Chapter 6

6

The Triangle and its Properties

Triangle

A **triangle** is a **closed figure** made of **three line segments**. Every **triangle** has **three sides, three angles,** and **three vertices**. These are known as the **parts of a triangle**. The sides and the angles of every triangle may differ from one another; therefore, they do not look alike.

Classification of Triangles

Triangles can be classified based on their **sides and angles**.

- Based on their sides, there are **equilateral**, **isosceles** and **scalene triangles**.
- Based on their angles, there are **acute**, **obtuse** and **right-angled triangles**.

Based on Sides:

a) **Equilateral triangle**: A triangle in which **all the sides are equal** is called an equilateral triangle. All the three angles of an equilateral triangle are also equal, and each measures **60°.**

b) **Isosceles triangle**: A triangle in which **any two sides are equal** is called an isosceles triangle. In an isosceles triangle, the **angles opposite the equal sides** are called the **base angles,** and they are also equal.

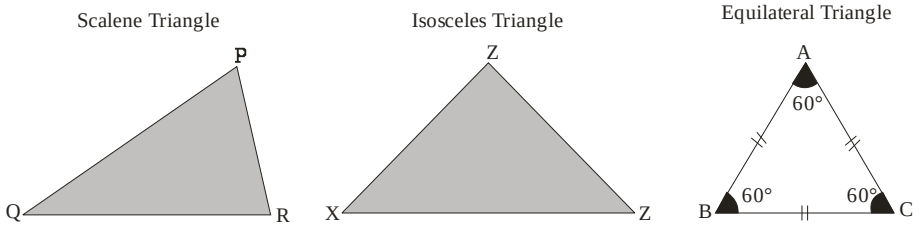
c) **Scalene triangle**: A triangle in which **no two sides are equal** is called a Scalene triangle.

Based on Angles:

a) **Acute-angled triangle**: A triangle with all its **angles less than 90°** is known as an acute-angled triangle.

b) **Obtuse-angled triangle**: A triangle with **one of its angles more than 90° and less than 180°** is known as an obtuse-angled triangle.

c) **Right-angled triangle**: A triangle with **one of its angles equal to 90°** is known as a right-angled triangle. The **side opposite the 90° angle** is called the **hypotenuse,** and is the **longest side of the triangle**.



Properties of Triangle

I. Angle sum property:

The sum of the three angles in a triangle is equal to 180°

Eg: If A, B and C are the angles of a triangle, then

$\angle A + \angle B + \angle C = 180°$

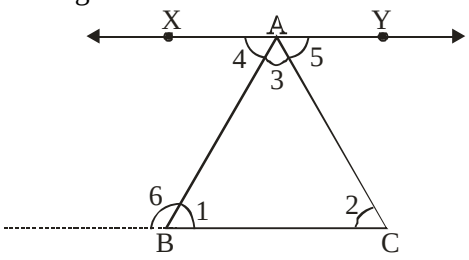
Suppose a line XY is parallel to side BC. AB is a transversal that cuts line XY and AB, at A and B, respectively. As the alternate interior angles are equal, $\angle 1 = \angle 4$ and also $\angle 2 = \angle 5$.

Also $\angle 2 = \angle 5$, $\angle 4, \angle 3$ and $\angle 5$ form a linear pair, and their sum is $180°$.

$\therefore \angle 4 + \angle 3 + \angle 5 = 180°$

$\angle 1 + \angle 3 + \angle 2 = 180°$

Hence sum of all interior angles at a Δ is 180°



II. **Exterior angle property:** An exterior angle of a triangle is equal to the sum of its opposite interior angles. Eg. In the figure given above, ∠ 6 is called the exterior angle to triangle ABC, and ∠ 6 = ∠ 3 + ∠2.

The sum of the lengths of any two sides of a triangle is greater than the third side.

