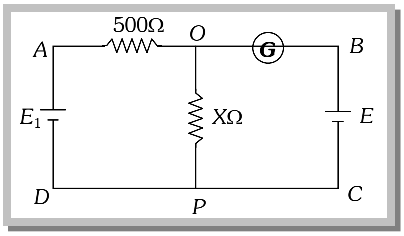
***Example*: 61** In the adjoining circuit, the battery E_1 has as emf of 12 *volt* and zero internal resistance, while the battery E has an emf of 2 *volt*. If the galvanometer reads zero, then the value of resistance X *ohm* is **[NCERT 1990]**



(a) 10

(b) 100

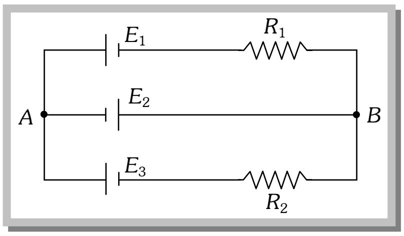
(c) 500

(d) 200

Solution : (b) For zero deflection in galvanometer the potential different across X should be $E = 2V$

In this condition $\frac{12X}{500 + X} = 2$ $\therefore X = 100\,\Omega$

***Example*: 62** In the circuit shown here $E_1 = E_2 = E_3 = 2V$ and $R_1 = R_2 = 4\,\Omega$. The current flowing between point A and B through battery E_2 is **[MP PET 2001]**



(a) Zero

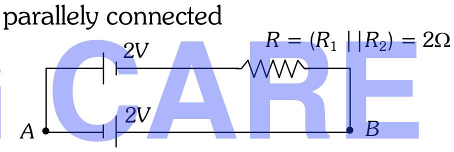
(b) 2 A from A to B

(c) 2 A from B to A

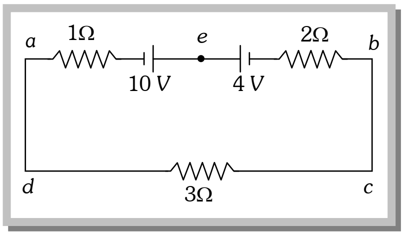
(d) None of these

Solution : (b) The equivalent circuit can be drawn as since E_1 & E_3 are parallely connected

So current $i = \frac{2+2}{2} = 2Amp$ from A to B.



***Example*: 63** The magnitude and direction of the current in the circuit shown will be **[CPMT 1986, 88]**



(a) $\frac{7}{3}A$ from a to b through e

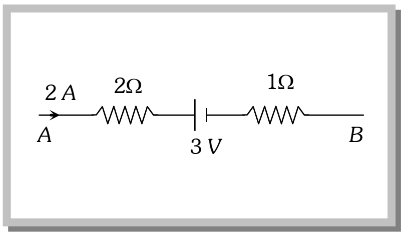
(b) $\frac{7}{3}A$ from b and a through e

(c) 1.0 A from b to a through e

(d) 1.0 A from a to b through e

Solution : (d) Current $i = \frac{10-4}{3+2+1} = 1A$ from a to b via e

***Example*: 64** Figure represents a part of the closed circuit. The potential difference between points A and B $(V_A - V_B)$ is



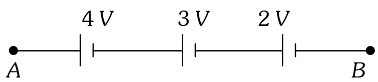
(a) + 9 V

(b) – 9 V

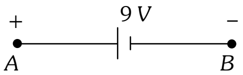
(c) + 3 V

(d) + 6 V

Solution : (a) The given part of a closed circuit can be redrawn as follows. It should be remember that product of current and resistance can be treated as an imaginary cell having emf = iR.

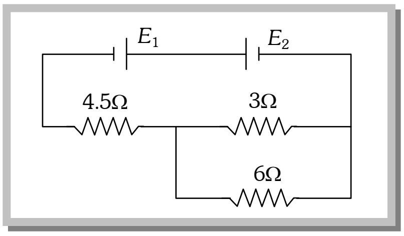


$\Rightarrow$



$\Rightarrow$ 7 Hence $V_A - V_B = +9\,V$

Example: 65 In the circuit shown below the cells E_1 and E_2 have emf's 4 *V* and 8 *V* and internal resistance 0.5 *ohm* and 1 *ohm* respectively. Then the potential difference across cell E_1 and E_2 will be



(a) 3.75 *V*, 7.5 *V*

(b) 4.25 *V*, 7.5 *V*

(c) 3.75 *V*, 3.5 *V*

(d) 4.25 *V*, 4.25 *V*

Solution : (b) In the given circuit diagram external resistance $R = \frac{3\times 6}{3+6} + 4.5 = 6.5\Omega$. Hence main current through the circuit $i = \frac{E_2 - E_1}{R + r_{eq}} = \frac{8-4}{6.5+0.5+0.5} = \frac{1}{2}\,amp.$

Cell 1 is charging so from it's emf equation $E_1 = V_1 - ir_1 \Rightarrow 4 = V_1 - \frac{1}{2}\times 0.5 \Rightarrow V_1 = 4.25\ volt$

Cell 2 is discharging so from it's emf equation $E_2 = V_2 + ir_2 \Rightarrow 8 = V_2 + \frac{1}{2}\times 1 \Rightarrow V_2 = 7.5\ volt$

Example: 66 A wire of length *L* and 3 identical cells of negligible internal resistances are connected in series. Due to this current, the temperature of the wire is raised by ΔT in time *t*. A number *N* of similar cells is now connected in series with a wire of the same material and cross-section but of length 2*L*. The temperature of wire is raised by same amount ΔT in the same time *t*. The value of *N* is **[IIT-JEE (Screening) 2001]**

(a) 4 (b) 6 (c) 8 (d) 9

Solution : (b) Heat $= mS\Delta T = i^2Rt$

Case I : Length $(L) \Rightarrow$ Resistance $= R$ and mass $= m$

Case II : Length $(2L) \Rightarrow$ Resistance $= 2R$ and mass $= 2m$

So $\frac{m_1S_1\Delta T_1}{m_2S_2\Delta T_2} = \frac{i_1^2R_1t_1}{i_2^2r_2t_2} \Rightarrow \frac{mS\Delta T}{2mS\Delta T} = \frac{i_1^2Rt}{i_2^2\,2Rt} \Rightarrow i_1 = i_2 \Rightarrow \frac{(3E)^2}{12} = \frac{(NE)^2}{2R} \Rightarrow N = 6$

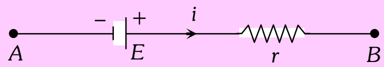
Tricky Example: 8

n identical cells, each of emf *E* and internal resistance *r*, are joined in series to form a closed circuit. The potential difference across any one cell is

(a) Zero (b) *E* (c) $\frac{E}{n}$ (d) $\left(\frac{n-1}{n}\right)E$

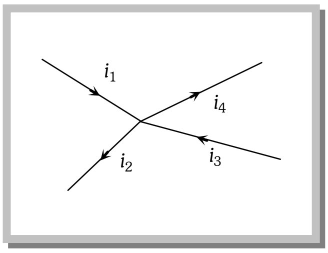
Solution: (a) Current in the circuit $i = \frac{nE}{nr} = \frac{E}{r}$

The equivalent circuit of one cell is shown in the figure. Potential difference across the cell $= V_A - V_B = -E + ir = -E + \frac{E}{r}.r = 0$



Kirchoff's Laws

(1) **Kirchoff's first law :** This law is also known as junction rule or current law (*KCL*). According to it the algebraic sum of currents meeting at a junction is zero *i.e.* $\sum i = 0$.



