

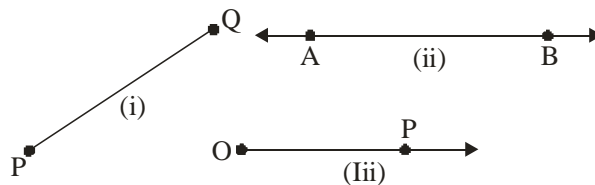
# Lines and Angles

## Lines

### Basic Terms:

- A **line** is a straight path extending indefinitely in both directions. It has no definite length.
- A **line segment** is a portion of a line having a definite length. It has two end points.
- A **ray** is a part of a line that has one fixed end point and extends indefinitely in the other direction.

*Example:*



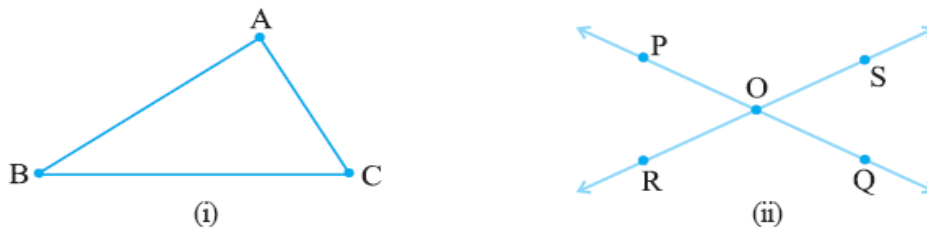
(i) shows a **line segment**, (ii) shows a **line** and (iii) is that of a **ray**.

**Note:** Three or more points which lie on the same line are called collinear points.

## Angles

An **angle** is formed when lines or line segments meet.

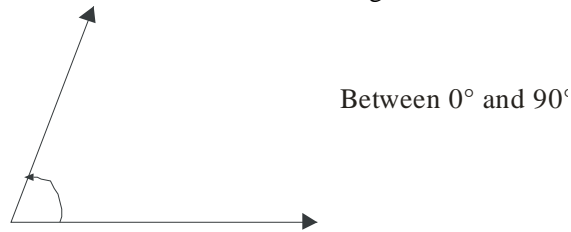
*Example:*



(i) line segments AB and BC intersect at B to form angle ABC, and again line segments BC and AC intersect at C to form angle ACB and so on. Whereas, in (ii) lines PQ and RS intersect at O to form four angles POS, SOQ, QOR and ROP. An angle ABC is represented by the symbol  $\angle ABC$ . Thus, in given fig (i), the three angles formed are  $\angle ABC$ ,  $\angle BCA$  and  $\angle BAC$ , and in (ii), the four angles formed are  $\angle POS$ ,  $\angle SOQ$ ,  $\angle QOR$  and  $\angle POR$ .

**Types of Angles:**

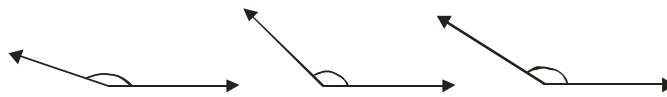
i) **Acute angle:** An angle whose measure is less than 90 degrees. The following are acute angle.



ii) **Right angle:** An angle whose measure is 90 degrees. The following is a right angle.



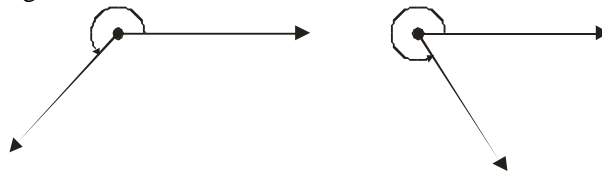
iii) **Obtuse angle:** An angle whose measure is more than 90 degrees but less than 180 degrees. Thus, it is between 90 degrees and 180 degrees. The following are obtuse angle.



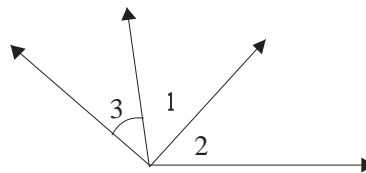
iv) **Straight angle:** An angle whose measure is 180 degrees. Thus, a straight angle look like a straight line. The following is a straight angle.



v) **Reflex angle:** An angle whose measure is more than 180 degrees but less than 360 degrees. The following is a reflex angle.