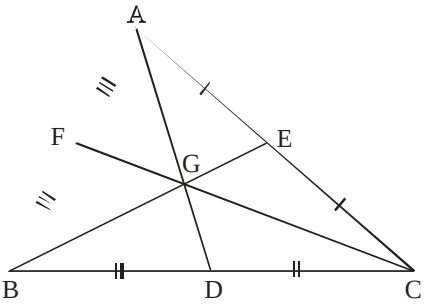
In triangle ABC, CA + AB > BC

BC + CA > AB

AB + BC > CA

Important terms of triangle:

i) **Median and Centroid:** A line joining the mid point of a side of triangle to the opposite vertex is called a median. D, E, F are the mid points of sides BC, AC and AB respectively. Hence, AD, BE and CF are the medians. A median divides a triangle into two parts of equal area.



The point where the three medians of a triangle meet is called the centroid of a triangle.

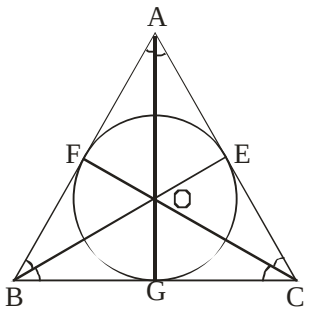
The centroid of the triangle divides each median in the ratio 2 : 1, i.e.,

$$\boxed{AG : GD = BG : GE = CG : GF = 2 : 1}$$

(ii) **Perpendicular bisector and circumcentre:** Perpendicular bisector to any side is the line that is perpendicular to that side of passes through its mid point. Perpendicular bisectors need not pass through the opposite vertex. The point of intersection of the three perpendicular bisectors of the triangle is called circumcentre. The circumcentre of a triangle is equidistance from its three

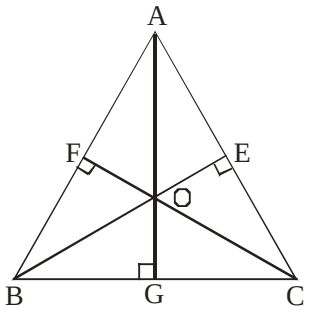
vertices. If we draw a circle with the circumcentre as the centre and the distance of any vertex from the circumcentre as radius, the circle passes through all the three vertices and the circle is called **circum circle.**

(iii) **Angle bisector and incentre:** An angle bisector is a line that divides the angle into two parts. The point of intersection of three angle bisectors of a triangle is called the **incentre.** It always lies inside the triangle. It is always equidistance from the sides of the triangle.



The circle drawn with incentre as centre and touching all the three sides of the triangle is called incircle.

(iv) **Altitude and orthocenter:** The perpendicular drawn a from the vertex of a triangle to the opposite side is called an **altitude.** The point of intersection of three altitudes of a triangle is called **orthocentre,** which can be inside or outside the triangle.



Types of triangles and their properties:

(i) **Scalene triangle:** All three sides are unequal

(ii) **Isosceles triangle:**

(a) Two sides are equal.

(b) The median drawn from a vertex to the opposite sides is also the perpendicular bisector of that side.

(c) In an isosceles ΔABC, all the four points, the centroid, the orthocentre, the circumcentre and the incentre lie on the altitude drawn from vertex A to base BC.

(iii) **Equilateral triangle:**

(a) All the three sides and angles are equal.

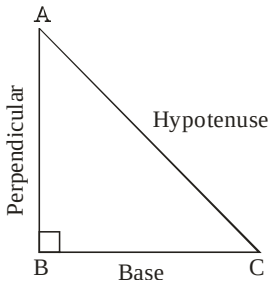
(b) The median, angle bisector, altitude and perpendicular bisector of sides are all represented by the same straight lines.

(iv) **Acute angled triangle:** A triangle with each angle less than 90°.

(v) **Right angled triangle:** A triangle with one angle equal to 90°.

(vi) **Obtuse angled triangle:** A triangle with one angle greater than 90°.

Pythagoras Theorem: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



$$\boxed{\begin{array}{l}(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2 \\ \text{or} \quad AC^2 = AB^2 + BC^2\end{array}}$$

In a right angle the hypotenuse is the longest side. Mid point of the hypotenuse of a right triangle is equidistance from the 3 vertices. If in a triangle, the square of the largest side is equal to the sum of the squares of the remaining two sides, then the angle opposite to the largest side is a right angle.