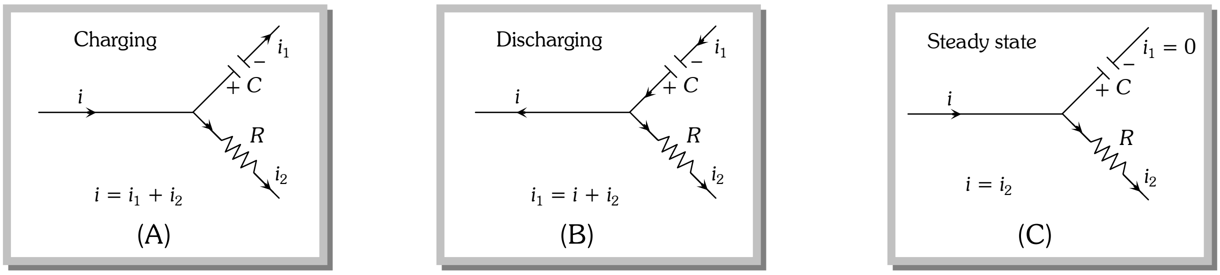
In a circuit, at any junction the sum of the currents entering the junction must equal the sum of the currents leaving the junction. $i_1 + i_3 = i_2 + i_4$

Here it is worthy to note that :

(i) If a current comes out to be negative, actual direction of current at the junction is opposite to that assumed, $i + i_1 + i_2 = 0$ can be satisfied only if at least one current is negative, *i.e.* leaving the junction.

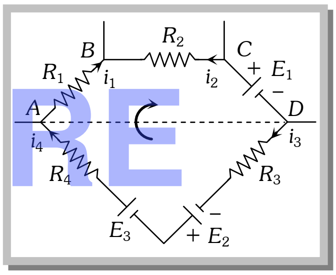
(ii) This law is simply a statement of "*conservation of charge*" as if current reaching a junction is not equal to the current leaving the junction, charge will not be conserved.

Note : ≅ This law is also applicable to a capacitor through the concept of displacement current treating the resistance of capacitor to be zero during charging or discharging and infinite in steady state as shown in figure.



(2) **Kirchoff's second law :** This law is also known as loop rule or voltage law (KVL) and according to it "the algebraic sum of the changes in potential in complete traversal of a mesh (closed loop) is zero", *i.e.* $\Sigma V = 0$

e.g. In the following closed loop.



$$-i_1R_1 + i_2R_2 - E_1 - i_3R_3 + E_2 + E_3 - i_4R_4 = 0$$

Here it is worthy to note that :

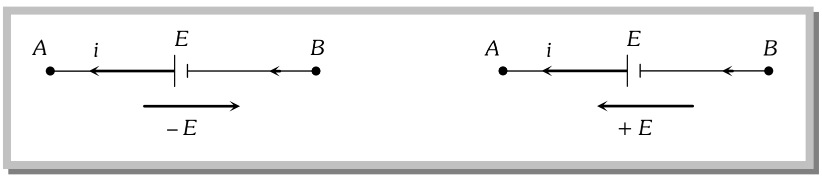
(i) This law represents "*conservation of energy*" as if the sum of potential changes around a closed loop is not zero, unlimited energy could be gained by repeatedly carrying a charge around a loop.

(ii) If there are n meshes in a circuit, the number of independent equations in accordance with loop rule will be $(n - 1)$.

(3) **Sign convention for the application of Kirchoff's law :** For the application of Kirchoff's laws following sign convention are to be considered

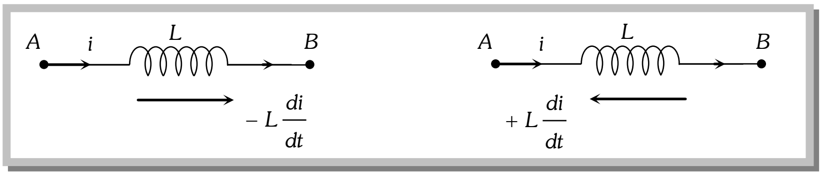
(i) The change in potential in traversing a resistance in the direction of current is $-iR$ while in the opposite direction $+iR$

(ii) The change in potential in traversing an emf source from negative to positive terminal is $+E$ while in the opposite direction $-E$ irrespective of the direction of current in the circuit.



(iii) The change in potential in traversing a capacitor from the negative terminal to the positive terminal is $+\frac{q}{C}$ while in opposite direction $-\frac{q}{C}$.

(iv) The change in voltage in traversing an inductor in the direction of current is $-L\frac{di}{dt}$ while in opposite direction it is $+L\frac{di}{dt}$.



(4) Guidelines to apply Kirchoff's law

(i) Starting from the positive terminal of the battery having highest emf, distribute current at various junctions in the circuit in accordance with '*junction rule*'. It is not always easy to correctly guess the direction of current, no problem if one assumes the wrong direction.

(ii) After assuming current in each branch, we pick a point and begin to walk (mentally) around a closed loop. As we traverse each resistor, capacitor, inductor or battery we must write down, the voltage change for that element according to the above sign convention.

(iii) By applying KVL we get one equation but in order to solve the circuit we require as many equations as there are unknowns. So we select the required number of loops and apply Kirchhoff's voltage law across each such loop.

(iv) After solving the set of simultaneous equations, we obtain the numerical values of the assumed currents. If any of these values come out to be negative, it indicates that particular current is in the opposite direction from the assumed one.

Note : ≅ The number of loops must be selected so that every element of the circuit must be included in at least one of the loops.

≅ While traversing through a capacitor or battery we do not consider the direction of current.

≅ While considering the voltage drop or gain across as inductor we always assume current to be in increasing function.

(5) Determination of equivalent resistance by Kirchoff's method : This method is useful when we are not able to identify any two resistances in series or in parallel. It is based on the two Kirchhoff's laws. The method may be described in the following guideline.

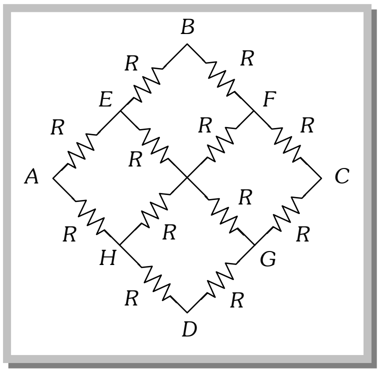
(i) Assume an imaginary battery of emf E connected between the two terminals across which we have to calculate the equivalent resistance.

(ii) Assume some value of current, say i, coming out of the battery and distribute it among each branch by applying Kirchhoff's current law.

(iii) Apply Kirchhoff's voltage law to formulate as many equations as there are unknowns. It should be noted that at least one of the equations must include the assumed battery.

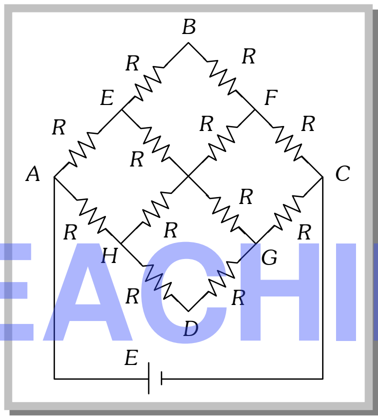
(iv) Solve the equations to determine $\frac{E}{i}$ ratio which is the equivalent resistance of the network.

e.g. Suppose in the following network of 12 identical resistances, equivalent resistance between point A and C is to be calculated.



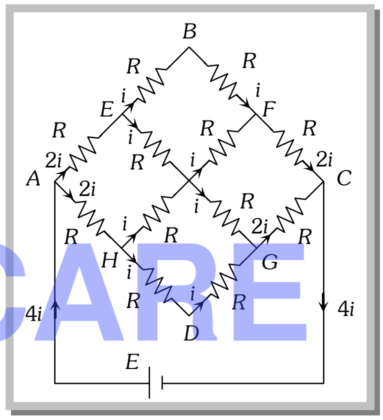
According to the above guidelines we can solve this problem as follows

Step (1)



An imaginary battery of emf E is assumed across the terminals A and C

Step (2)



The current in each branch is distributed by assuming $4i$ current coming out of the battery.