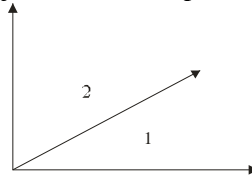


vi) **Adjacent angles:** Angle with a common vertex and one common side.  $\angle 1$  and  $\angle 2$ , are adjacent angles. However,  $\angle 2$  and  $\angle 3$  is not a pair of adjacent angles:

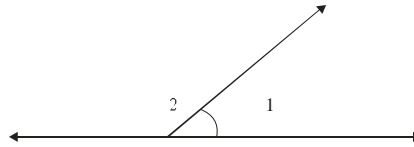


vii) **Complementary angles:** Two angles whose measures add to 90 degrees.  $\angle 1$  and  $\angle 2$  are complementary angles because together they form a right angle:

**Note:**  $\angle 1$  and  $\angle 2$  do not have to be adjacent to be complementary as long as they add up to 90 degrees



**viii) Supplementary angles:** Two angles whose measures add to 180 degrees. The following are supplementary angles. Angles, forming a pair of supplementary angles, need not to be adjacent angles.

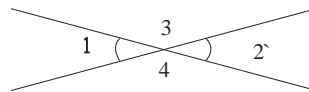


**ix) Vertically angles:** Angles that have a common vertex and whose sides are formed by the same lines. The following angles are pair of vertically opposite angles:

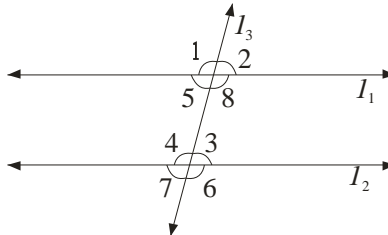
$\angle 1$  and  $\angle 2$

$\angle 3$  and  $\angle 4$

Hence  $\angle 1 = \angle 2$  and  $\angle 3 = \angle 4$



When two lines  $l_1$  and  $l_2$  are crossed by a third line  $l_3$  (Transversal), 8 angles are formed. Take a look at the following figure



Angles 3,4,5,8 are interior angles

Angles 1,2,6,7 are exterior angles

**x) Alternate interior angles:** Pairs of interior angles on opposite sides of the transversal. For instance, angle 3 and angle 5 form a pair of alternate interior angles. Angle 4 and angle 8 form a pair of alternate interior angles.

**xi) Alternate exterior angles:** Pairs of exterior angles on opposite sides of the transversal. Angle 2 and angle 7 form a pair of alternate exterior angles. Similarly  $\angle 1$  and  $\angle 6$  also

**xii) Corresponding angles:** Pairs of angles that are in similar positions.

Angle 3 and angle 2 form a pair of corresponding angles.

Angle 5 and angle 7 form a pair of corresponding angles

**xiii) Converse of Property:** If a transversal intersect two lines, then they are parallel if any one of the following is true.

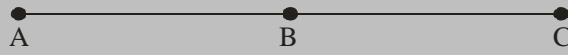
- a) Pairs of corresponding angles are equal.
- b) Pairs of alternate angles are equal.
- c) Co-interior angles are supplementary.

### THINGS TO REMEMBER

1. A triangle whose two sides are equal, is called an isosceles triangle.
2. A triangle whose all sides are equal, is called an equilateral triangle.
3. A triangle whose no two sides are equal, is called a scalene triangle.
4. A triangle whose all the angles are acute is called an acute triangle.
5. A triangle whose one of the angles is a right angle is called a right triangle.
6. A triangle whose one of the angles is an obtuse angle is called an obtuse triangle.
7. The interior of a triangle is made up of all such points P of the plane, as are enclosed by the triangle.
8. In a right triangle, if  $a, b$  are the lengths of the sides and  $c$  that of the hypotenuse, then  $c^2 = a^2 + b^2$ .
9. If the sides of a triangle are of lengths  $a, b$  and  $c$  such that  $c^2 = a^2 + b^2$ , then the triangle is right-angled and the side of length  $c$  is the hypotenuse.
10. Three positive numbers  $a, b, c$  in this order are said to form a Pythagorean triplet if  $c^2 = a^2 + b^2$ . Triplets (3, 4, 5), (5, 12, 13), (8, 15, 17), (7, 24, 25) and (12, 35, 37) are some Pythagorean triplet.
11. A line which intersects two or more given lines at distinct points is called a transversal to the given lines.
12. Lines in a plane are parallel if they do not intersect when produced indefinitely in either direction.
13. The distance between two intersecting lines is zero.
14. The distance between two parallel lines is the same everywhere and is equal to the perpendicular distance between them.
15. If two parallel lines are intersected by a transversal then:
  - i) Pairs of alternate (interior or exterior) angles are equal.
  - ii) pairs of corresponding angles are equal.
  - iii) Interior angles on the same side of the transversal are supplementary.
16. If two non-parallel lines are intersected by transversal then none of (i), (ii) and (iii) of 15 hold true.
17. If two lines are intersected by a transversal, then they are parallel if any one of the following is true:
  - i) the pair of corresponding angles are equal.
  - ii) The pair of alternate interior angles are equal.
  - iii) The pair of interior angles on the same side of the transversal are supplementary.