Of all the line segments that can be drawn to a given line from a point outside it, the perpendicular line segment is the shortest.

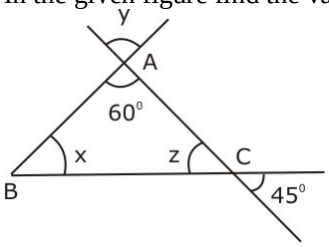
Note: If the angles of a right triangle are 30°, 60° and 90°, the hypotenuse is equal to twice the side opposite to the 30° angle, i.e. AC = 2BC.

Converse of Pythagorean Theorem

If the Pythagoras property holds, then the triangle must be right-angled. That is, if there is a triangle such that the sum of the squares on two of its sides is equal to the square of the third side, then it must be a right-angled triangle. The angle opposite to the third side is right angle.

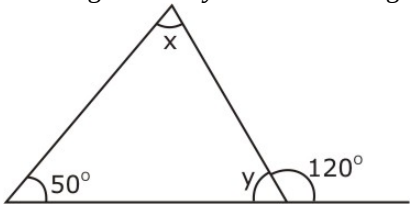
SOLVED PROBLEMS

Problem 1: In the given figure find the value of x and y.



Solution: In the given figure,
y = 60°(vertically opposite angles)
z = 45°(vertically opposite angles)
In triangle ABC
$\angle A + \angle B + \angle C = 180°$ (By angle sum property of triangle)
$\Rightarrow 60° + x + 45° = 180°$
$\Rightarrow x + 105° = 180° \Rightarrow x = 180° - 105° \Rightarrow x = 75°$

Problem 2: Find angles *x* and *y* in the following figure.



Solution: In the given figure,

$\angle y + 120^\circ = 180^\circ$ (since they are linear pair)

$\Rightarrow \angle y = 180^\circ - 120^\circ = 60^\circ$

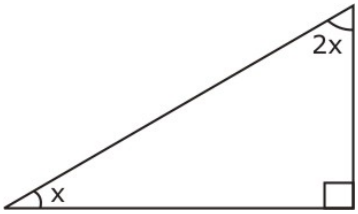
Also $\angle x + 50^\circ = 120^\circ$ (by exterior angle property of triangle)

$\Rightarrow \angle x = 120^\circ - 50^\circ$

$\Rightarrow \angle x = 70^\circ$

$\therefore \angle x = 70^\circ$ and $\angle y = 60^\circ$

Problem 3: Find angle *x* in the following figure:



Solution: Sum of the angles of a triangle = 180°

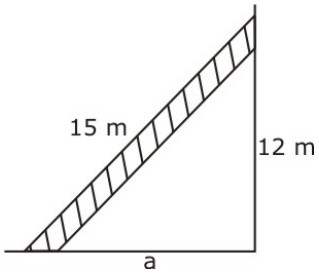
$2x + x + 90^\circ = 180^\circ$

$\Rightarrow 3x = 90^\circ$

$\Rightarrow x = 30^\circ$

Problem 4: A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance 'a' metre from the wall. Find the distance of the foot of the ladder from the wall.

Solution:



By using Pythagoras theorem,

We have

$\Rightarrow a^2 + 12^2 = 15^2 \Rightarrow a^2 = 225 - 144$

$\Rightarrow a^2 = 81 \Rightarrow a^2 = 9^2$

$\Rightarrow a = 9m$

Thus the ladder is 9 m away from the foot of the wall.

Example 5: If in a right angle isosceles triangle area is 32 cm^2 . Find the sides of the triangle.

Solution: Area of triangle = $\frac{1}{2}$ (base × altitude)

Base = Altitude = x

$\frac{1}{2}(x \times x) = 32 \Rightarrow x^2 = 64$

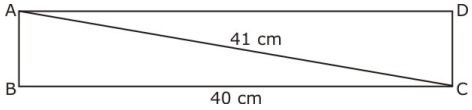
$x = 8\ cm$

Therefore two sides containing right angle are equal to 8 cm

Third side of triangle = $\sqrt{8^2 + 8^2} = 8\sqrt{2} cm$

Problem 6: Find the perimeter of a rectangle whose length is 40 cm and the length of one of the diagonal is 41 cm.

