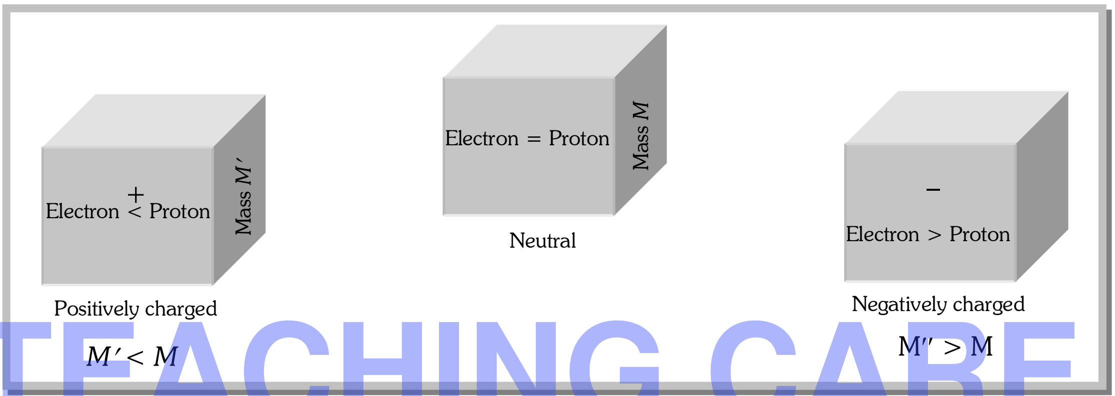
Electric Charge

Definition : Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects.

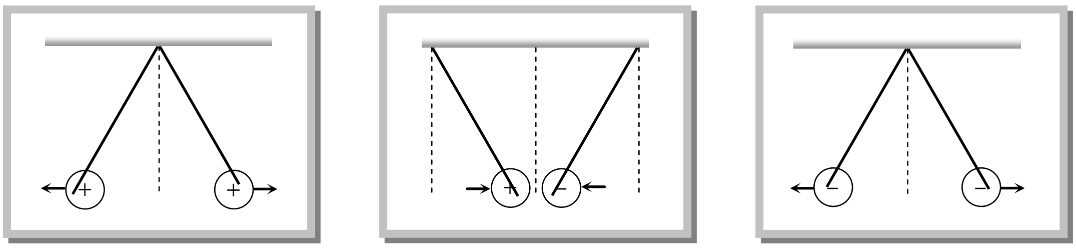
Origin of electric charge : It is known that every atom is electrically neutral, containing as many electrons as the number of protons in the nucleus.

Charged particles can be created by disturbing neutrality of an atom. Loss of electrons gives positive charge (as then $n_p > n_e$) and gain of electrons gives negative charge (as then $n_e > n_p$) to a particle. When an object is negatively charged it gains electrons and therefore its mass increases negligibly. Similarly, on charging a body with positive electricity its mass decreases. Change in mass of object is equal to $n \times m_e$. Where, n is the number of electrons transferred and m_e is the mass of electron $= 9.1 \times 10^{-31}\, Kg$.



Type : There exists two types of charges in nature (i) Positive charge (ii) Negative charge

Charges with the same electrical sign repel each other, and charges with opposite electrical sign attract each other.



Unit and dimensional formula : Rate of flow of electric charge is called electric current *i.e.*, $\boldsymbol{i} = \frac{\boldsymbol{dQ}}{\boldsymbol{dt}}$ $\Rightarrow \boldsymbol{dQ} = \boldsymbol{idt}$, hence S.I. unit of charge is – *Ampere* × *sec* = *coulomb* (*C*), smaller S.I. units are *mC*, *μC*, *nC* $(1mC = 10^{-3}C, 1\mu C = 10^{-6}C, 1nC = 10^{-9}C)$. C.G.S. unit of charge is – *Stat coulomb* or *e.s.u.* Electromagnetic unit of charge is – *ab coulomb* $1C = 3 \times 10^9$ *stat coulomb* $= \frac{1}{10}$ *ab coulomb*. Dimensional formula $[Q] = [AT]$

Note : ≅Benjamin Franklin was the first to assign positive and negative sign of charge.

≅ The existence of two type of charges was discovered by Dufog.

≅ Franklin (*i.e.*, *e.s.u.* of charge) is the smallest unit of charge while faraday is largest (1 *Faraday* = 96500 *C*).

≅ The *e.s.u.* of charge is also called stat coulomb or Franklin (*Fr*) and is related to *e.m.u.* of charge through the relation $\frac{\text{emu of charge}}{\text{esu of charge}} = 3 \times 10^{10}$

Point charge : A finite size body may behave like a point charge if it produces an inverse square electric field. For example an isolated charged sphere behave like a point charge at very large distance as well as very small distance close to it's surface.

Properties of charge

(i) **Charge is transferable :** If a charged body is put in contact with an uncharged body, uncharged body becomes charged due to transfer of electrons from one body to the other.

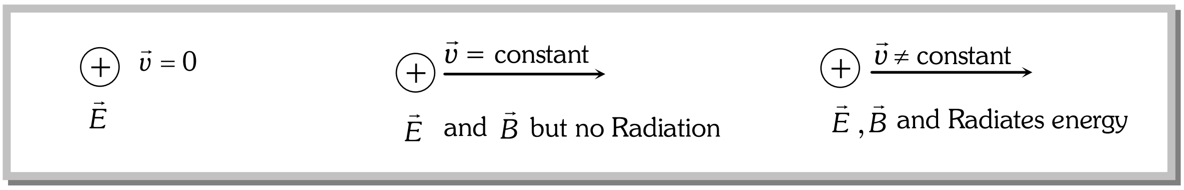
(ii) **Charge is always associated with mass**, *i.e.*, charge can not exist without mass though mass can exist without charge.

(iii) **Charge is conserved :** Charge can neither be created nor be destroyed. *e.g.* In radioactive decay the uranium nucleus (charge $= +92e$) is converted into a thorium nucleus (charge $= +90e$) and emits an α-particle (charge $= +2e$)

$_{92}U^{238} \rightarrow _{90}Th^{234} + _2He^4$. Thus the total charge is $+92e$ both before and after the decay.

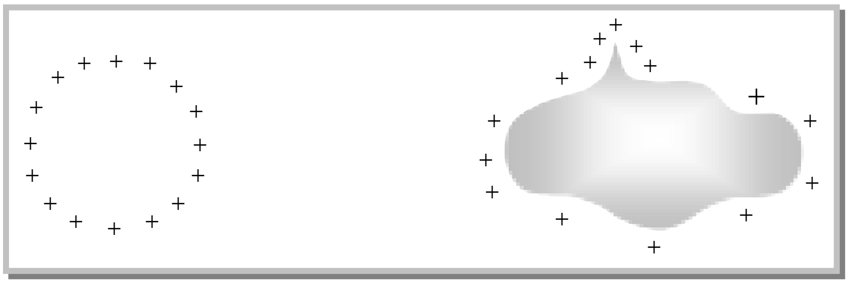
(iv) **Invariance of charge** : The numerical value of an elementary charge is independent of velocity. It is proved by the fact that an atom is neutral. The difference in masses on an electron and a proton suggests that electrons move much faster in an atom than protons. If the charges were dependent on velocity, the neutrality of atoms would be violated.