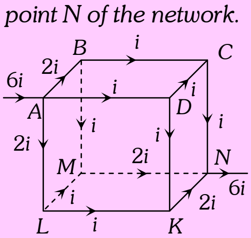
Step (3) Applying KVL along the loop including the nodes A, B, C and the battery E. Voltage equation is $-2iR - iR - iR - 2iR + E = 0$

Step (4) After solving the above equation, we get $6iR = E \Rightarrow$ equivalent resistance between A and C is $R = \frac{E}{4i} = \frac{6iR}{4i} = \frac{3}{2}R$

Concepts

- *Using Kirchoff's law while dividing the current having a junction through different arms of a network, it will be same through different arms of same resistance if the end points of these arms are equilocated w.r.t. exit point for current in network and will be different through different arms if the end point of these arms are not equilocated w.r.t. exit point for current of the network.*

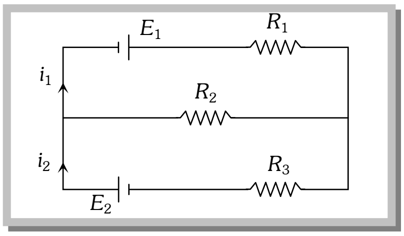
e.g. In the following figure the current going in arms AB, AD and AL will be same because the location of end points B, D and L of these arms are symmetrically located w.r.t. exit point N of the network.



Example

Example: 67 In the following circuit $E_1 = 4V$, $R_1 = 2\Omega$ **[MP PET 2003]**

$E_2 = 6V$, $R_2 = 2\Omega$ and $R_3 = 4\Omega$. The current i_1 is



(a) 1.6 *A*

(b) 1.8 *A*

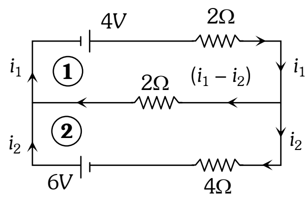
(c) 2.25 *A*

(d) 1 *A*

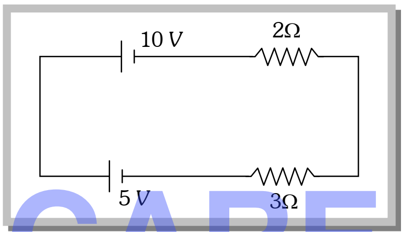
Solution : (b) For loop (1) $-2i_1 - 2(i_1 - i_2) + 4 = 0 \Rightarrow 2i_1 - i_2 = 2$ ...... (i)

For loop (2) $-4i_2 + 2(i_1 - i_2) + 6 = 0 \Rightarrow 3i_2 - i_1 = 3$ ...... (ii)

After solving equation (i) and (ii) we get $i_1 = 1.8A$ and $i_2 = 1.6A$



Example: 68 Determine the current in the following circuit



(a) 1 *A*

(b) 2.5 *A*

(c) 0.4 *A*

(d) 3 *A*

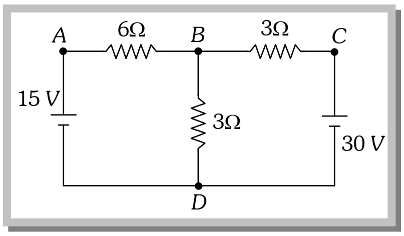
Solution : (a) Applying KVL in the given circuit we get $-2i + 10 - 5 - 3i = 0 \Rightarrow i = 1A$

Second method : Similar plates of the two batteries are connected together, so the net emf = 10 – 5 = 5*V*

Total resistance in the circuit = 2 + 3 = 5Ω

$\therefore \quad i = \frac{\Sigma V}{\Sigma R} = \frac{5}{5} = 1A$

Example: 69 In the circuit shown in figure, find the current through the branch *BD*



(a) 5 *A*

(b) 0 *A*

(c) 3 *A*

(d) 4 *A*

Solution : (a) The current in the circuit are assumed as shown in the fig.

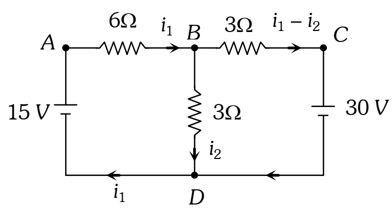
Applying KVL along the loop *ABDA*, we get

$-6i_1 - 3i_2 + 15 = 0$ or $2i_1 + i_2 = 5$ ...... (i)

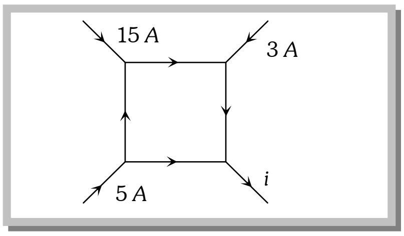
Applying KVL along the loop *BCDB*, we get

$-3(i_1 - i_2) - 30 + 3i_2 = 0$ or $-i_1 + 2i_2 = 10$ ...... (ii)

Solving equation (i) and (ii) for i_2, we get $i_2 = 5\ A$



Example: 70 The figure shows a network of currents. The magnitude of current is shown here. The current *i* will be **[MP PMT 1995]**



(a) 3 *A*

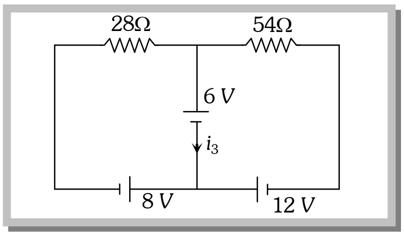
(b) 13 *A*

(c) 23 *A*

(d) – 3 *A*

Solution : (c) $i = 15 + 3 + 5 = 23A$

Example: 71 Consider the circuit shown in the figure. The current i_3 is equal to **[AMU 1995]**



(a) 5 *amp*

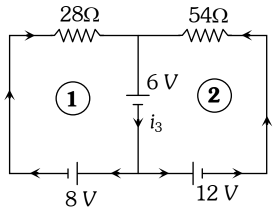
(b) 3 *amp*

(c) – 3 *amp*

(d) – 5/6 *amp*

Solution : (d) Suppose current through different paths of the circuit is as follows.

After applying *KVL* for loop (1) and loop (2)

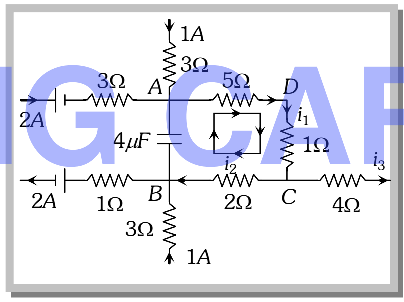


We get $28i_1 = -6-8 \Rightarrow i_1 = -\frac{1}{2}A$

and $54i_2 = -6-12 \Rightarrow i_2 = -\frac{1}{3}A$

Hence $i_3 = i_1 + i_2 = -\frac{5}{6}A$

Example: 72 A part of a circuit in steady state along with the current flowing in the branches, with value of each resistance is shown in figure. What will be the energy stored in the capacitor C_0 **[IIT-JEE 1986]**



(a) $6 \times 10^{-4}\ J$

(b) $8 \times 10^{-4}\ J$

(c) $16 \times 10^{-4}\ J$

(d) Zero

Solution : (b) Applying Kirchhoff's first law at junctions *A* and *B* respectively we have $2 + 1 - i_1 = 0$ *i.e.*, $i_1 = 3A$