Solution:



Now in DABC, AB ⊥ BC

$\therefore AB^2 + BC^2 = AC^2$

$\Rightarrow AB^2 + 40^2 = 41^2 \Rightarrow AB^2 = 1681 - 1600$

$\Rightarrow AB^2 = 81 \Rightarrow AB^2 = 9^2$ or AB = 9

$\therefore$ Perimeter = 2 (AB + BC) = 2(9 + 40) = 2 × 49 = 98 cm

Problem 7: Can 5cm, 7cm and 5cm be the sides of a triangle?

Solution: Check whether the sum of any two sides is greater than the third side or not:

5+ 7 =12 cm > 5 cm.

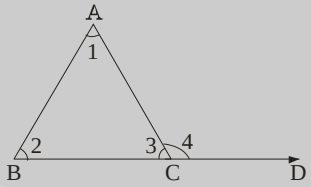
7+5 =12 cm > 5 cm

5 + 5 = 10 cm > 7 cm

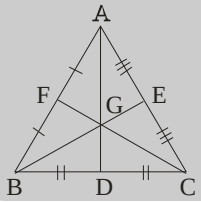
Therefore, these measurements can be the sides of a triangle.

THINGS TO REMEMBER

1. In triangle ABC
   $\angle A + \angle B + \angle C = 180°$
2. In triangle ABC
   CA + AB > BC
   BC + CA > AB
   AB + BC > CA
3. In triangle ABC
   $\angle 4 = \angle 1 + \angle 2$

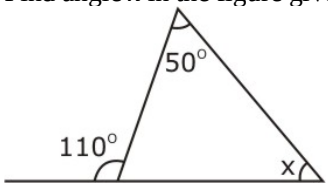


4. In a given triangle ABC, G is the centroid then
   $\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = \frac{2}{1}$

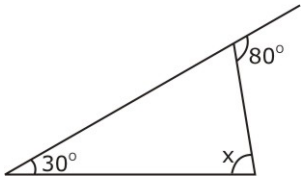


PART – I: MISCELLANEOUS DOMAIN

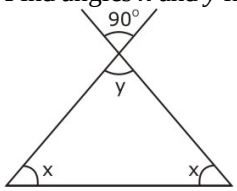
1. Find angle *x* in the figure given below:



2. Find the measure of *x* in the following figure:



3. Find angles *x* and *y* in the following figure:



4. Find the perimeter of a rectangle whose length is 40 cm and the length of one of the diagonal is 41 cm.

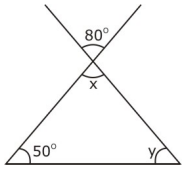
Fill in the blanks:

5. Exterior anlge of a triangle is always equal to sum of two _____.

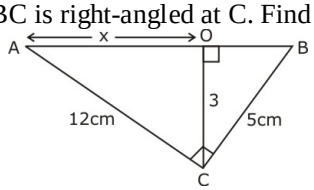
6. If in a triangle Pythagoras property holds, the triangle must be _____.

7. Define the median of a triangle.

8. Find angles *x* and *y* in the following figure.



9. In the given figure, triangle ABC is right-angled at C. Find *x*.



10. A tree is broken at a height of 5 m from the ground and is bent so that its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree?

11. Fill in the blank
Sum of three angles of a triangle is equal to **_____.**

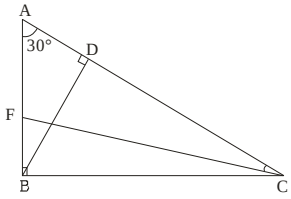
12. In a right-angled triangle if two anlges other than right angle are equal. Find all the anlges of the trianlge.