(v) **Charge produces electric field and magnetic field** : A charged particle at rest produces only electric field in the space surrounding it. However, if the charged particle is in unaccelerated motion it produces both electric and magnetic fields. And if the motion of charged particle is accelerated it not only produces electric and magnetic fields but also radiates energy in the space surrounding the charge in the form of electromagnetic waves.



(vi) **Charge resides on the surface of conductor :** Charge resides on the outer surface of a conductor because like charges repel and try to get as far away as possible from one another and stay at the farthest distance from each other which is outer surface of the conductor. This is why a solid and hollow conducting sphere of same outer radius will hold maximum equal charge and a **soap bubble expands on charging.**

(vii) **Charge leaks from sharp points :** In case of conducting body no doubt charge resides on its outer surface, if surface is uniform the charge distributes uniformly on the surface and for irregular surface the distribution of charge, *i.e.*, charge density is not uniform. It is maximum where the radius of curvature is minimum and vice versa. *i.e.*, $\sigma \propto (1/R)$. This is why charge leaks from sharp points.



(viii) **Quantization of charge** : When a physical quantity can have only discrete values rather than any value, the quantity is said to be quantised. The smallest charge that can exist in nature is the charge of an electron. If the charge of an electron $(=1.6\times10^{-19}\,C)$ is taken as elementary unit *i.e.* quanta of charge the charge on any body will be some integral multiple of e *i.e.*,

$\boxed{\boldsymbol{Q=\pm ne}}$ with $n=1,2,3\ldots.$

Charge on a body can never be $\pm\frac{2}{3}e$, $\pm17.2e$ or $\pm10^{-5}e$ *etc.*

Note : ≅Recently it has been discovered that elementary particles such as proton or neutron are composed of quarks having charge $(\pm1/3)e$ and $(\pm2/3)e$. However, as quarks do not exist in free state, the quanta of charge is still e.

≅ Quantization of charge implies that there is a maximum permissible magnitude of charge.

Comparison of Charge and Mass.

We are familiar with role of mass in gravitation, and we have just studied some features of electric charge. We can compare the two as shown below

Charge	Mass
(1) Electric charge can be positive, negative or zero.	(1) Mass of a body is a positive quantity.
(2) Charge carried by a body does not depend upon velocity of the body.	(2) Mass of a body increases with its velocity as $m=\frac{m_0}{\sqrt{1-v^2/c^2}}$ where c is velocity of light in vacuum, m is the mass of the body moving with velocity v and m_0 is rest mass of the body.
(3) Charge is quantized.	(3) The quantization of mass is yet to be established.
(4) Electric charge is always conserved.	(4) Mass is not conserved as it can be changed into energy and vice-versa.
(5) Force between charges can be attractive or repulsive, according as charges are unlike or like charges.	(5) The gravitational force between two masses is always attractive.

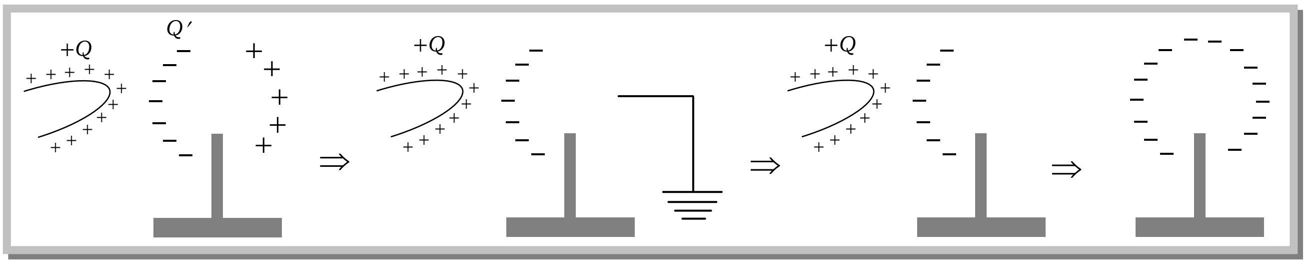
Methods of Charging.

A body can be charged by following methods :

(1) **By friction :** In friction when two bodies are rubbed together, electrons are transferred from one body to the other. As a result of this one body becomes positively charged while the other negatively charged,

e.g., when a glass rod is rubbed with silk, the rod becomes positively charged while the silk negatively. However, ebonite on rubbing with wool becomes negatively charged making the wool positively charged. Clouds also become charged by friction. In charging by friction in accordance with conservation of charge, both positive and negative charges in equal amounts appear simultaneously due to transfer of electrons from one body to the other.

(2) **By electrostatic induction :** If a charged body is brought near an uncharged body, the charged body will attract opposite charge and repel similar charge present in the uncharged body. As a result of this one side of neutral body (closer to charged body) becomes oppositely charged while the other is similarly charged. This process is called electrostatic induction.



Note : ≅Inducting body neither gains nor loses charge.

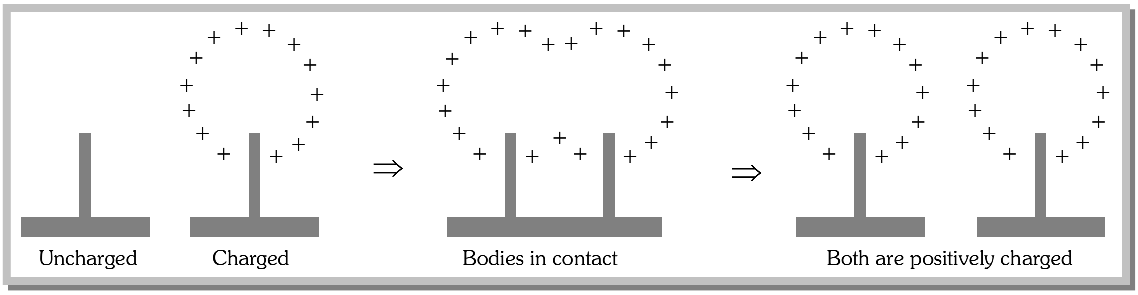
≅ Induced charge can be lesser or equal to inducing charge (but never greater) and its maximum value is given by $\boldsymbol{Q'} = -\boldsymbol{Q}\left[\mathbf{1} - \frac{\mathbf{1}}{\boldsymbol{K}}\right]$ where Q is the inducing charge and K is the dielectric constant of the material of the uncharged body. Dielectric constant of different media are shown below

Medium	K
Vacuum / air	1
Water	80
Mica	6
Glass	5–10
Metal	∞

≅Dielectric constant of an insulator can not be ∞

≅For metals in electrostatics $K = \infty$ and so $Q' = -Q$; i.e. in metals induced charge is equal and opposite to inducing charge.