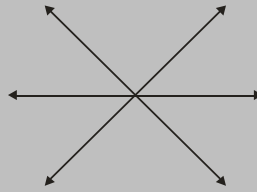
**TIPS**

1. Three or more points which lie on the same line are called **collinear points**.



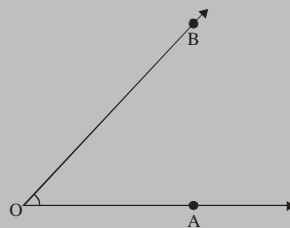
Collinear points A, B, C

2. Three or more lines in a plane passing through the same point are called **concurrent lines**.



Concurrent lines

3. The unit of angle measure is degree denoted by:  
 1 rotation =  $360^\circ$ ,  $1^\circ = 60'$  (60 minutes),  $1^\circ = 60''$  (60 seconds)

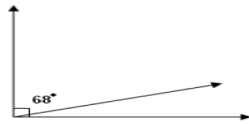


Angle AOB vertex O  
Arms OA and OB

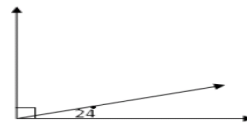
4. A transversal is a line that cuts across (intersects) two or more lines in distinct points.

**PART – I: MISCELLANEOUS DOMAIN**

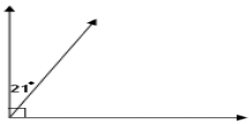
1. Find the missing angles in the following complementary angles:



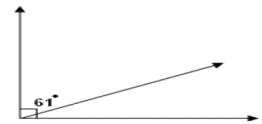
Answer: \_\_\_\_\_



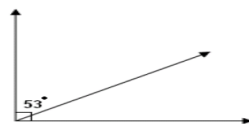
Answer: \_\_\_\_\_



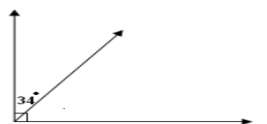
Answer: \_\_\_\_\_



Answer: \_\_\_\_\_



Answer: \_\_\_\_\_



Answer: \_\_\_\_\_

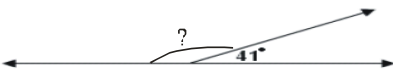
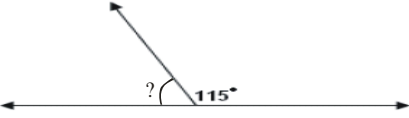
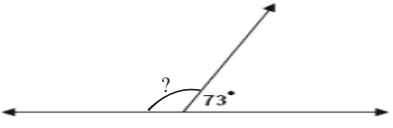
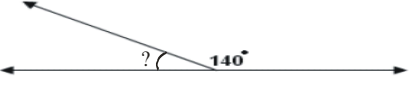
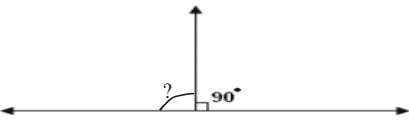
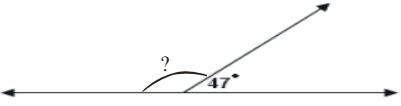
2. Match the complementary angles:

Complement of $54^\circ$	$46^\circ$
Complement of $28^\circ$	$57^\circ$
Complement of $61^\circ$	$29^\circ$
Complement of $15^\circ$	$64^\circ$
Complement of $26^\circ$	$75^\circ$
Complement of $44^\circ$	$36^\circ$
Complement of $71^\circ$	$62^\circ$
Complement of $33^\circ$	$19^\circ$

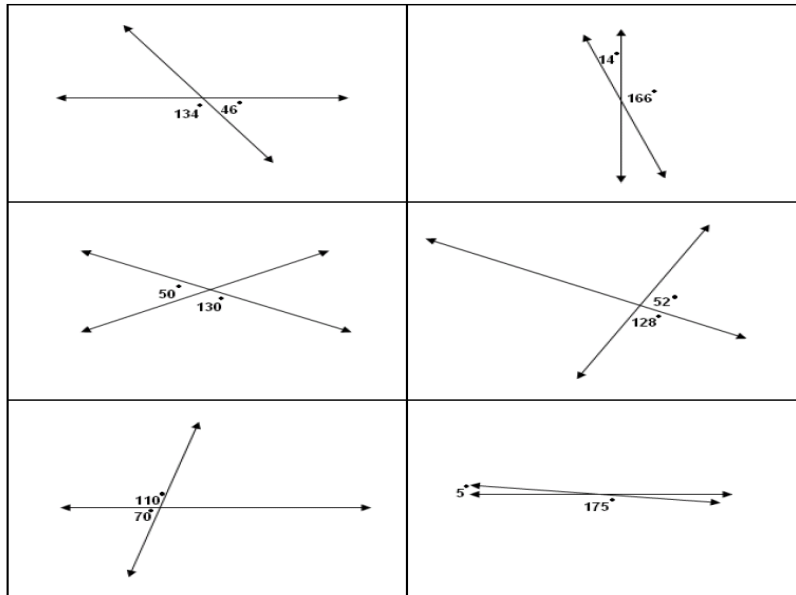
3. Match the complementary angles:

Supplement of $120^\circ$	$79^\circ$
Supplement of $96^\circ$	$25^\circ$
Supplement of $155^\circ$	$43^\circ$
Supplement of $101^\circ$	$84^\circ$
Supplement of $56^\circ$	$60^\circ$
Supplement of $137^\circ$	$124^\circ$

4. Use linear pair theorem to find the missing angles.

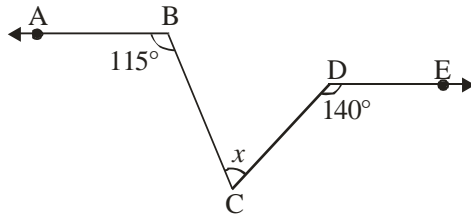
	
	
	

5. Write the missing angles:

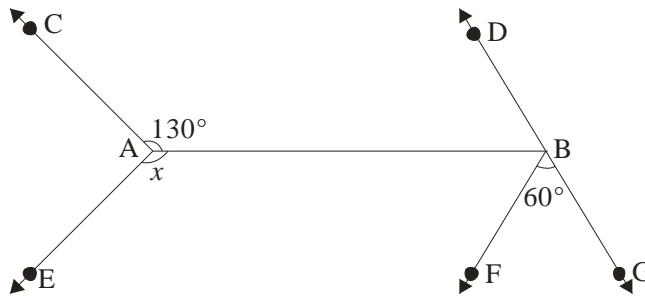


6. The difference in the measures of two complementary angles is  $12^\circ$ . Find the measures of the angles.
7. Among two supplementary angles the measure of the larger angle is  $44^\circ$  more than the measure of the smaller. Find their measures.
- Can two acute angles form a linear pair?
  - Can two obtuse angles form a linear pair?
  - Can two right angles form a linear pair?
8. Identify which of the following pairs of angles are complementary and which are supplementary.  
 (i)  $65^\circ, 115^\circ$  (ii)  $63^\circ, 27^\circ$  (iii)  $112^\circ, 68^\circ$  (iv)  $130^\circ, 50^\circ$  (v)  $45^\circ, 45^\circ$  (vi)  $80^\circ, 10^\circ$
9. An angle is greater than  $45^\circ$ . Is its complementary angle greater than  $45^\circ$  or equal to  $45^\circ$  or less than  $45^\circ$ ?
10. Fill in the blanks:
- If two angles are complementary, then the sum of their measures is \_\_\_\_\_.
  - If two angles are supplementary, then the sum of their measures is \_\_\_\_\_.
  - Two angles forming a linear pair are \_\_\_\_\_.
  - If two adjacent angles are supplementary, they form a \_\_\_\_\_.
  - If two lines intersect at a point, then the vertically opposite angles are always \_\_\_\_\_.
  - If two lines intersect at a point, and if one pair of vertically opposite angles are acute angles, then the other pair of vertically opposite angles are \_\_\_\_\_.

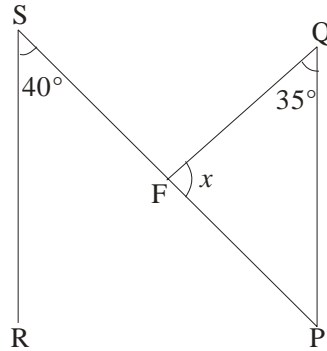
11. Given that  $AB \parallel DE$ ,  $\angle ABC = 115^\circ$ ,  $\angle EDC = 140^\circ$ , then find  $x$ ?



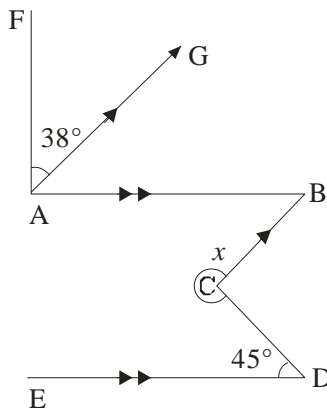
12. In the given figure  $AC \parallel BD$  and  $AE \parallel BF$ . Find the measure of  $\angle x$ ?



13. In the given figure,  $PQ \parallel RS$ ,  $\angle RSF = 40^\circ$ ,  $\angle PQF = 35^\circ$  and  $\angle QFP = x^\circ$ . What is the value of  $x$ ?

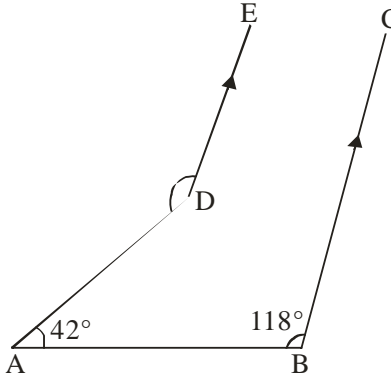


14. Given,  $AB \parallel ED$ ,  $AG \parallel CB$  and  $AF \perp AB$ .  $\angle FAG = 38^\circ$ ,  $\angle CDE = 45^\circ$ . Find the value of  $x$ ?