13. In a ΔABC, if the bisector of the angle BAC meets BC and D, then which one of the following is correct?

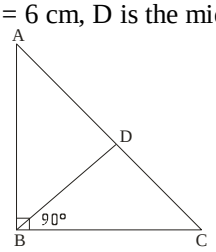
(a) $AB \leq BD$ (b) $\angle BDA > \frac{1}{2}\angle BAC$

(c) ΔABD can never be isosceles (d) IF BD = AD, then ΔABD will be equilateral

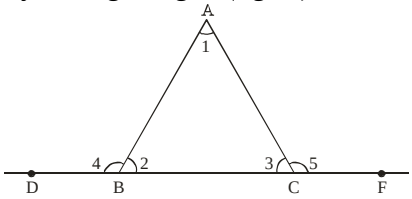
14. AB $\perp$ BC, BD $\perp$ AC and CE bisects $\angle C$, $\angle A = 30°$. Then what is $\angle CED$?



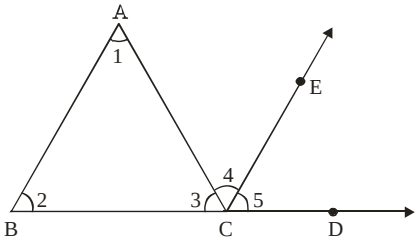
15. In DABC, $\angle B$ is a right angle, AC = 6 cm, D is the mid point of AC. The length of BD is



16. The side BC of ΔABC is produced on both sides. Show that the sum of the exterior angles so formed is greater than $\angle A$ by two right angles (figure).

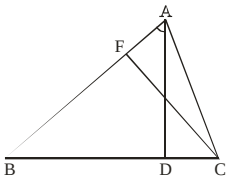


17. In figure, the side BC of ΔABC is produced to form ray BD. Ray CE is drawn parallel to BA. Show directly, without using the angle sum property of a triangle that $\angle ACD = \angle A + \angle B$ and deduce that $\angle A + \angle B + \angle C = 180°$.



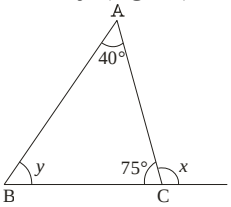
18. One of the exterior angles of a triangle is 80°, and the interior opposite angles are equal to each other. What is the measure of each of these two angles?

19. In figure AD and CF are respectively perpendiculars to sides BC and AB of DABC. It ∠FCD = 50°, find ∠BAD?



20. Explain the concept of interior and exterior angles and in each of the figures given below, find *x* and *y* (figure)?

(i)



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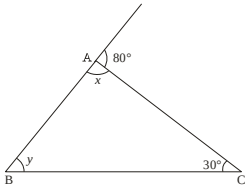
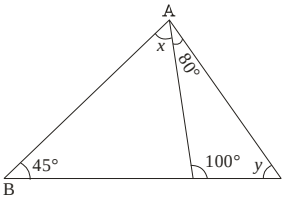


(iii)



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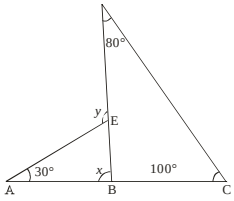


21. ABC is an isosceles right triangle, right angled at C. Prove that: $AB^2 = 2AC^2$.

22. In a ΔABC, AD ⊥ BC and $AD^2 = BD \times CD$. Prove that ΔABC is a right triangle.

23. The hypotenuse of a triangle is 2.5 cm. If one of the sides is 1.5 cm, find the length of the other side?

24. A ladder 25 m long reaches a window of a building 20 m above the ground. Determine the distance of the foot of the ladder from the building?

25. If the sides of a triangle are 3 cm, 4 cm and 6 cm long, determine whether the triangle is right angled triangle.

26. The hypotenuse of a right triangle is 17 cm long. If one of the remaining two sides is of length 8 cm, find the length of another side?

HIGHER ORDER THINKING SKILLS (HOTS)

1. A ladder 15 m long reaches a window which is 9 m above the ground on one side of a street. Keeping its foot at the same point, the ladder is turned to other side of the street to reach a window 12 m high. Find the width of the street.