(3) **Charging by conduction :** Take two conductors, one charged and other uncharged. Bring the conductors in contact with each other. The charge (whether $-ve$ or $+ve$) under its own repulsion will spread over both the conductors. Thus the conductors will be charged with the same sign. This is called as charging by conduction (through contact).

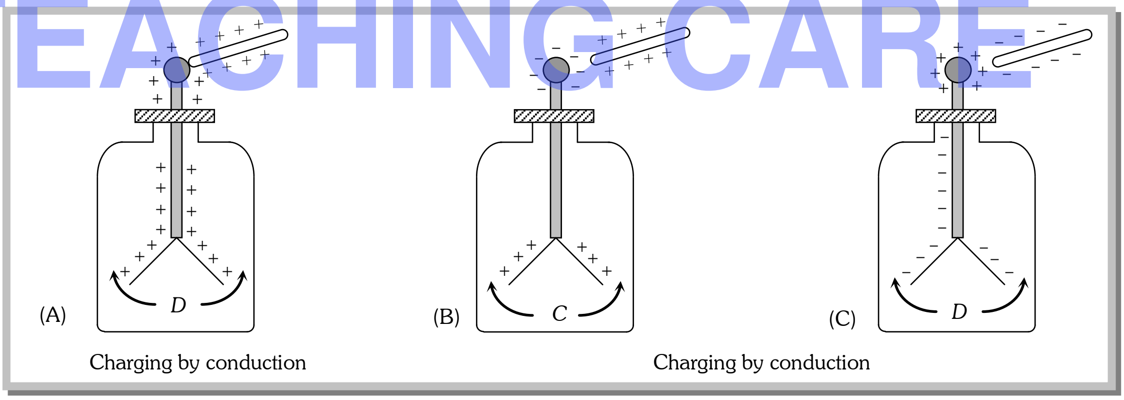


Note : ≅A truck carrying explosives has a metal chain touching the ground, to conduct away the charge produced by friction.

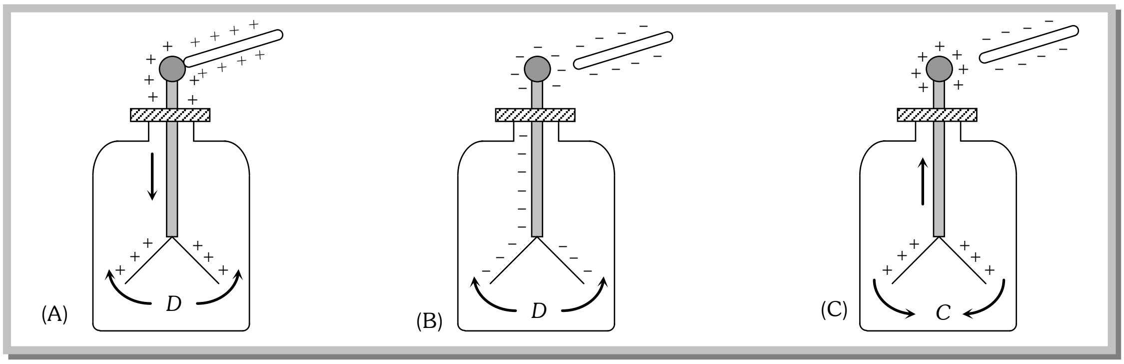
Electroscope.

It is a simple apparatus with which the presence of electric charge on a body is detected (see figure). When metal knob is touched with a charged body, some charge is transferred to the gold leaves, which then diverges due to repulsion. The separation gives a rough idea of the amount of charge on the body. If a charged body brought near a charged electroscope the leaves will further diverge. If the charge on body is similar to that on electroscope and will usually converge if opposite. If the induction effect is strong enough leaves after converging may again diverge.

(1) **Uncharged electroscope**

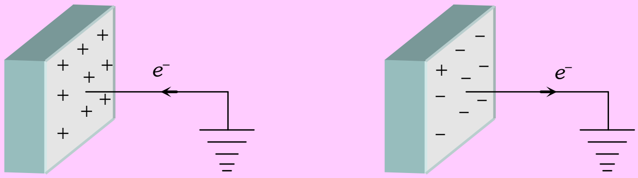


(2) **Charged electroscope**

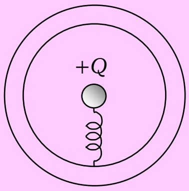


Concepts

- *After earthing a positively charged conductor electrons flow from earth to conductor and if a negatively charged conductor is earthed then electrons flows from conductor to earth.*



- *When a charged spherical conductor placed inside a hollow insulated conductor and connected if through a fine conducting wire the charge will be completely transferred from the inner conductor to the outer conductor.*



- *Lightening-rods arrestors are made up of conductors with one of their ends earthed while the other sharp, and protects a building from lightening either by neutralising or conducting the charge of the cloud to the ground.*
- *With rise in temperature dielectric constant of liquid decreases.*
- *Induction takes place only in bodies (either conducting or non-conducting) and not in particles.*
- *If X-rays are incident on a charged electroscope, due to ionisation of air by X-rays the electroscope will get discharged and hence its leaves will collapse. However, if the electroscope is evacuated. X-rays will cause photoelectric effect with gold and so the leaves will further diverge if it is positively charged (or uncharged) and will converge if it is negatively charged.*
- *If only one charge is available than by repeating the induction process, it can be used to obtain a charge many times greater than it's equilibrium. (High voltage generator)*

Examples based on properties of charge

Example: 1 **A soap bubble is given negative charge. Its radius will** [DCE 2000; RPMT 1997; CPMT 1997; MNR 1988]

(a) Increase **(b) Decrease** **(c) Remain unchanged** **(d) Fluctuate**

Solution: (a) Due to repulsive force.

Example: 2 **Which of the following charge is not possible**

(a) $1.6\times10^{-18}C$ (b) $1.6\times10^{-19}C$ (c) $1.6\times10^{-20}C$ (d) None of these

Solution: (c) $1.6\times10^{-20}C$, because this is $\frac{1}{10}$ of electronic charge and hence not an integral multiple.

Example: 3 **Five balls numbered 1 to 5 balls suspended using separate threads. Pair (1,2), (2,4) and (4,1) show electrostatic attraction, while pair (2,3) and (4,5) show repulsion. Therefore ball 1 must be** [NCERT 1980]

(a) Positively charged **(b) Negatively charged** **(c) Neutral** **(d) Made of metal**

Solution: (c) Since 1 does not enter the list of repulsion, it is just possible that it may not be having any charge. Moreover, since ball no. 1 is being attracted by 2 and 4 both. So 2 and 4 must be similarly charged, but it is also given that 2 and 4 also attract each other. So 2 and 4 are certainly oppositely charged.

Since 1 is attracting 2, either 1 or 2 must be neutral but since 2 is already in the list of balls repelling each other, it necessarily has some charge, similarly 4 must have some charge. It means that though 1 is attracting 2 and 4 it does not have any charge.