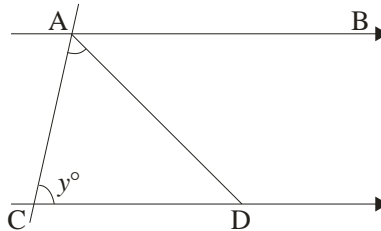




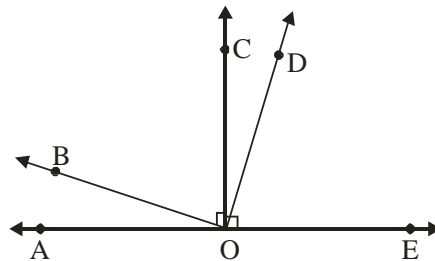
15. In the given figure,  $DE \parallel BC$ ,  $\angle ABC = 118^\circ$ ,  $\angle DAB = 42^\circ$ , then find  $\angle ADE$ ?



16. In the given figure, ray AD is the bisector of  $\angle CAB$  and the measure of  $\angle ACD$  is  $y$ . What must be the measure of  $\angle ADC$  in order for the line AB to be parallel to line CD.



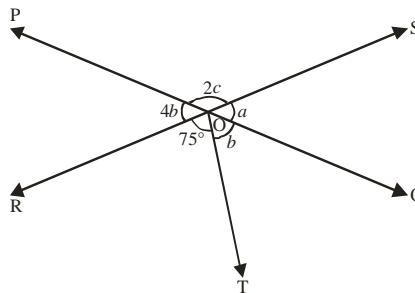
17. In the given figure,  $\angle COE$  and  $\angle BOD$  are right angles. If the measure of  $\angle BOC$  is four times the measure of  $\angle COD$ , what is the measure of  $\angle AOB$ ?



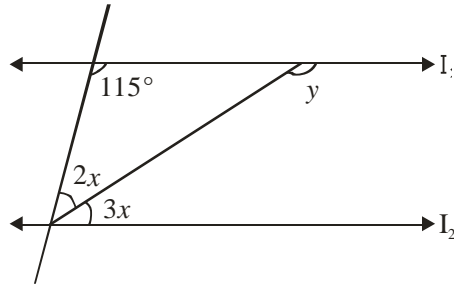
18. Lines  $m$  and  $n$  are cut by a transversal so that  $\angle 1$  and  $\angle 3$  are corresponding angles. If  $\angle 1 = 26x - 7^\circ$  and  $\angle 5 = 20x + 17^\circ$ . What value of  $x$  makes the lines  $m$  and  $n$  parallel?

- (a) 5                      (b) 4                      (c)  $4\frac{1}{2}$                       (d)  $3\frac{1}{4}$

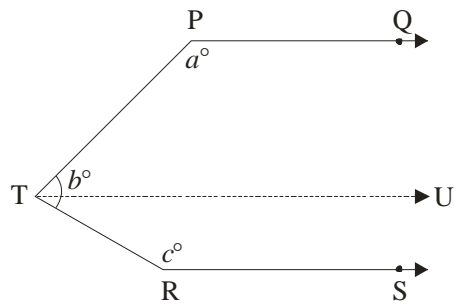
19. In the given figure PQ and RS intersect each other at O. If  $\angle SOT = 75^\circ$ , find the value of  $a$ ,  $b$  and  $c$ ?



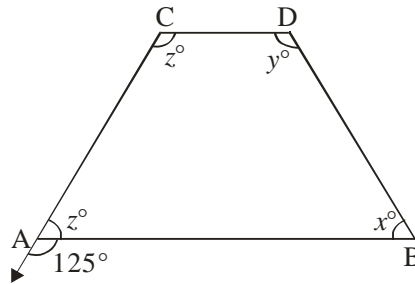
20. If line  $I_1$  is parallel to the  $I_2$  in the given figure, what is the value of  $y$ ?



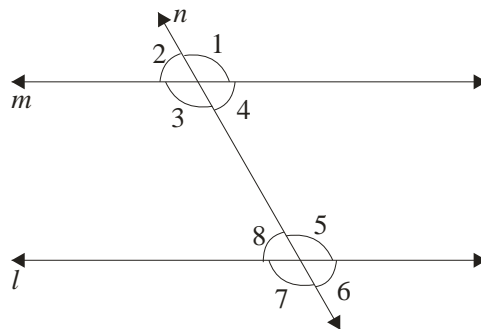
21. If ray PQ and RS are parallel as given in the figure, then find  $a + b + c$ ?