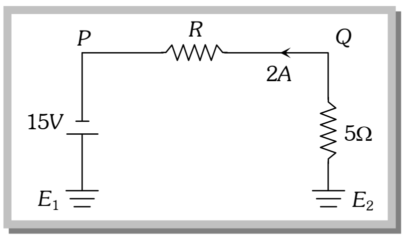
and $i_2 + 1 - 2 - 0 = 0$ *i.e.*, $i_2 = 1A$

Now applying Kirchhoff's second law to the mesh *ADCBA* treating capacitor as a seat of emf *V* in open circuit

$-3 \times 5 - 3 \times 1 - 1 \times 2 + V = 0$ *i.e.* $V(= V_A - V_B) = 20\ V$

So, energy stored in the capacitor $U = \frac{1}{2}CV^2 = \frac{1}{2}(4 \times 10^{-6}) \times (20)^2 = 8 \times 10^{-4}\ J$

Example: 73 In the following circuit the potential difference between *P* and *Q* is



(a) 15 *V*

(b) 10 *V*

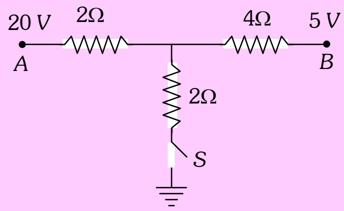
(c) 5 *V*

(d) 2.5 *V*

Solution : (c) By using KVL $-5 \times 2 - V_{PQ} + 15 = 0 \Rightarrow V_{PQ} = 5V$

Tricky Example: 9

As the switch S is closed in the circuit shown in figure, current passed through it is



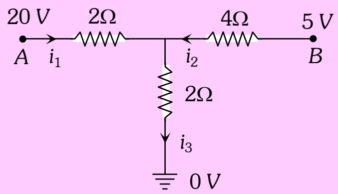
(a) 4.5 A (b) 6.0 A (c) 3.0 A (d) Zero

Solution : (a) Let V be the potential of the junction as shown in figure. Applying junction law, we have

or $\frac{20-V}{2}+\frac{5-V}{4}=\frac{V-0}{2}$ or $40-2V+5-V=2V$

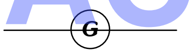
or $5V=45 \Rightarrow V=9V$

$\therefore \quad i_3=\frac{V}{2}=4.5A$



Different Measuring Instruments.

(1) **Galvanometer :** It is an instrument used to detect small current passing through it by showing deflection. Galvanometers are of different types *e.g.* moving coil galvanometer, moving magnet galvanometer, hot wire galvanometer. In dc circuit usually moving coil galvanometer are used.

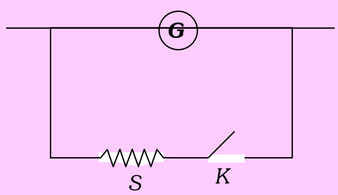
(i) **It's symbol :**  ; where G is the total internal resistance of the galvanometer.

(ii) **Principle :** In case of moving coil galvanometer deflection is directly proportional to the current that passes through it *i.e.* $i \propto$ □ .

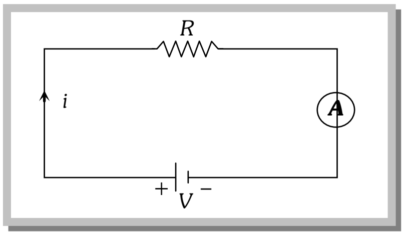
(iii) **Full scale deflection current :** The current required for full scale deflection in a galvanometer is called full scale deflection current and is represented by i_g.

(iv) **Shunt :** The small resistance connected in parallel to galvanometer coil, in order to control current flowing through the galvanometer is known as shunt.

Merits of shunt	Demerits of shunt
(a) To protect the galvanometer coil from burning (b) It can be used to convert any galvanometer into ammeter of desired range.	Shunt resistance decreases the sensitivity of galvanometer.



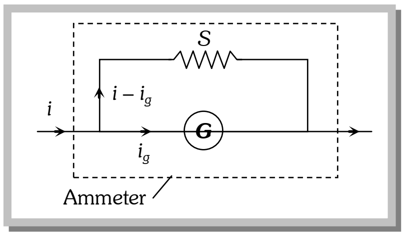
(2) **Ammeter :** It is a device used to measure current and is always connected in series with the 'element' through which current is to be measured.



(i) The reading of an ammeter is always lesser than actual current in the circuit.

(ii) Smaller the resistance of an ammeter more accurate will be its reading. An ammeter is said to be ideal if its resistance r is zero.

(iii) **Conversion of galvanometer into ammeter :** A galvanometer may be converted into an ammeter by connecting a low resistance (called shunt S) in parallel to the galvanometer G as shown in figure.



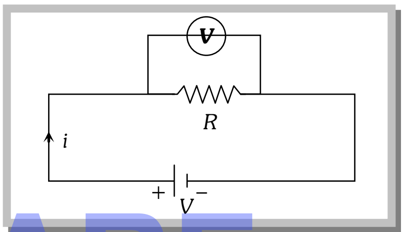
(a) Equivalent resistance of the combination $= \dfrac{GS}{G+S}$

(b) G and S are parallel to each other hence both will have equal potential difference *i.e.* $i_g G = (i - i_g)S$; which gives

Required shunt $\boldsymbol{S} = \dfrac{\boldsymbol{i_g}}{(\boldsymbol{i} - \boldsymbol{i_g})}\boldsymbol{G}$

(c) To pass nth part of main current (*i.e.* $i_g = \dfrac{i}{n}$) through the galvanometer, required shunt $\boldsymbol{S} = \dfrac{\boldsymbol{G}}{(\boldsymbol{n}-\boldsymbol{1})}$.

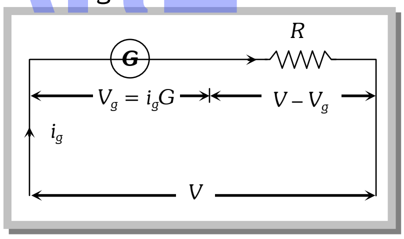
(3) **Voltmeter :** It is a device used to measure potential difference and is always put in parallel with the 'circuit element' across which potential difference is to be measured.



(i) The reading of a voltmeter is always lesser than true value.

(ii) Greater the resistance of voltmeter, more accurate will be its reading. A voltmeter is said to be ideal if its resistance is infinite, *i.e.*, it draws no current from the circuit element for its operation.

(iii) **Conversion of galvanometer into voltmeter :** A galvanometer may be converted into a voltmeter by connecting a large resistance R in series with the galvanometer as shown in the figure.



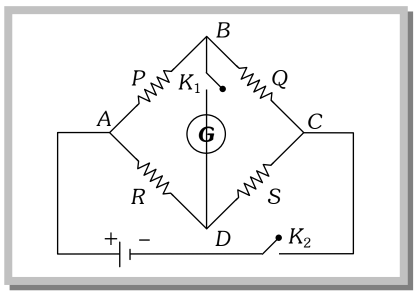
(a) Equivalent resistance of the combination $= G + R$

(b) According to ohm's law $V = i_g\,(G + R)$; which gives

Required series resistance $\boldsymbol{R} = \dfrac{\boldsymbol{V}}{\boldsymbol{i_g}} - \boldsymbol{G} = \left(\dfrac{\boldsymbol{V}}{\boldsymbol{V_g}} - \boldsymbol{1}\right)\boldsymbol{G}$

(c) If n^{th} part of applied voltage appeared across galvanometer (*i.e.* $V_g = \dfrac{V}{n}$) then required series resistance $\boldsymbol{R} = (\boldsymbol{n}-\boldsymbol{1})\,\boldsymbol{G}$.