Example: 4 **If the radius of a solid and hollow copper spheres are same which one can hold greater charge**

[BHU 1999; KCET 1994; IIT-JEE 1974]

(a) Solid sphere **(b)Hollow sphere**

(c) Both will hold equal charge **(d) None of these**

Solution: (c) Charge resides on the surface of conductor, since both the sphere having similar surface area so they will hold equal charge.

Example: 5 **Number of electrons in one coulomb of charge will be**

(a) 5.46×10^{29} (b) 6.25×10^{18} (c) 1.6×10^{19} (d) 9×10^{11}

Solution: (b) By using $Q = ne \Rightarrow n = \dfrac{Q}{e} \Rightarrow n = \dfrac{1}{1.6\times10^{-19}} = 6.25\times10^{18}$

Example: 6 **The current produced in wire when 10^7 electron/sec are flowing in it** **[CPMT 1994]**

(a) 1.6×10^{-26} *amp* (b) 1.6×10^{12} *amp* (c) 1.6×10^{26} *amp* (d) 1.6×10^{-12} *amp*

Solution: (d) $i = \dfrac{Q}{t} = \dfrac{ne}{t} = 10^7\times1.6\times10^{-19} = 1.6\times10^{-12}\ amp$

Example: 7 **A table-tennis ball which has been covered with a conducting paint is suspended by a silk thread so that it hangs between two metal plates. One plate is earthed. When the other plate is connected to a high voltage generator, the ball**

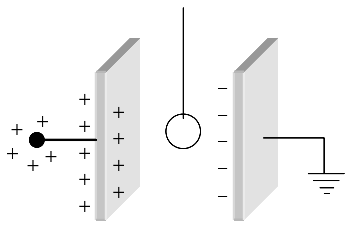
(a) Is attracted to the high voltage plate and stays there

(b) Hangs without moving

(c) Swings backward and forward hitting each plate in turn

(d) None of these

Solution: (c) The table tennis ball when slightly displaced say towards the positive plate gets attracted towards the positive plate due to induced negative charge on its near surface.



The ball touches the positive plate and itself gets positively charged by the process of conduction from the plate connected to high voltage generator. On getting positively charged it is repelled by the positive plate and therefore the ball touches the other plate (earthed), which has negative charge due to induction. On touching this plate, the positive charge of the ball gets neutralized and in turn the ball shares negative charge of the earthed plate and is again repelled from this plate also, and this process is repeated again and again.

Here it should be understood that since the positive plate is connected to high voltage generator, its potential and hence its charge will always remain same, as soon as this plate gives some of its charge to ball, excess charge flows from generator to the plate, and an equal negative charge is always induced on the other plate.

Tricky example: 1

In 1 *gm* of a solid, there are 5×10^{21} *atoms*. If one electron is removed from everyone of 0.01% atoms of the solid, the charge gained by the solid is (given that electronic charge is 1.6×10^{-19} *C*)

(a) + 0.08 *C* (b) + 0.8 *C* (c) – 0.08 *C* (d) – 0.8 *C*

Solution: (a) To calculate charge, we will apply formula $Q = ne$ for this, we must have number of electrons. Here, number of electrons $n = .01\%$ of 5×10^{21}

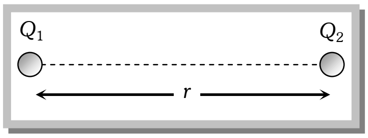
i.e. $n = \frac{5 \times 10^{21} \times .01}{100} = 5 \times 10^{21} \times 10^{-4} = 5 \times 10^{17}$

So $Q = 5 \times 10^{17} \times 1.6 \times 10^{-19} = 8 \times 10^{-2} = 0.08\ C$

Since electrons have been removed, charge will be positive *i.e.* $Q = +0.08\ C$

Coulomb's Law.

If two stationary and point charges Q_1 and Q_2 are kept at a distance *r*, then it is found that force of attraction



or repulsion between them is $F \propto \frac{Q_1 Q_2}{r^2}$ *i.e.*, $F = \frac{kQ_1Q_2}{r^2}$; (*k* = Proportionality constant)

(1) **Dependence of *k* :** Constant *k* depends upon system of units and medium between the two charges.

(i) **Effect of units**

(a) In C.G.S. for air $k = 1$, $F = \frac{Q_1 Q_2}{r^2}$ *Dyne*

(b) In S.I. for air $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{N-m^2}{C^2}$, $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1Q_2}{r^2}$ *Newton* (1 *Newton* = 10^5 *Dyne*)

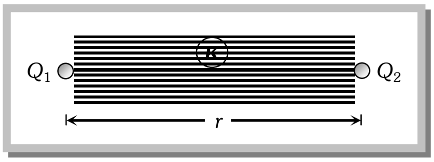
Note : $\cong \varepsilon_0$ = Absolute permittivity of air or free space = $8.85 \times 10^{-12} \frac{C^2}{N-m^2} \left(= \frac{Farad}{m}\right)$. It's

Dimension is $[ML^{-3}T^4A^2]$

$\cong$ ε_0 Relates with absolute magnetic permeability (μ_0) and velocity of light (*c*) according to the following relation $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

(ii) **Effect of medium**

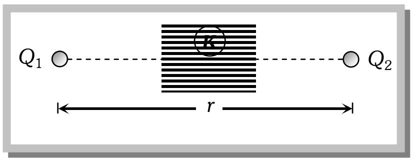
(a) When a dielectric medium is completely filled in between charges rearrangement of the charges inside the dielectric medium takes place and the force between the same two charges decreases by a factor of *K* known as **dielectric constant** or specific inductive capacity **(SIC)** of the medium, K is also called relative permittivity ε_r of the medium (relative means with respect to free space).



Hence in the presence of medium $F_m = \frac{F_{air}}{K} = \frac{1}{4\pi\varepsilon_0 K}.\frac{Q_1Q_2}{r^2}$

Here $\varepsilon_0 K = \varepsilon_0\ \varepsilon_r = \varepsilon$ (permittivity of medium)

(b) If a dielectric medium (dielectric constant K, thickness t) is partially filled between the charges then effective air separation between the charges becomes $(r - t + t\sqrt{K})$



Hence force $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{(r - t + t\sqrt{K})^2}$

(2) **Vector form of coulomb's law :** Vector form of Coulomb's law is $\vec{F}_{12} = K.\frac{q_1q_2}{r^3}\vec{r}_{12} = K.\frac{q_1q_2}{r^2}\hat{r}_{12}$, where $\hat{r}_{12}$ is the unit vector from first charge to second charge along the line joining the two charges.

(3) **A comparative study of fundamental forces of nature**

S.No.	**Force**	**Nature and formula**	**Range**	**Relative strength**
(i)	Force of gravitation between two masses	Attractive $F = Gm_1m_2/r^2$, obey's Newton's third law of motion, it's a conservative force	Long range (between planets and between electron and proton)	1
(ii)	Electromagnetic force (for stationary and moving charges)	Attractive as well as repulsive, obey's Newton's third law of motion, it's a conservative force	Long (upto few *kelometers*)	10^{37}
(iii)	Nuclear force (between nucleons)	Exact expression is not known till date. However in some cases empirical formula U_0e^{r/r_0} can be utilized for nuclear potential energy U_0 and r_0 are constant.	Short (of the order of nuclear size 10^{-15} *m*)	10^{39} (strongest)
(iv)	Weak force (for processes like β decay)	Formula not known	Short (upto 10^{-15}*m*)	10^{24}

Note : ≅Coulombs law is not valid for moving charges because moving charges produces magnetic field also.