PART – II: MULTIPLE CHOICE QUESTIONS

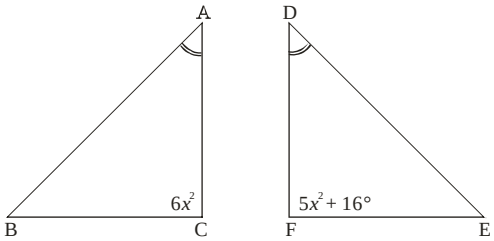
1. If the sides of a right triangle are x, $x + 1$ and $x - 1$ then the hypotenuse is:

(a) 5 (b) 4 (c) 1 (d) 0

2. The green measure of each of the three angles of a triangles is an integer. Which of the following could NOT be the ratio of their measures?

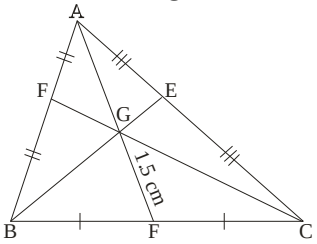
(a) 2 : 3 : 4 (b) 3 : 4 : 5 (c) 5 : 6 : 7 (d) 6 : 7 : 8

3. $\Delta ACB \cong \Delta DFE$. Find $\angle F$:



(a) 4° (b) 96° (c) 100° (d) 60°

4. In ΔABC medians BE and CF intersect at G. If the straight line AGD meets BC in D in such a way that GD = 1.5 cm, then the length of AD is:

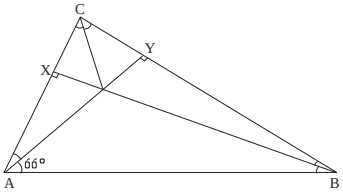


(a) 2.5 cm (b) 3 cm (c) 4 cm (d) 4.5 cm

5. If PL, QM and RN are the altitudes of ΔPQR whose orthocentre is O, then P is the orthocentre of :

(a) ΔPQO (b) ΔPQL (c) ΔQLO (d) ΔQRO

6. P is the incentre of ΔABC. If $\angle CAB = 66°$, and $\angle CPY = 46°$, find $\angle PBA$?

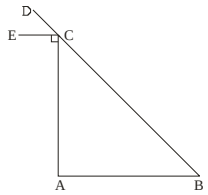


(a) ?? (b) ?? (c) ?? (d) ??

7. In a ΔPQR, the sides PQ and PR are produced to S and T respectively. Bisectors of $\angle SQR$ and $\angle QRT$ meet the point O. If $\angle P = 66°$, then what is the value of $\angle QOR$?

(a) 47° (b) 50° (c) 57° (d) 67°

8. In the given figure, ABC is a triangle in which BC is produced to D. If $\angle A : \angle B : \angle C : 3 : 2 : 1$ and $AC \perp CE$, then $\angle ECD$ is :



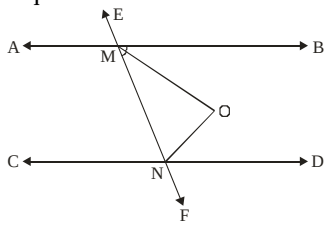
(a) 30° (b) 45° (c) 60° (d) 72°

9. In which one of the following triangles does the orthocentre lie in the exterior of the triangle.
(a) ΔABC, wherein, $\angle A = \angle B = \angle C = 60°$
(b) ΔPQR, wherein, $\angle P = 40°$, $\angle Q = 30°$, $\angle R = 110°$
(c) ΔPQR, wherein, $\angle X = 80°$, $\angle Y = 60°$, $\angle Z = 40°$
(d) ΔDEF, wherein, $\angle D = 52°$, $\angle E = 90°$, $\angle F = 38°$

10. A man gives to a garden and runs in the following manner. From the starting point, he goes west 25 m, then due north 60 m, then the last 80 m and finally due south 12 m. The distance between the finishing point and the starting point is:
(a) 177 m (b) 103 m (c) 83 m (d) 73 m