(4) **Wheatstone bridge :** Wheatstone bridge is an arrangement of four resistance which can be used to measure one of them in terms of rest. Here arms AB and BC are called ratio arm and arms AC and BD are called conjugate arms



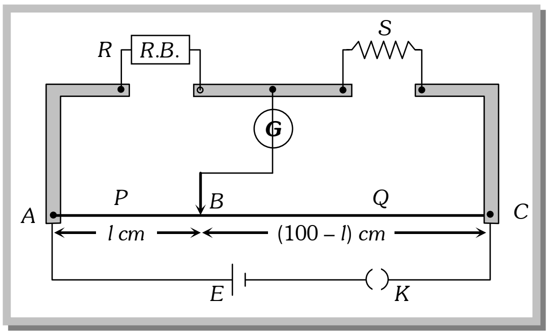
(i) **Balanced bridge :** The bridge is said to be balanced when deflection in galvanometer is zero *i.e.* no current flows through the galvanometer or in other words $V_B = V_D$. In the balanced condition $\dfrac{\boldsymbol{P}}{\boldsymbol{Q}} = \dfrac{\boldsymbol{R}}{\boldsymbol{S}}$, on mutually changing the position of cell and galvanometer this condition will not change.

(ii) **Unbalanced bridge :** If the bridge is not balanced current will flow from D to B if $V_D > V_B$ *i.e.* $(V_A - V_D) < (V_A - V_B)$ which gives $PS > RQ$.

(iii) **Applications of wheatstone bridge :** Meter bridge, post office box and Carey Foster bridge are instruments based on the principle of wheatstone bridge and are used to measure unknown resistance.

(5) **Meter bridge :** In case of meter bridge, the resistance wire *AC* is 100 *cm* long. Varying the position of tapping point *B*, bridge is balanced. If in balanced position of bridge *AB* = *l*, *BC* (100 – *l*) so that $\frac{Q}{P}=\frac{(100-l)}{l}$.

Also $\frac{P}{Q}=\frac{R}{S} \Rightarrow S=\frac{(100-l)}{l}R$



Concepts

- *Wheatstone bridge is most sensitive if all the arms of bridge have equal resistances i.e. P = Q = R = S*
- *If the temperature of the conductor placed in the right gap of metre bridge is increased, then the balancing length decreases and the jockey moves towards left.*
- *In Wheatstone bridge to avoid inductive effects the battery key should be pressed first and the galvanometer key afterwards.*
- *The measurement of resistance by Wheatstone bridge is not affected by the internal resistance of the cell.*

Example

Example: 74 The scale of a galvanometer of resistance 100 Ω contains 25 divisions. It gives a deflection of one division on passing a current of 4×10^{-4} *A*. The resistance in *ohms* to be added to it, so that it may become a voltmeter of range 2.5 *volt* is **[EAMCET 2003]**

(a) 100 (b) 150 (c) 250 (d) 300

Solution : (b) Current sensitivity of galvanometer = 4×10^{-4} *Amp/div*.

So full scale deflection current (i_g) = Current sensitivity × Total number of division = $4 \times 10^{-4} \times 25 = 10^{-2}$ *A*

To convert galvanometer in to voltmeter, resistance to be put in series is $R=\frac{V}{i_g}-G=\frac{2.5}{10^{-2}}-100=150\,\Omega$

Example: 75 A galvanometer, having a resistance of 50 Ω gives a full scale deflection for a current of 0.05 *A*. the length in meter of a resistance wire of area of cross-section 2.97×10^{-2} cm^2 that can be used to convert the galvanometer into an ammeter which can read a maximum of 5*A* current is : (Specific resistance of the wire = 5×10^{-7} Ω*m*) **[EAMCET 2003]**

(a) 9 (b) 6 (c) 3 (d) 1.5

Solution : (c) Given *G* = 50 Ω, i_g = 0.05 *Amp.*, *i* = 5*A*, $A = 2.97 \times 10^{-2}$ cm^2 and $\rho = 5 \times 10^{-7}$Ω-*m*

By using $\frac{i}{i_g}=1+\frac{G}{S} \Rightarrow S=\frac{G.i_g}{(i-i_g)} \Rightarrow \frac{\rho\, l}{A}=\frac{Gi_g}{(i-i_g)} \Rightarrow l=\frac{Gi_gA}{(i-i_g)\rho}$ on putting values *l* = 3 *m*.

Example: 76 100 *mA* current gives a full scale deflection in a galvanometer of resistance 2 Ω. The resistance connected with the galvanometer to convert it into a voltmeter of 5 *V* range is

[KCET 2002; UPSEAT 1998; MNR 1994 Similar to MP PMT 1999]

(a) 98 Ω (b) 52 Ω (c) 80 Ω (d) 48 Ω

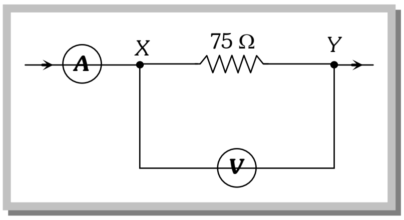
Solution : (d) $R=\frac{V}{I_g}-G=\frac{5}{100\times 10^{-3}}-2=50-2=48\,\Omega$.

Example: 77 A milliammeter of range 10 *mA* has a coil of resistance 1 Ω. To use it as voltmeter of range 10 *volt*, the resistance that must be connected in series with it will be **[KCET (Engg./Med.) 2001]**

(a) 999 Ω (b) 99 Ω (c) 1000 Ω (d) None of these

Solution : (a) By using $R=\frac{V}{ig}-G \Rightarrow R=\frac{10}{10\times 10^{-3}}-1=999\Omega$

Example: 78 In the following figure ammeter and voltmeter reads 2 *amp* and 120 *volt* respectively. Resistance of voltmeter is



(a) 100 Ω

(b) 200 Ω

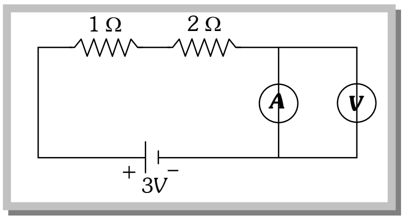
(c) 300 Ω

(d) 400 Ω

Solution : (c) Let resistance of voltmeter be R_V. Equivalent resistance between X and Y is $R_{XY}=\frac{75R_V}{75+R_V}$

Reading of voltmeter = potential difference across X and Y = $120=i\times R_{XY}=2\times\frac{75R_V}{75+R_V}\Rightarrow R_V=300\Omega$

Example: 79 In the circuit shown in figure, the voltmeter reading would be



(a) Zero

(b) 0.5 *volt*

(c) 1 *volt*

(d) 2 *volt*

Solution : (a) Ammeter has no resistance so there will be no potential difference across it, hence reading of voltmeter is zero.

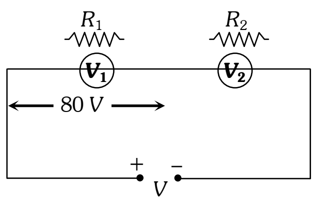
Example: 80 Voltmeters V_1 and V_2 are connected in series across a d.c. line. V_1reads 80 *V* and has a per *volt* resistance of 200 Ω, V_2 has a total resistance of 32 *k*Ω. The line voltage is **[MNR 1992]**

(a) 120 *V* (b) 160 *V* (c) 220 *V* (d) 240 *V*

Solution : (d) Resistance of voltmeter V_1 is $R_1=200\times 80=16000\,\Omega$ and resistance of voltmeter V_2 is $R_2=32000\,\Omega$

By using relation $V'=\left(\frac{R'}{R_{eq}}\right)V$; where V' = potential difference across any resistance R' in a series grouping.