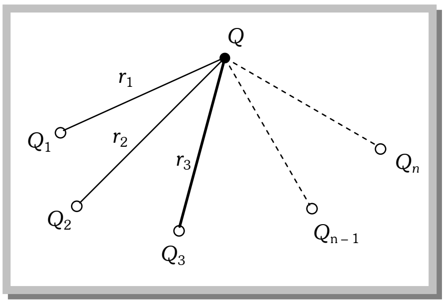
≅ Coulombs law is valid at a distance greater than $10^{-15}\,m$.

≅ A charge Q_1 exert some force on a second charge Q_2. If third charge Q_3 is brought near, the force of Q_1 exerted on Q_2 remains unchanged.

≅ Ratio of gravitational force and electrostatic force between (i) Two electrons is $\mathbf{10^{-43}/1}$. (ii) Two protons is $\mathbf{10^{-36}/1}$ (iii) One proton and one electron $\mathbf{10^{-39}/1}$.

≅ Decreasing order to fundamental forces $F_{Nuclear} > F_{Electromagnetic} > F_{Weak} > F_{Gravitational}$

(4) **Principle of superposition :** According to the principle of super position, total force acting on a given charge due to number of charges is the vector sum of the individual forces acting on that charge due to all the charges.



Consider number of charge Q_1, Q_2, Q_3 ...are applying force on a charge Q

Net force on Q will be

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \text{.........} + \vec{F}_{n-1} + \vec{F}_n$$

Concepts

- *Two point charges separated by a distance r in vacuum and a force F acting between them. After filling a dielectric medium having dielectric constant K completely between the charges, force between them decreases. To maintain the force as before separation between them changes to $r\sqrt{K}$. This distance known as effective air separation.*

Examples based on Coulomb's

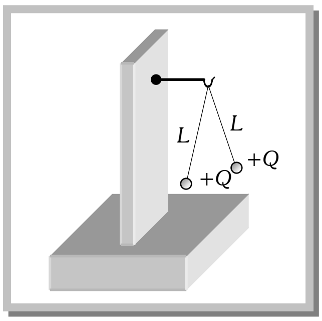
Example: 8 **Two point charges $+3\mu C$ and $+8\mu C$ repel each other with a force of 40 N. If a charge of $-5\mu C$ is added to each of them, then the force between them will become**

(a) $-10N$ (b) $+10N$ (c) $+20N$ (d) $-20N$

Solution: (a) Initially $F = k \times \frac{3\times8\times10^{-12}}{r^2}$ and Finally $F' = -k\frac{2\times3\times10^{-12}}{r^2}$ so $\frac{F'}{F} = -\frac{1}{4} \Rightarrow F' = -10N$

Example: 9 **Two small balls having equal positive charge Q (coulomb) on each are suspended by two insulated string of equal length L meter, from a hook fixed to a stand. The whole set up is taken in satellite into space where there is no gravity (state of weight less ness). Then the angle between the string and tension in the string is**

[IIT-JEE 1986]



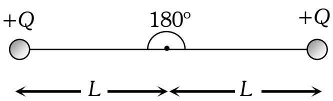
(a) $180^o, \frac{1}{4\pi\varepsilon_0}.\frac{Q^2}{(2L)^2}$

(b) $90^\circ, \frac{1}{4\pi\varepsilon_0}.\frac{Q^2}{L^2}$

(c) $180^\circ, \frac{1}{4\pi\varepsilon_0}.\frac{Q^2}{2L^2}$

(d) $180^o, \frac{1}{4\pi\varepsilon_0}.\frac{QL}{4L^2}$

Solution: (a) In case to weight less ness following situation arises



So angle $\theta = 180^\circ$ and force $F = \frac{1}{4\pi\varepsilon_0}.\frac{Q^2}{(2L)^2}$

Example: 10 **Two point charges 1 μC & $5\mu C$ are separated by a certain distance. What will be ratio of forces acting on these two**

(a) 1 : 5 (b) 5 : 1 (c) 1 : 1 (d) 0

Solution: (c) Both the charges will experience same force so ratio is 1:1

Example: 11 **Two charges of $40\mu C$ and $-20\mu C$ are placed at a certain distance apart. They are touched and kept at the same distance. The ratio of the initial to the final force between them is**

(a) $8:1$ (b) $4:1$ (c) $1:8$ (d) $1:1$

Solution: (a) Since only magnitude of charges are changes that's why $F \propto q_1 q_2 \Rightarrow \frac{F_1}{F_2} = \frac{q_1 q_2}{q'_1 q'_2} = \frac{40\times 20}{10\times 10} = \frac{8}{1}$

Example: 12 **A total charge Q is broken in two parts Q_1 and Q_2 and they are placed at a distance R from each other. The maximum force of repulsion between them will occur, when**

(a) $Q_2 = \frac{Q}{R}, Q_1 = Q - \frac{Q}{R}$ (b) $Q_2 = \frac{Q}{4}, Q_1 = Q - \frac{2Q}{3}$ (c) $Q_2 = \frac{Q}{4}, Q_1 = \frac{3Q}{4}$ (d) $Q_1 = \frac{Q}{2}, Q_2 = \frac{Q}{2}$

Solution: (d) Force between charges Q_1 and Q_2 $F = k\frac{Q_1 Q_2}{R^2} = k\frac{Q_1(Q-Q_1)}{R^2}$

For F to be maximum, $\frac{dF}{dQ_1} = 0$ i.e., $\frac{d}{dQ_1}\left\{k\frac{(Q_1 Q - Q_1^2)}{R^2}\right\} = 0$ or $Q - 2Q_1 = 0, Q_1 = \frac{Q}{2}$

Hence $Q_1 = Q_2 = \frac{Q}{2}$

Example: 13 **The force between two charges 0.06*m* apart is 5 *N*. If each charge is moved towards the other by 0.01*m*, then the force between them will become**

(a) 7.20 *N* (b) 11.25 *N* (c) 22.50 *N* (d) 45.00 *N*

Solution: (b) Initial separation between the charges = 0.06*m*

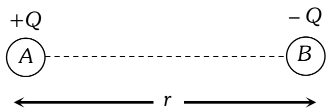
Final separation between the charges = 0.04*m*

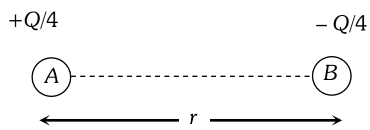
Since $F \propto \frac{1}{r^2} \Rightarrow \frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)^2 \Rightarrow \frac{5}{F_2} = \left(\frac{0.04}{0.06}\right)^2 = \frac{4}{9} \Rightarrow F_2 = 11.25N$