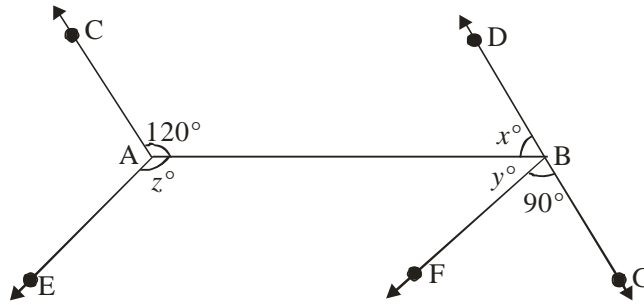
22. Figure  $AB \parallel CD$ . Find the values of  $x, y, z$ ?



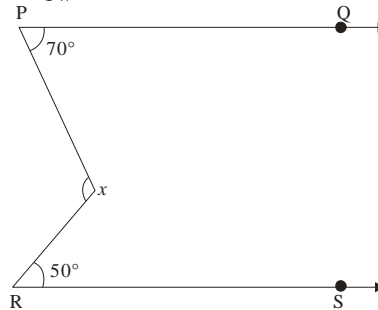
23. In figure transversal  $n$  cuts two lines  $l$  and  $m$  such that  $\angle 2 = 65^\circ$  and  $\angle 8 = 65^\circ$ . Are the lines parallel?



24. In figure  $AB \parallel CD$  and  $AE \parallel CF$ ,  $\angle FOG = 90^\circ$  and  $\angle BAC = 120^\circ$ . Find the values of  $x$ ,  $y$  and  $z$ .

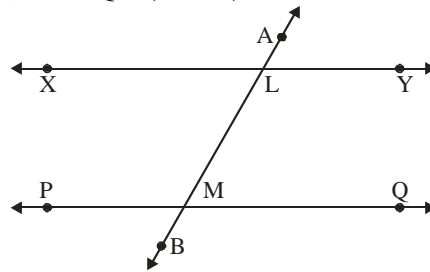


25. In figure, find out  $\angle PXR$ , if  $PQ \parallel RS$

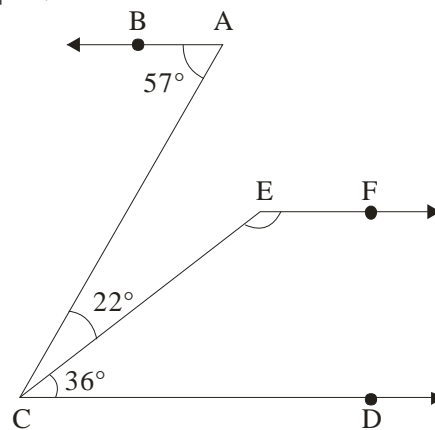


26. In figure 78, we have:

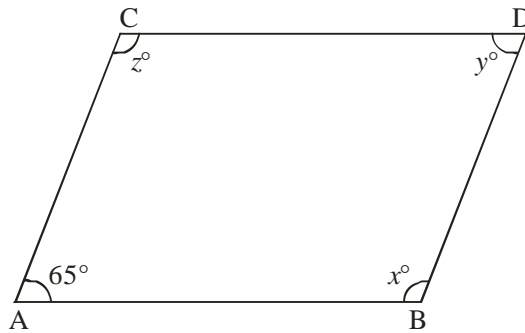
- i)  $\angle MLY = 2\angle LMQ$ , find  $\angle LMQ$ ?
- ii)  $\angle XLM = (2x - 10)^\circ$  and  $\angle LMQ = x + 30^\circ$ , find  $x$ ?
- iii)  $\angle XLM = \angle PML$ , find  $\angle ALY$ ?
- iv)  $\angle ALY = (2x - 15)^\circ$ ,  $\angle LMQ = (x + 40)^\circ$ , find  $x$ ?



27. In figure show that  $AB \parallel EF$ .



28. Figure  $AB \parallel CD$  and  $AC \parallel BD$ . Find the values of  $x, y, z$ ?

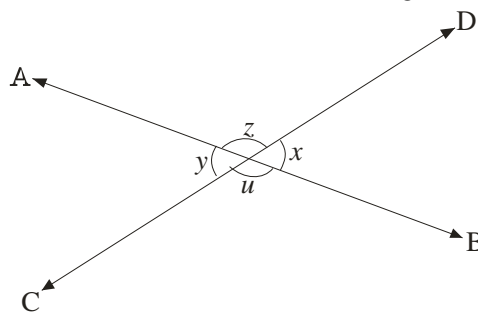


### HIGHER ORDER THINKING SKILLS (HOTS)

1. Draw a triangle ABC, with  $AC = 4$  cm,  $BC = 3$  cm and  $\angle C = 80^\circ$ . Measure AB. Is  $(AB)^2 = (AC)^2 + (BC)^2$ ? If not, which one of the following is true:  $(AB)^2 > (AC)^2 + (BC)^2$  or  $(AB)^2 < (AC)^2 + (BC)^2$ ?
2. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.
3. A ladder 50 dm long when set against the wall of a house just reaches a window at a height of 48 dm. How far is the lower end of the ladder from the base of the wall?
4. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its top reach.