So for voltmeter V_1 potential difference across it is



$$80=\left(\frac{R_1}{R_1+R_2}\right).V \Rightarrow V=240\ V$$

Example: 81 The resistance of 1 *A* ammeter is 0.018 Ω. To convert it into 10 *A* ammeter, the shunt resistance required will be **[MP PET 1982]**

(a) 0.18 Ω (b) 0.0018 Ω (c) 0.002 Ω (d) 0.12 Ω

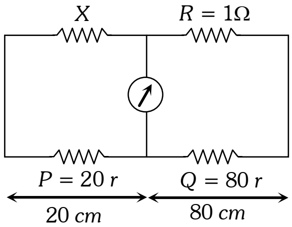
Solution : (c) By using $\frac{i}{i_g} = 1 + \frac{4}{S} \Rightarrow \frac{10}{1} = 1 + \frac{0.018}{S} \Rightarrow S = 0.002\ \Omega$

***Example*: 82** In meter bridge the balancing length from left and when standard resistance of 1 Ω is in right gas is found to be 20 *cm*. The value of unknown resistance is **[CBSE PMT 1999]**

(a) 0.25 Ω (b) 0.4 Ω (c) 0.5 Ω (d) 4 Ω

Solution: (a) The condition of wheatstone bridge gives $\frac{X}{R} = \frac{20r}{80r}$, *r*- resistance of wire per *cm*, *X*- unknown resistance

$\therefore \quad X = \frac{20}{80} \times R = \frac{1}{4} \times 1 = 0.25\,\Omega$



***Example*: 83** A galvanometer having a resistance of 8 Ω is shunted by a wire of resistance 2 Ω. If the total current is 1 *amp*, the part of it passing through the shunt will be **[CBSE PMT 1998]**

(a) 0.25 *amp* (b) 0.8 *amp* (c) 0.2 *amp* (d) 0.5 *amp*

Solution: (b) Fraction of current passing through the galvanometer

$$\frac{i_g}{i} = \frac{S}{S+G} \text{ or } \frac{i_g}{i} = \frac{2}{2+8} = 0.2$$

So fraction of current passing through the shunt

$$\frac{i_s}{i} = 1 - \frac{i_g}{i} = 1 - 0.2 = 0.8\,amp$$

***Example*: 84** A moving coil galvanometer is converted into an ammeter reading upto 0.03 *A* by connecting a shunt of resistance 4*r* across it and into an ammeter reading upto 0.06 *A* when a shunt of resistance *r* connected across it. What is the maximum current which can be through this galvanometer if no shunt is used **[MP PMT 1996]**

(a) 0.01 *A* (b) 0.02 *A* (c) 0.03 *A* (d) 0.04 *A*

Solution: (b) For ammeter, $S = \frac{i_g}{(i - i_g)}G \quad \Rightarrow i_g G = (i - i_g)S$

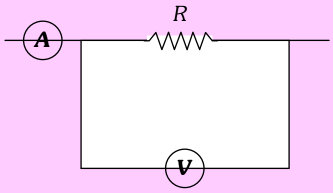
So $i_g G = (0.03 - i_g)4r$ ...... (i) and $i_g G = (0.06 - i_g)r$ ...... (ii)

Dividing equation (i) by (ii) $1 = \frac{(0.03 - i_g)4}{0.06 - i_g} \Rightarrow 0.06 - i_g = 0.12 - 4i_g$

$\Rightarrow 3i_g = 0.06 \Rightarrow i_g = 0.02\,A$

Tricky Example: 10

The ammeter *A* reads 2 *A* and the voltmeter *V* reads 20 *V*. The value of resistance *R* is **[JIPMER 1999]**



(a) Exactly 10 *ohm* (b) Less than 10 *ohm*

(c) More than 10 *ohm* (d) We cannot definitely say

Solution: (c) If current goes through the resistance R is i then $iR = 20$ *volt* $\Rightarrow R = \frac{20}{i}$. Since $i < 2A$ so $R > 10\Omega$.

Potentiometer.

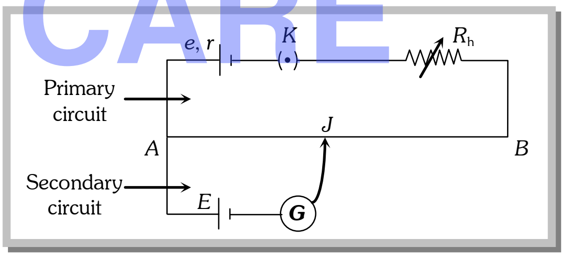
Potentiometer is a device mainly used to measure emf of a given cell and to compare emf's of cells. It is also used to measure internal resistance of a given cell.

(1) **Superiority of potentiometer over voltmeter** : An ordinary voltmeter cannot measure the emf accurately because it does draw some current to show the deflection. As per definition of emf, it is the potential difference when a cell is in open circuit or no current through the cell. Therefore voltmeter can only measure terminal voltage of a give n cell.

Potentiometer is based on no deflection method. When the potentiometer gives zero deflection, it does not draw any current from the cell or the circuit *i.e.* potentiometer is effectively an ideal instrument of infinite resistance for measuring the potential difference.

(2) **Circuit diagram :** Potentiometer consists of a long resistive wire AB of length L (about 6m to 10 m long) made up of mangnine or constantan. A battery of known voltage e and internal resistance r called supplier battery or driver cell. Connection of these two forms primary circuit.

One terminal of another cell (whose emf E is to be measured) is connected at one end of the main circuit and the other terminal at any point on the resistive wire through a galvanometer G. This forms the secondary circuit. Other details are as follows



J = Jockey

K = Key

R = Resistance of potentiometer wire,

ρ = Specific resistance of potentiometer wire.

R_h = Variable resistance which controls the current through the wire AB

(3) **Points to be remember**

(i) The specific resistance (ρ) of potentiometer wire must be high but its temperature coefficient of resistance (α) must be low.

(ii) All higher potential points (terminals) of primary and secondary circuits must be connected together at point A and all lower potential points must be connected to point B or jockey.

(iii) The value of known potential difference must be greater than the value of unknown potential difference to be measured.

(iv) The potential gradient must remain constant. For this the current in the primary circuit must remain constant and the jockey must not be slided in contact with the wire.

(v) The diameter of potentiometer wire must be uniform everywhere.

(4) **Potential gradient (x) :** Potential difference (or fall in potential) per unit length of wire is called potential gradient *i.e.* $x = \frac{V}{L}\frac{volt}{m}$ where $V = iR = \left(\frac{e}{R + R_h + r}\right).R$. So $\boldsymbol{x = \frac{V}{L} = \frac{iR}{L} = \frac{i\theta}{A} = \frac{e}{(R + R_h + r)}.\frac{R}{L}}$

(i) Potential gradient directly depends upon

(a) The resistance per unit length (R/L) of potentiometer wire.

(b) The radius of potentiometer wire (*i.e.* Area of cross-section)

(c) The specific resistance of the material of potentiometer wire (*i.e.* ρ)

(d) The current flowing through potentiometer wire (i)

(ii) x indirectly depends upon

(a) The emf of battery in the primary circuit (*i.e.* e)

(b) The resistance of rheostat in the primary circuit (*i.e.* R_h)

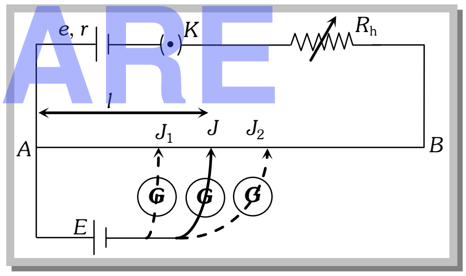
Note : ≅When potential difference V is constant then $\frac{x_1}{x_2} = \frac{L_2}{L_1}$

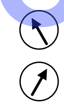
≅ Two different wire are connected in series to form a potentiometer wire then $\frac{x_1}{x_2} = \frac{R_1}{R_2} \cdot \frac{L_2}{L_1}$

≅ If the length of a potentiometer wire and potential difference across it's ends are kept constant and if it's diameter is changed from $d_1 \rightarrow d_2$ then potential gradient remains unchanged.