Example: 14 **Two charges equal in magnitude and opposite in polarity are placed at a certain distance apart and force acting between them is *F*. If 75% charge of one is transferred to another, then the force between the charges becomes**

(a) $\frac{F}{16}$ (b) $\frac{9F}{16}$ (c) F (d) $\frac{15}{16}F$

Solution: (a)





Initially $F = k\frac{Q^2}{r^2}$ Finally $F' = \frac{k\left(\frac{Q}{4}\right)^2}{r^2} = \frac{F}{16}$

Example: 15 **Three equal charges each +*Q*, placed at the corners of on equilateral triangle of side a what will be the force on any charge $\left(k = \frac{1}{4\pi\varepsilon_0}\right)$**

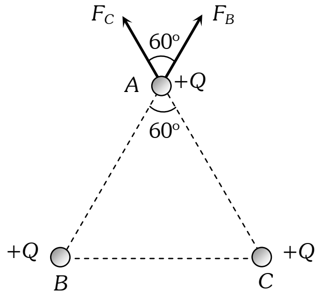
(a) $\frac{kQ^2}{a^2}$ (b) $\frac{2kQ^2}{a^2}$ (c) $\frac{\sqrt{2}kQ^2}{a^2}$ (d) $\frac{\sqrt{3}kQ^2}{a^2}$

Solution: (d) Suppose net force is to be calculated on the charge which is kept at *A*. Two charges kept at *B* and *C* are applying force on that particular charge, with direction as shown in the figure.

Since $F_b = F_c = F = k\dfrac{Q^2}{a^2}$

So, $F_{net} = \sqrt{F_B^2 + F_C^2 + 2F_BF_C \cos 60}$

$F_{net} = \sqrt{3}F = \dfrac{\sqrt{3}kQ^2}{a^2}$



Example: 16 **Equal charges Q are placed at the four corners A, B, C, D of a square of length a. The magnitude of the force on the charge at B will be**

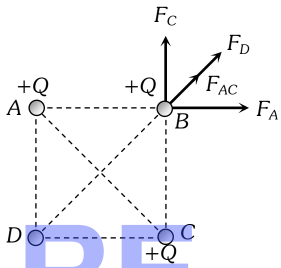
(a) $\dfrac{3Q^2}{4\pi\varepsilon_0 a^2}$ **(b)** $\dfrac{4Q^2}{4\pi\varepsilon_0 a^2}$ **(c)** $\left(\dfrac{1+1\sqrt{2}}{2}\right)\dfrac{Q^2}{4\pi\varepsilon_0 a^2}$ **(d)** $\left(2+\dfrac{1}{\sqrt{2}}\right)\dfrac{Q^2}{4\pi\varepsilon_0 a^2}$

Solution: (c) After following the guidelines mentioned above

$F_{net} = F_{AC} + F_D = \sqrt{F_A^2 + F_C^2} + F_D$

Since $F_A = F_C = \dfrac{kQ^2}{a^2}$ and $F_D = \dfrac{kQ^2}{(a\sqrt{2})^2}$

$F_{net} = \dfrac{\sqrt{2}kQ^2}{a^2} + \dfrac{kQ^2}{2a^2} = \dfrac{kQ^2}{a^2}\left(\sqrt{2} + \dfrac{1}{2}\right) = \dfrac{Q^2}{4\pi\varepsilon_0 a^2}\left(\dfrac{1+2\sqrt{2}}{2}\right)$



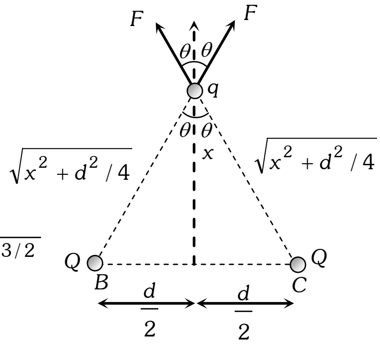
Example: 17 **Two equal charges are separated by a distance d. A third charge placed on a perpendicular bisector at x distance, will experience maximum *coulomb* force when**

(a) $x = \dfrac{d}{\sqrt{2}}$ **(b)** $x = \dfrac{d}{2}$ **(c)** $x = \dfrac{d}{2\sqrt{2}}$ **(d)** $x = \dfrac{d}{2\sqrt{3}}$

Solution: (c) Suppose third charge is similar to Q and it is q

So net force on it $F_{net} = 2F\cos\theta$

Where $F = \dfrac{1}{4\pi\varepsilon_0}.\dfrac{Qq}{\left(x^2 + \dfrac{d^2}{4}\right)}$ and $\cos\theta = \dfrac{x}{\sqrt{x^2 + \dfrac{d^2}{4}}}$



$\therefore \quad F_{net} = 2 \times \dfrac{1}{4\pi\varepsilon_0}.\dfrac{Qq}{\left(x^2 + \dfrac{d^2}{4}\right)} \times \dfrac{x}{\left(x^2 + \dfrac{d^2}{4}\right)^{1/2}} = \dfrac{2Qqx}{4\pi\varepsilon_0\left(x^2 + \dfrac{d^2}{4}\right)^{3/2}}$

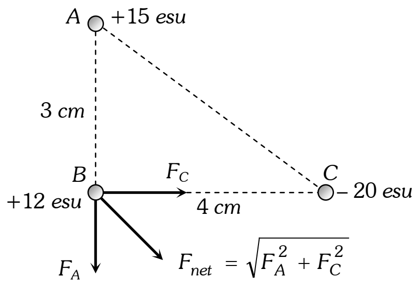
for F_{net} to be maximum $\dfrac{dF_{net}}{dx} = 0$ i.e. $\dfrac{d}{dx}\left[\dfrac{2Qqx}{4\pi\varepsilon_0\left(x^2 + \dfrac{d^2}{4}\right)^{3/2}}\right] = 0$

or $\left[\left(x^2 + \dfrac{d^2}{4}\right)^{-3/2} - 3x^2\left(x^2 + \dfrac{d^2}{4}\right)^{-5/2}\right] = 0$ i.e. $x = \pm\dfrac{d}{2\sqrt{2}}$

Example: 18 ***ABC* is a right angle triangle in which *AB* = 3 *cm*, *BC* = 4 cm and $\angle ABC = \frac{\pi}{2}$. The three charges +15, +12 and – 20 e.s.u. are placed respectively on *A*, *B* and *C*. The force acting on *B* is**

(a) 125 *dynes* **(b) 35 *dynes*** **(c) 25 *dynes*** **(d) Zero**

Solution: (c) Net force on B $F_{net} = \sqrt{F_A^2 + F_C^2}$



$$F_A = \frac{15 \times 12}{(3)^2} = 20 \ dyne$$

$$F_C = \frac{12 \times 20}{(4)^2} = 15 \ dyne$$

$$F_{net} = 25 \ dyne$$

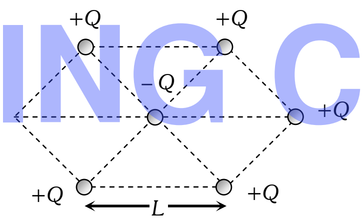
Example: 19 **Five point charges each of value +*Q* are placed on five vertices of a regular hexagon of side *L*. What is the magnitude of the force on a point charge of value – *q* placed at the centre of the hexagon** **[IIT-JEE 1992]**