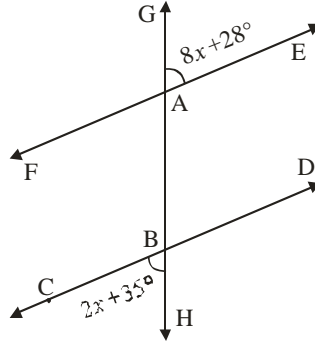
### PART – II: MULTIPLE CHOICE QUESTIONS

1. If  $x + z + y = 240^\circ$ , find the value of each of the four angles in the order  $x, y, z, u$ .



- |  |  |
|--|--|
| (a) $30^\circ, 50^\circ, 130^\circ, 130^\circ$ | (b) $60^\circ, 60^\circ, 120^\circ, 120^\circ$ |
| (c) $60^\circ, 70^\circ, 110^\circ, 120^\circ$ | (d) $50^\circ, 60^\circ, 130^\circ, 130^\circ$ |

2.  $\angle ABC$  is equal to



- (a)  $75^\circ$                       (b)  $105^\circ$                       (c)  $115^\circ$                       (d)  $85^\circ$

3. Two lines in a plane are cut by a transversal. Which condition does NOT imply that the two lines are parallel?

- (a) A pair of alternate interior angles are congruent.  
 (b) A pair of co-interior angles are supplementary.  
 (c) A pair of corresponding angles are congruent.  
 (d) A pair of alternate exterior angles are complementary.

4. Find the complement of an angle whose measure is  $3x - 8^\circ$ ?

- (a)  $3x - 98^\circ$                       (b)  $82^\circ - 3x$                       (c)  $98^\circ + 3x$                       (d)  $3x - 82^\circ$

5. How many degrees are there in an angle which equals one-fifth of the supplement?

- (a)  $15^\circ$                       (b)  $30^\circ$                       (c)  $75^\circ$                       (d)  $150^\circ$

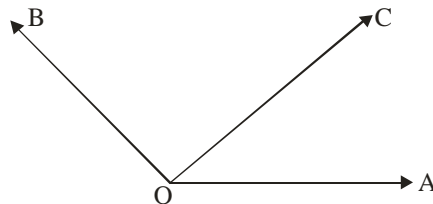
6. Two complementary angles are such that two times the measure of one is equal to three times the measure of the other. The measure of the larger angle is?

- (a)  $72^\circ$                       (b)  $108^\circ$                       (c)  $36^\circ$                       (D)  $54^\circ$

7. If the arms of one angle are respectively parallel to the arms of another angle, then the two angles are:

- (a) neither equal nor supplementary                      (b) not equal but supplementary  
 (c) equal but not supplementary                      (d) either equal or supplementary

8. In the given figure, if  $\angle BOC = 7x + 20^\circ$  and  $\angle COA = 3x$ , then the value of  $x$  for which AOB becomes a straight line is:

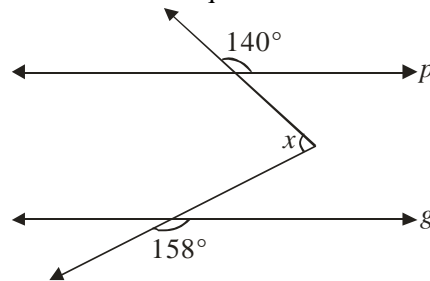


- (a)  $16^\circ$                       (b)  $14^\circ$                       (c)  $20^\circ$                       (d)  $21^\circ$

9. A and B are two points and C is any point collinear with A and B. If  $AB = 10$ ,  $BC = 5$ , then AC is equal to

- (a) either 15 or 5      (b) necessarily 5      (c) necessarily 16      (d) none of these

10. Answer p/g in the figure shown. Then 'x' equals.



- (a)  $18^\circ$       (b)  $22^\circ$   
 (c)  $62^\circ$       (d) cannot be determined

11. If X, Y and Z are three points such that  $XY = 2YZ$  and  $XZ = 3YZ$ , then the three points are:

- (a) not collinear      (b) collinear and X lies between Y and Z  
 (c) collinear and Y lies between X and Z      (d) collinear and Z lies between X and Y