≅ The value of x does not change with any change effected in the secondary circuit.

(5) **Working :** Suppose jocky is made to touch a point J on wire then potential difference between A and J will be $V = xl$





At this length (l) two potential difference are obtained

(i) V due to battery e and

(ii) E due to unknown cell

If $V > E$ then current will flow in galvanometer circuit in one direction

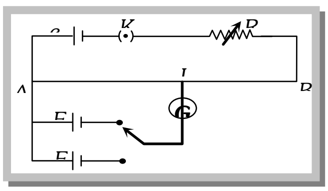
If $V < E$ then current will flow in galvanometer circuit in opposite direction

If $V = E$ then no current will flow in galvanometer circuit this condition to known as null deflection position, length l is known as balancing length.

In balanced condition $\boldsymbol{E = xl}$ or $\boldsymbol{E = xl = \frac{V}{L} l = \frac{iR}{L} l = \left(\frac{e}{R + R_h + r}\right) \cdot \frac{R}{L} \times l}$

Note : ≅ If V is constant then $L \propto l$ $\Rightarrow \frac{L_1}{L_2} = \frac{l_1}{l_2}$

(6) **Standardization of potentiometer :** The process of determining potential gradient experimentally is known as standardization of potentiometer.



Let the balancing length for the standard emf E_0 is l_0 then by the principle of potentiometer $E_0 = xl_0 \Rightarrow x = \frac{E_0}{l_0}$

(7) **Sensitivity of potentiometer :** A potentiometer is said to be more sensitive, if it measures a small potential difference more accurately.

(i) The sensitivity of potentiometer is assessed by its potential gradient. The sensitivity is inversely proportional to the potential gradient.

(ii) In order to increase the sensitivity of potentiometer

(a) The resistance in primary circuit will have to be decreased.

(b) The length of potentiometer wire will have to be increased so that the length may be measured more accuracy.

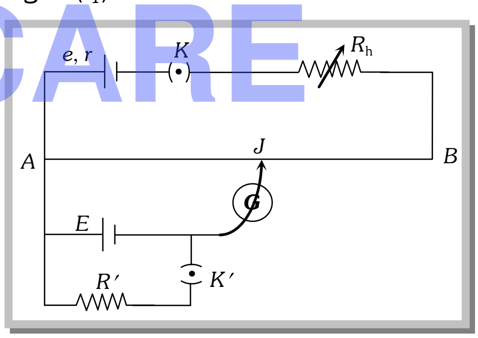
(8) **Difference between voltmeter and potentiometer**

	Voltmeter	Potentiometer
(i)	It's resistance is high but finite	Its resistance is high but infinite
(ii)	It draws some current from source of emf	It does not draw any current from the source of known emf
(iii)	The potential difference measured by it is lesser than the actual potential difference	The potential difference measured by it is equal to actual potential difference
(iv)	Its sensitivity is low	Its sensitivity is high
(v)	It is a versatile instrument	It measures only emf or potential difference
(vi)	It is based on deflection method	It is based on zero deflection method

Application of Potentiometer.

(1) **To determine the internal resistance of a primary cell**

(i) Initially in secondary circuit key K' remains open and balancing length (l_1) is obtained. Since cell E is in open circuit so it's emf balances on length l_1 i.e. $E = xl_1$ .... (i)

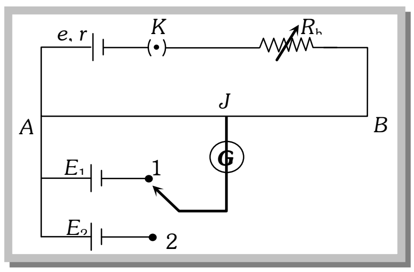


(ii) Now key K' is closed so cell E comes in closed circuit. If the process is repeated again then potential difference V balances on length l_2 i.e. $V = xl_2$ .... (ii)

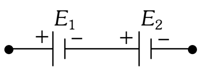
(iii) By using formula internal resistance $r = \left(\frac{E}{V} - 1\right).R'$

$$r = \left(\frac{l_1 - l_2}{l_2}\right).R'$$

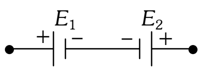
(2) **Comparison of emf's of two cell** : Let l_1 and l_2 be the balancing lengths with the cells E_1 and E_2 respectively then $E_1 = xl_1$ and $E_2 = xl_2 \Rightarrow \frac{E_1}{E_2} = \frac{l_1}{l_2}$



Note : ≅ Let $E_1 > E_2$ and both are connected in series. If balancing length is l_1 when cell assist each other and it is l_2 when they oppose each other as shown then :



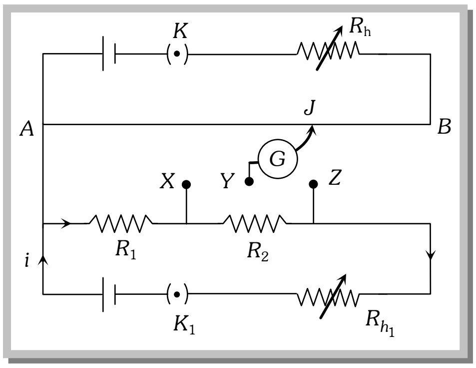
$(E_1 + E_2) = xl_1$



$(E_1 - E_2) = xl_2$

$$\Rightarrow \qquad \frac{E_1 + E_2}{E_1 - E_2} = \frac{l_1}{l_2} \qquad \text{or} \qquad \frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2}$$

(3) **Comparison of resistances :** Let the balancing length for resistance R_1 (when XY is connected) is l_1 and let balancing length for resistance $R_1 + R_2$ (when YZ is connected) is l_2.

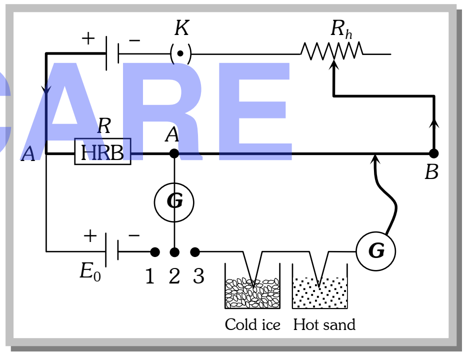


Then $\qquad iR_1 = xl_1$ and $i(R_1 + R_2) = xl_2$

$$\Rightarrow \qquad \frac{R_2}{R_1} = \frac{l_2 - l_1}{l_1}$$

(4) **To determine thermo emf**

(i) The value of thermo-emf in a thermocouple for ordinary temperature difference is very low (10^{-6} *volt*). For this the potential gradient x must be also very low (10^{-4} V/m). Hence a high resistance (R) is connected in series with the potentiometer wire in order to reduce current.



(ii) The potential difference across R must be equal to the emf of standard cell *i.e.* $iR = E_0$ $\therefore$ $i = \frac{E_0}{R}$

(iii) The small thermo emf produced in the thermocouple $e = xl$

(iv) $x = i\rho = \frac{iR}{L}$ $\quad \therefore \quad e = \frac{iRl}{L}$ $\quad$ where L = length of potentiometer wire, ρ = resistance per unit length, l = balancing length for e

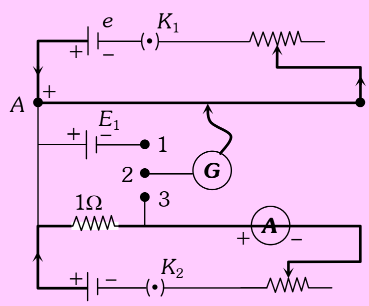
(5) **To calibrate ammeter and voltmeter**

Calibration of ammeter

(i) If p.d. across 1Ω resistance is measured by potentiometer, then current through this (indirectly measured) is thus known or if R is known then $i = V/R$ can be found.