(a) $k\frac{Q^2}{L^2}$ **(b)** $k\frac{Q^2}{4L^2}$

(c) Zero **(d) Information is insufficient**

Solution: (a) Four charges cancels the effect of each other, so the net force on the charge placed at centre due to remaining fifth charge is

$$F = k\frac{Q^2}{L^2}$$



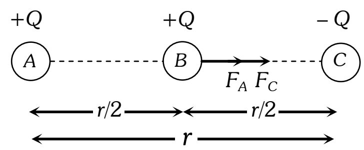
Example: 20 **Two small, identical spheres having +*Q* and – *Q* charge are kept at a certain distance. *F* force acts between the two. If in the middle of two spheres, another similar sphere having +*Q* charge is kept, then it experience a force in magnitude and direction as**

(a) Zero having no direction **(b) 8*F* towards +*Q* charge**

(c) 8*F* towards – *Q* charge **(d) 4*F* towards +*Q* charge**

Solution: (c) Initially, force between *A* and *C* $F = k\frac{Q^2}{r^2}$

When a similar sphere *B* having charge +*Q* is kept at the mid point of line joining *A* and *C*, then Net force on *B* is



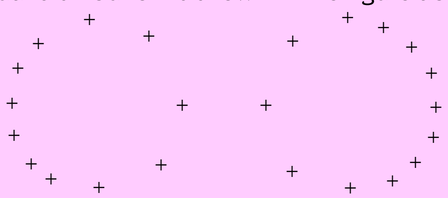
$F_{net} = F_A + F_C = k\frac{Q^2}{(r/2)^2} + \frac{kQ^2}{(r/2)^2} = 8\frac{kQ^2}{r^2} = 8F$. (Direction is shown in figure)

Tricky example: 2

Two equal spheres are identically charged with q units of electricity separately. When they are placed at a distance $3R$ from centre-to-centre where R is the radius of either sphere the force of repulsion between them is

(a) $\frac{1}{4\pi\varepsilon_0}.\frac{q^2}{R^2}$ (b) $\frac{1}{4\pi\varepsilon_0}.\frac{q^2}{9R^2}$ (c) $\frac{1}{4\pi\varepsilon_0}.\frac{q^2}{4R^2}$ (d) None of these

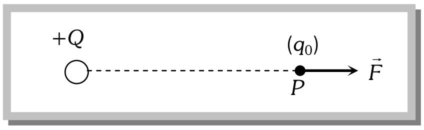
Solution: (a) Generally students give the answer $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(3R)^2}$ but it is not true. Since the charges are not uniformly distributed, they cannot be treated as point charges and so we cannot apply coulombs law which is a law for point charges. The actual distribution is shown in the figure above.



Electrical Field.

A positive charge or a negative charge is said to create its field around itself. If a charge Q_1 exerts a force on charge Q_2 placed near it, it may be stated that since Q_2 is in the field of Q_1, it experiences some force, or it may also be said that since charge Q_1 is inside the field of Q_2, it experience some force. Thus space around a charge in which another charged particle experiences a force is said to have electrical field in it.

(1) **Electric field intensity** $(\vec{E})$**:** The electric field intensity at any point is defined as the force experienced by a unit positive charge placed at that point. $\vec{E} = \frac{\vec{F}}{q_0}$



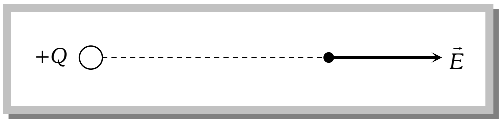
Where $q_0 \to 0$ so that presence of this charge may not affect the source charge Q and its electric field is not changed, therefore expression for electric field intensity can be better written as $\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$

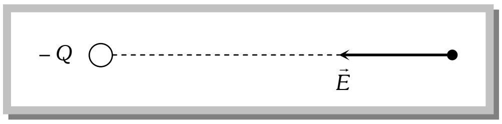
(2) **Unit and Dimensional formula :** It's S.I. unit – $\frac{Newton}{coulomb} = \frac{volt}{meter} = \frac{Joule}{coulomb \times meter}$ and

C.G.S. unit – *Dyne/stat coulomb*.

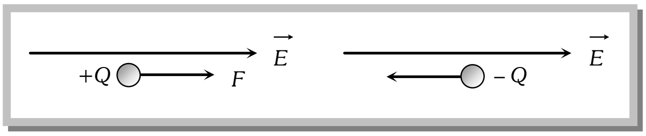
Dimension : $[E] = [MLT^{-3}A^{-1}]$

(3) **Direction of electric field :** Electric field (intensity) $\vec{E}$ is a vector quantity. Electric field due to a positive charge is always away from the charge and that due to a negative charge is always towards the charge





(4) **Relation between electric force and electric field :** In an electric field $\vec{E}$ a charge (Q) experiences a force $F = QE$. If charge is positive then force is directed in the direction of field while if charge is negative force acts on it in the opposite direction of field

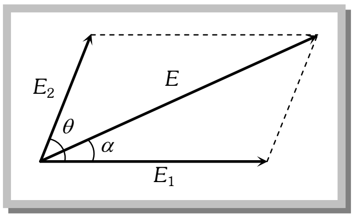


(5) **Super position of electric field** (electric field at a point due to various charges) : The resultant electric field at any point is equal to the vector sum of electric fields at that point due to various charges.

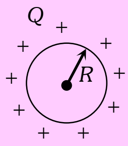
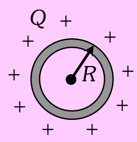
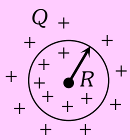
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$$

The magnitude of the resultant of two electric fields is given by

$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta}$ and the direction is given by $\tan\alpha = \dfrac{E_2 \sin\theta}{E_1 + E_2\cos\theta}$



(6) **Electric field due to continuous distribution of charge :** A system of closely spaced electric charges forms a continuous charge distribution

Continuous charge distribution		
Linear charge distribution	**Surface charge distribution**	**Volume charge distribution**
In this distribution charge distributed on a line.	In this distribution charge distributed on the surface.	In this distribution charge distributed in the whole volume of the body.
For example : charge on a wire, charge on a ring etc. Relevant parameter is λ which is called linear charge density *i.e.*,	For example : Charge on a conducting sphere, charge on a sheet etc. Relevant parameter is σ which is called surface charge density *i.e.*,	For example : Non conducting charged sphere. Relevant parameter is ρ which is called volume charge density *i.e.*,
$\lambda = \dfrac{\text{charge}}{\text{length}}$ $\lambda = \dfrac{Q}{2\pi R}$  Circular charged ring	$\sigma = \dfrac{\text{charge}}{\text{area}}$ $\sigma = \dfrac{Q}{4\pi R^2}$  Spherical shall	$\rho = \dfrac{\text{charge}}{\text{volume}}$ $\rho = \dfrac{Q}{\frac{4}{3}\pi R^3}$  Non conducting sphere

To find the field of a continuous charge distribution, we divide the charge into infinitesimal charge elements. Each infinitesimal charge element is then considered, as a point charge and electric field $\overrightarrow{dE}$ is determined due to this charge at given point. The Net field at the given point is the summation of fields of all the elements. *i.e.*, $\vec{E} = \int \overrightarrow{dE}$

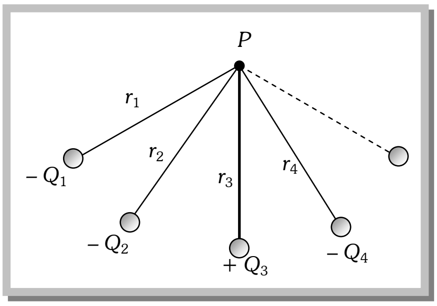
Electric Potential.