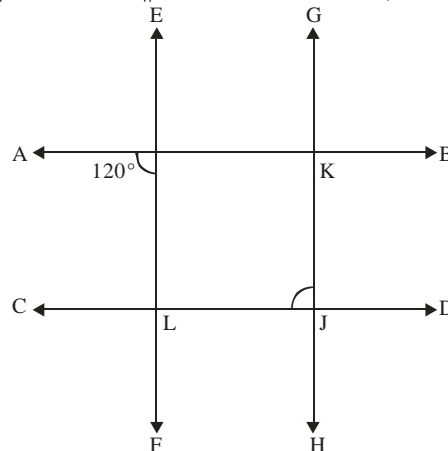
12. LM is a straight line and O is a point on LM. Line ON is drawn as coinciding with OL or OM> If  $\angle MON$  is three times  $\angle LON$ , then  $\angle MON$  is equal to:

- (a)  $45^\circ$       (b)  $60^\circ$       (c)  $105^\circ$       (d)  $135^\circ$

13. Three lines intersect at a point generating six angles. If one of these angles is  $90^\circ$ , then the number of other distinct angles is:

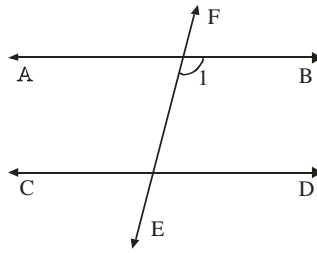
- 1 or 2      (b) 1 or 3      (c) 2 or 3      (d) 2 or 4

14. In the given figure  $AB \parallel CD$  and  $EF \parallel GH$ . If  $\angle AIF = 120^\circ$ , then  $\angle CJG$  is :

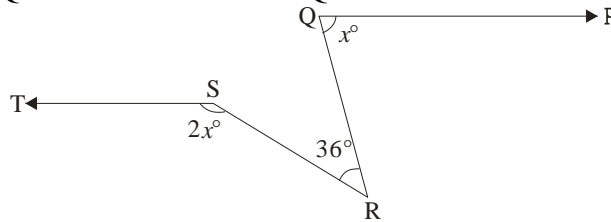


- (a)  $80^\circ$       (b)  $100^\circ$       (c)  $\frac{1}{3}$  of a right angle      (d)  $\frac{2}{3}$  of a right angle

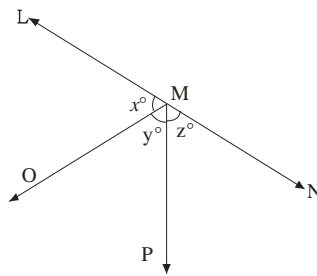
15. For two parallel lines and a transversal,  $\angle 1 = 74^\circ$ . For which pair of angle measures is the sum is least?



- (a)  $\angle 1$  and its corresponding angle  
 (b)  $\angle 1$  and the co interior angle  
 (c)  $\angle 1$  and its supplement  
 (d)  $\angle 1$  and its complement
16. AB is a straight line and O is a point lying on AB. A line OC is drawn from O such that  $\angle COA = 36^\circ$ , OD is a line within  $\angle COA$  such that  $\angle DOA = \frac{1}{3} \angle COA$ . If OE is a line within the  $\angle BOC$ ,  $\angle BOE = \frac{1}{4} \angle BOC$ , then  $\angle DOE$  must be:
- (a)  $60^\circ$                       (b)  $132^\circ$                       (c)  $144^\circ$                       (d)  $108^\circ$
17. The bisector of an angle is produced backwards. It bisects which angle at the same vertex?
- (a) acute                      (b) obtuse                      (c) reflex                      (d) complete
18.  $QP \parallel TS$  and  $\angle QRS = 36^\circ$ . Calculate  $\angle PQR$ ?

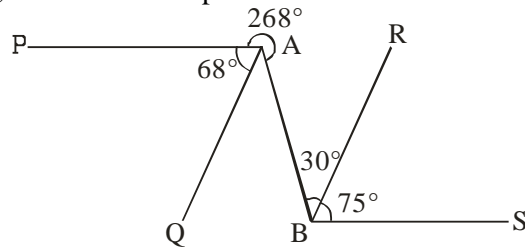


- (a)  $75^\circ$                       (b)  $70^\circ$                       (c)  $72^\circ$                       (d)  $71^\circ$
19. Find y if  $x + z + y$  :



- (a)  $80^\circ$                       (b)  $100^\circ$                       (c)  $90^\circ$                       (d)  $120^\circ$

20. State, with reasons, which lines are parallel.



- (a) AP, BS
  - (b) AQ, RB
  - (c) AP, RB
  - (d) Cannot be determined
21. Draw four lines OA, OB, OC, OD in order,  $\angle AOB = 48^\circ$ ,  $\angle COD = 34^\circ$ . OP bisects  $\angle AOB$ , OQ bisects  $\angle COD$ . If OQ is perpendicular to OP, calculate  $\angle BOC$ ?
- (a)  $49^\circ$
  - (b)  $50^\circ$
  - (c)  $51^\circ$
  - (d) None of these