11. D, E, F are the mid points of BC< CA and AB of ΔABC. If AD and BE intersect in G, then AG + BG + CG is equal to:
(a) AD = BE = CF (b) $\frac{2}{3}(AD + BE + CF)$ (c) $\frac{3}{2}(AD + BE + CF)$ (d) $\frac{1}{3}(AD + BE + CF)$

12. AB is parallel to CD, EF intersects them at M and N. The bisectors of $\angle M$ and $\angle N$ meet at Q. If $\angle AME = 80°$. Then $\angle MQN$ is equal to:



(a) 60° (b) 70° (c) 80° (d) 90°

13. The side opposite to an obtuse angle of a triangle is:
(a) smallest (b) greatest
(c) half of the perimeter (d) none of these

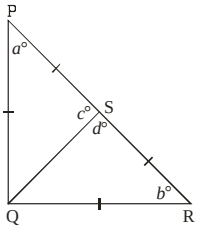
14. In a ΔABC, the sum of exterior angles at B and C is equal to:
(a) $180° - \angle BAC$ (b) $180° + \angle BAC$ (c) $180° - 2\angle BAC$ (d) $180° + 2\angle BAC$

15. Ankita wants to prove that $\Delta ABC \cong \Delta DEF$ using SAS. She knows AB = DE and AC = DF. What additional piece of information does she need?
(a) $\angle A = \angle D$ (b) $\angle C = \angle F$ (c) $\angle B = \angle E$ (d) $\angle A = \angle B$

16. The point of intersection of the right bisectors of a triangle is called:

(a) in-centre (b) circumcentre (c) orthocentre (d) centroid

17. PQ = QR = PS. Calculate the size of the labeled angles.



(a) $a = 40°, b = 50°, c = 70°, d = 110°$
(b) $a = 42°, b = 48°, c = 69°, d = 111°$
(c) $a = 45°, b = 45°, c = 67.5°, d = 112.5°$
(d) $a = 50°, b = 40°, c = 65°, d = 115°$

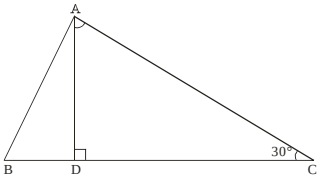
18. Match correctly:

(1) Each angle of an equilateral triangle	(a) greater than 2
(2) Measure of the vertical angle of a Δ if Each base angle is double the vertical angle	(b) incentre
(3) Sum of any two exterior angles of a D	(c) 36°
(4) Point of intersection of the angle bisectors	(d) 60°

19. If the straight line which bisects the vertical angle of a triangle is perpendicular to the basic the triangle is:

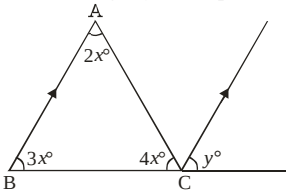
(a) equilateral (b) isosceles (c) scalene (d) right angled

20. Which of the following is not a correct statement?



(a) AC = 2AD (b) ∠ACD = ∠CAB + ∠ABC
(c) $BD^2 = AB^2 - AD^2$ (d) AC > AB + BC

21. In the figure CD is parallel to AB. The angle y is equal to:

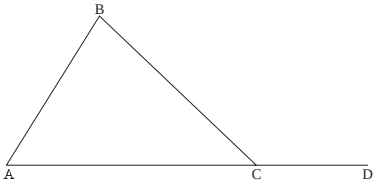


(a) 40° (b) 60° (c) 80° (d) 100°

22. If the angles of a triangle are in the ratio 5 : 3 : 2, then the triangle could be

(a) 1 : 2 : 3 (b) $1:1:\sqrt{2}$ (c) $1:\sqrt{3}:3$ (d) $1:\sqrt{2}:2$

23. ∠A and ∠B are the interior opposite angles of ∠BCD is ΔABC, which of these statements is true?



(a) ∠BCA = ∠BCD – ∠A (b) ∠A – 180° = ∠B
(c) ∠A = 90° – ∠B (d) ∠B = ∠BCD – ∠A

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