value of $\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \frac{9}{4^2 \cdot 5^2} + \frac{11}{5^2 \cdot 6^2} + \frac{13}{6^2 \cdot 7^2} + \frac{15}{7^2 \cdot 8^2} + \frac{17}{8^2 \cdot 9^2} + \frac{19}{9^2 \cdot 10^2}$?
4. A woman sells to the first customer half her stock of apples and half an apple, to the second customer sells half her remaining stock and half an apple, and so on to the third and to the fourth customer. Such that she has now 15 apples left. How many apples did she have before she started selling?
5. Find x if $5\frac{1}{6} - \left[1\frac{1}{5} + \left\{ 2\frac{3}{4} \div 5\frac{1}{2} \div x - \left(\frac{5}{6} - \frac{2}{3} \right) \right\} \right] = 2\frac{61}{120}$.

HIGHER ORDER THINKING SKILLS (HOTS)

1. Simplify: $\left[3\frac{1}{4} \div \left\{ 1\frac{1}{4} - \frac{1}{2} \left(2\frac{1}{2} - \frac{1}{4} - \frac{1}{6} \right) \right\} \right] \div \left(\frac{1}{2} \text{ of } 4\frac{1}{3} \right)$.
2. Find the value of $\frac{5\frac{5}{6}}{\frac{3}{7}}$ of $\frac{6\frac{7}{9}}{\frac{1}{8}} \div 8 \left(2\frac{3}{11} + \frac{13}{22} \right)$ of $\frac{3}{5}$.
3. Calculate: $\left(1 - \frac{1}{2^2} \right) \left(1 - \frac{1}{3^2} \right) \left(1 - \frac{1}{4^2} \right) \left(1 - \frac{1}{5^2} \right) \dots \left(1 - \frac{1}{9^2} \right) \left(1 - \frac{1}{10^2} \right)$.
4. A student was asked to solve the fraction $\frac{\frac{7}{3} + 1\frac{1}{2} \text{ of } \frac{5}{3}}{2 + 1\frac{2}{3}}$ and his answer was $\frac{1}{4}$. By how much was his answer wrong?
5. In the expression $2.5 + 0.05 - [1 - 6 - \{3.2 - (3.2 + 2.1 \div x)\}] = 0.65$, find the value of x ?

12. Which of the following sets of fractions is in the correct sequence of ascending order of their values?
 (a) $-\frac{1}{2}, \frac{5}{6}, -\frac{4}{3}$ (b) $-\frac{3}{7}, -\frac{5}{6}, \frac{3}{5}$ (c) $-\frac{1}{2}, -\frac{4}{9}, \frac{5}{6}$ (d) $-\frac{4}{9}, \frac{5}{6}, \frac{1}{6}$
13. The GCD of $\frac{3}{16}, \frac{5}{12}, \frac{7}{18}$ is:
 (a) $\frac{105}{48}$ (b) $\frac{1}{4}$ (c) $\frac{1}{48}$ (d) None of these
14. Suppose $a = \frac{2}{3}b$, $b = \frac{2}{3}c$, and $c = \frac{2}{3}d$, what was be the value of b as a fraction of d ?
 (a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{4}{9}$ (d) $\frac{8}{27}$
15. $[(5.\overline{88} - 4.\overline{58}) - (0.\overline{64} + 0.\overline{36})]$ is equal to:
 (a) $1.\overline{01}$ (b) $1.\overline{30}$ (c) $1.\overline{39}$ (d) $0.\overline{29}$
16. Simplify: $[0.9 - \{2.3 - 3.2 - (7.1 - 5.4 - 3.5)\}]$
 (a) 0.18 (b) 1.8 (c) 0 (d) 2.6
17. Simplify: $\frac{3.6 \times 0.48 \times 2.50}{0.12 \times 0.09 \times 0.5}$
 (a) 80 (b) 800 (c) 8000 (d) 80,000
18. $\frac{24.23 \times 1,423 \times 34,21}{521.3 \times 413.32 \times 2.53}$ is same as
 (a) $\frac{2423 \times 1423 \times 3421}{5213 \times 41332 \times 253}$ (b) $\frac{242.3 \times 142.3 \times 342.1}{5213 \times 4133.2 \times 2.53}$
 (c) $\frac{2.423 \times 14.23 \times 342.1}{521.3 \times 4133.2 \times 2.53}$ (d) $\frac{24.23 \times 14.23 \times 3.421}{5.213 \times 41332 \times 0.253}$
19. $1 + \frac{1}{4 \times 3} + \frac{1}{4 \times 3^2} + \frac{1}{4 \times 3^3}$ is equal to:
 (a) 1.120 (b) 1.250 (c) 1.140 (d) 1.160
20. $2.8\overline{768}$ expressed as a rational number is:
 (a) $2\frac{878}{999}$ (b) $2\frac{9}{10}$ (c) $2\frac{292}{333}$ (d) $2\frac{4394}{4995}$
21. 5.63 divided by 0.01 is equal to:
 (a) 563 (b) 56.3 (c) 0.563 (d) 5630
22. $58.326 \times 463.9 \times 0.0081$ is the same as:
 (a) $5.8326 \times 4.639 \times 8.1$ (b) $5.8326 \times 4.639 \times 0.81$
 (c) $58326 \times 4639 \times 0.00000081$ (d) None of these
23. The value of $(0.44\overline{67} + 0.14\overline{44})$ is:
 (a) 0.59 (b) $0.59\overline{12}$ (c) $0.5\overline{9}$ (d) None of these

24. Consider the following statements:
1. $\frac{1}{22}$ cannot be written as a terminating decimal.
 2. $\frac{2}{15}$ can be written as a terminating decimal.
 3. $\frac{1}{16}$ can be written as a terminating decimal.
- Which of the statements given above is/are correct?
- (a) 1 only (b) 2 only (c) 1 and 3 (d) 2 and 3
25. If $2.5252525\dots = \frac{p}{q}$ (in the lowest form), then what is the value of $\frac{q}{p}$?
- (a) 0.4 (b) 0.42525 (c) 0.0396 (d) 0.396
26. If $47.2506 = 4A + \frac{7}{B} + 2C + \frac{5}{D} + 6E$, then the value of $5A + 3B + 6C + D + 3E$ is:
- (a) 53.6003 (b) 53.603 (c) 153.6003 (d) 213.0003
27. Which pair of operations will make the equation below true when inserted into the blank spaces in the shown?
- $$2\frac{3}{10} \text{ _____ } 1.5 \text{ _____ } 2 = 1.8$$
- (a) – and + (b) \times and + (c) + and – (d) \times and –
28. $\frac{4}{15}$ of $\frac{5}{7}$ of a number is greater than $\frac{4}{9}$ of $\frac{2}{5}$ of the same number by 8. What is half of the number?
- (a) 630 (b) 315 (c) 210 (d) 105
29. If two-third of three-fourth of a number added to three-fourth of the fourth-fifth of the number is x times the number, the value of x is
- (a) $\frac{11}{10}$ (b) $1\frac{1}{11}$ (c) $\frac{10}{11}$ (d) $\frac{9}{11}$
30. What would be the reciprocal of the sum reciprocals of the numbers $\frac{3}{5}$ and $\frac{7}{3}$?
- (a) $\frac{1}{42}$ (b) $\frac{21}{44}$ (c) $\frac{4}{5}$ (d) $\frac{36}{55}$