(1) **Definition :** Potential at a point in a field is defined as the amount of work done in bringing a unit positive test charge, from infinity to that point along any arbitrary path (infinity is point of zero potential). Electric potential is a scalar quantity, it is denoted by V; $\boldsymbol{V = \dfrac{W}{q_0}}$

(2) **Unit and dimensional formula :** S. I. unit – $\frac{Joule}{Coulomb} = volt$ C.G.S. unit – *Stat volt* (e.s.u.); **1 *volt*** $= \frac{1}{300}$ ***Stat volt*** Dimension – $[V] = [ML^2T^{-3}A^{-1}]$

(3) **Types of electric potential :** According to the nature of charge potential is of two types

(i) Positive potential : Due to positive charge. (ii) Negative potential : Due to negative charge.



(4) **Potential of a system of point charges :** Consider *P* is a point at which net electric potential is to be determined due to several charges. So net potential at *P*

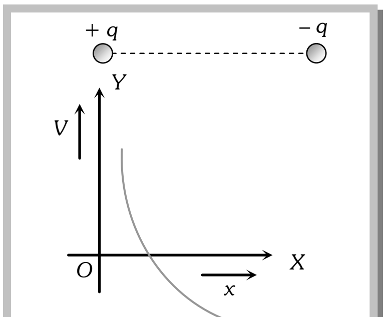
$$V = k\frac{Q_1}{r_1} + k\frac{Q_2}{r_2} + k\frac{Q_3}{r_3} + k\frac{(-Q_4)}{r_4} + \dots \text{ In general } V = \sum_{i=1}^{X} \frac{kQ_i}{r_i}$$

Note : ≅At the centre of two equal and opposite charge $V = 0$ but $E \neq 0$

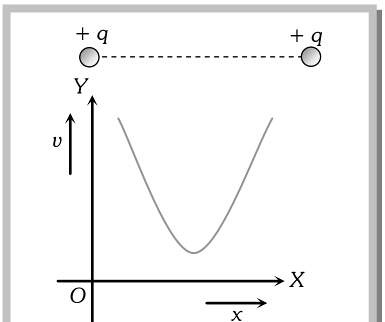
≅ At the centre of the line joining two equal and similar charge $V \neq 0, E = 0$

(5) **Electric potential due to a continuous charge distribution :** The potential due to a continuous charge distribution is the sum of potentials of all the infinitesimal charge elements in which the distribution may be divided *i.e.*, $\boldsymbol{V = \int dV, = \int \frac{dQ}{4\pi\varepsilon_0 r}}$

(6) **Graphical representation of potential :** When we move from a positive charge towards an equal negative charge along the line joining the two then initially potential decreases in magnitude and at centre become zero, but this potential is throughout positive because when we are nearer to positive charge, overall potential must be positive. When we move from centre towards the negative charge then though potential remain always negative but increases in magnitude fig. (A). As one move from one charge to other when both charges are like, the potential first decreases, at centre become minimum and then increases Fig. (B).



(A)



(B)

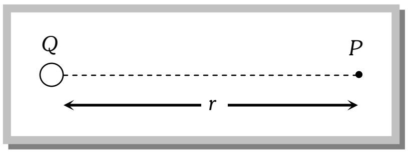
(7) **Potential difference :** In an electric field potential difference between two points *A* and *B* is defined as equal to the amount of work done (by external agent) in moving a unit positive charge from point *A* to point *B*.

i.e., $V_B - V_A = \frac{W}{q_0}$ in general $W = Q.\Delta V$; ΔV = Potential difference through which charge Q moves.

Electric Field and Potential Due to Various Charge Distribution.

(1) **Point charge :** Electric field and potential at point P due to a point charge Q is

$$\boldsymbol{E} = \boldsymbol{k}\frac{\boldsymbol{Q}}{\boldsymbol{r^2}} \text{ or } \vec{E} = k\frac{Q}{r^2}\hat{r} \quad \left(k = \frac{1}{4\pi\varepsilon_0}\right), \quad V = k\frac{Q}{r}$$

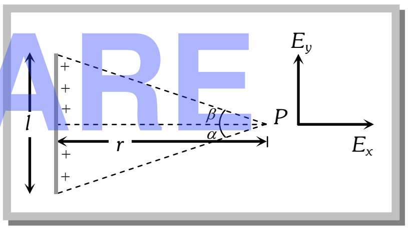


Note : ≅ Electric field intensity and electric potential due to a point charge q, at a distance $t_1 + t_2$ where t_1 is thickness of medium of dielectric constant K_1 and t_2 is thickness of medium of dielectric constant K_2 are :

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(t_1\sqrt{K_1} + t_2\sqrt{K_2})^2}; \qquad \boldsymbol{V} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(t_1\sqrt{K_1} + t_2\sqrt{K_2})}$$

(2) **Line charge**

(i) **Straight conductor :** Electric field and potential due to a charged straight conducting wire of length l and charge density λ



(a) **Electric field :** $E_x = \frac{k\lambda}{r}(\sin\alpha + \sin\beta)$ and $E_y = \frac{k\lambda}{r}(\cos\beta - \cos\alpha)$

If $\alpha = \beta$; $\quad E_x = \frac{2k\lambda}{r}\sin\alpha$ and $E_y = 0$

If $l \to \infty$ i.e. $\alpha = \beta = \frac{\pi}{2}$; $E_x = \frac{2k\lambda}{r}$ and $E_y = 0$ so $E_{net} = \frac{\lambda}{2\pi\varepsilon_0 r}$

If $\alpha = 0,\ \beta = \frac{\pi}{2}$; $\quad |E_x| = |E_y| = \frac{k\lambda}{r}$ so $E_{net} = \sqrt{E_x^2 + E_y^2} = \frac{\sqrt{2}\,k\lambda}{r}$

(b) **Potential :** $V = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left[\frac{\sqrt{r^2 + l^2} - 1}{\sqrt{r^2 + l^2} + 1}\right]$ for infinitely long conductor $V = \frac{-\lambda}{2\pi\varepsilon_0}\log_e r + c$