(ii) **Circuit and method**

(a) Standardisation is required and per formed as already described earlier. ($x = E_0/l_0$)



(b) The current through R or 1Ω coil is measured by the connected ammeter and same is calculated by potentiometer by finding a balancing length as described blow.

Let i' current flows through 1Ω resistance giving p.d. as $V' = i'(1) = xl_1$ where l_1 is the balancing length. So error can be found as [i (measured by ammeter) $\Delta i' = i - i'\left[= xl_1 = \left(\frac{E_0}{l_0}\right) l_1 \right]$

Calibration of voltmeter

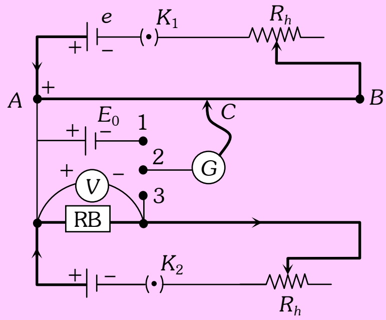
(i) Practical voltmeters are not ideal, because these do not have infinite resistance. The error of such practical voltmeter can be found by comparing the voltmeter reading with calculated value of p.d. by potentiometer.

(ii) **Circuit and procedure**

(a) **Standardisation :** If l_0 is balancing length for E_0 the emf of standard cell by connecting 1 and 2 of bi-directional key, then $x = E_0/l_0$.

(b) The balancing length l_1 for unknown potential difference V' is given by (by closing 2 and 3) $V' = xl_1 = (E_0 / l_0)l_1$.

If the voltmeter reading is V then the error will be $(V - V')$ which may be $+ve$, $-ve$ or zero.



Concepts

- *In case of zero deflection in the galvanometer current flows in the primary circuit of the potentiometer, not in the galvanometer circuit.*
- *A potentiometer can act as an ideal voltmeter.*

Example

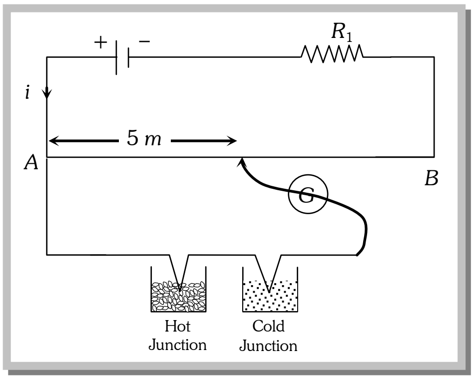
Example: 85 A battery with negligible internal resistance is connected with 10*m* long wire. A standard cell gets balanced on 600 *cm* length of this wire. On increasing the length of potentiometer wire by 2*m* then the null point will be displaced by

(a) 200 *cm* (b) 120 *cm* (c) 720 *cm* (d) 600 *cm*

Solution : (b) By using $\frac{L_1}{L_2} = \frac{l_1}{l_2} \Rightarrow \frac{10}{12} = \frac{600}{l_2} \Rightarrow l_2 = 720\,cm$.

Hence displacement = 720 – 600 = 120 *cm*

Example: 86 In the following circuit a 10 *m* long potentiometer wire with resistance 1.2 *ohm/m*, a resistance R_1 and an accumulator of emf 2 *V* are connected in series. When the emf of thermocouple is 2.4 *mV* then the deflection in galvanometer is zero. The current supplied by the accumulator will be



(a) $4 \times 10^{-4} A$ (b) $8 \times 10^{-4} A$ (c) $4 \times 10^{-3} A$ (d) $8 \times 10^{-3} A$

Solution : (a) $E = xl = i\rho l$ $\therefore\ i = \frac{E}{\rho l} = \frac{E}{\rho l} = \frac{2.4 \times 10^{-3}}{1.2 \times 5} = 4 \times 10^{-4} A$.

***Example*: 87** The resistivity of a potentiometer wire is 40×10^{-8} Ωm and its area of cross section is 8×10^{-6} m^2. If 0.2 *amp*. Current is flowing through the wire, the potential gradient will be **[MP PET/PMT 1998]**

(a) 10^{-2} *volt/m* (b) 10^{-1} *volt/m* (c) 3.2×10^{-2} *volt/m* (d) 1 *volt/m*

Solution : (a) Potential gradient $= \frac{V}{L} = \frac{iR}{L} = \frac{i\rho L}{AL} = \frac{i\rho}{A} = \frac{0.2 \times 40 \times 10^{-8}}{8 \times 10^{-6}} = 10^{-2} V/m$

***Example*: 88** A deniel cell is balanced on 125 *cm* length of a potentiometer wire. When the cell is short circuited with a 2 Ω resistance the balancing length obtained is 100 *cm*. Internal resistance of the cell will be **[RPMT 1998]**

(a) 1.5 Ω (b) 0.5 Ω (c) 1.25 Ω (d) 4/5 Ω

Solution: (b) By using $r = \frac{l_1 - l_2}{l_2} \times R'$ $\Rightarrow r = \frac{125 - 100}{100} \times 2 = \frac{1}{2} = 0.5\,\Omega$

***Example*: 89** A potentiometer wire of length 10 *m* and a resistance 30 Ω is connected in series with a battery of emf 2.5 *V* and internal resistance 5 Ω and an external resistance *R*. If the fall of potential along the potentiometer wire is 50 *μV/mm*, the value of *R* is (in Ω) **[KCET 1998]**

(a) 115 (b) 80 (c) 50 (d) 100

Solution : (a) By using $x = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L}$

$\Rightarrow \frac{50 \times 10^{-6}}{10^{-3}} = \frac{2.5}{(30 + R + 5)} \times \frac{30}{10} \Rightarrow R = 115$

***Example*: 90** A 2 *volt* battery, a 15 Ω resistor and a potentiometer of 100 *cm* length, all are connected in series. If the resistance of potentiometer wire is 5 Ω, then the potential gradient of the potentiometer wire is **[AIIMS 1982]**

(a) 0.005 *V/cm* (b) 0.05 *V/cm* (c) 0.02 *V/cm* (d) 0.2 *V/cm*

Solution : (a) By using $x = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L}$ $\Rightarrow x = \frac{2}{(5 + 15 + 0)} \times \frac{5}{1} = 0.5\,V/m = 0.005\,V/cm$

***Example*: 91** In an experiment to measure the internal resistance of a cell by potentiometer, it is found that the balance point is at a length of 2 *m* when the cell is shunted by a 5 Ω resistance; and is at a length of 3 *m* when the cell is shunted by a 10 Ω resistance. The internal resistance of the cell is, then **[Haryana CEE 1996]**

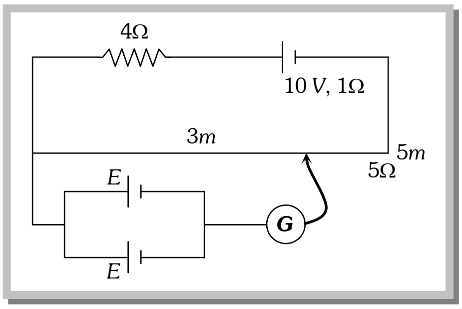
(a) 1.5 Ω (b) 10 Ω (c) 15 Ω (d) 1 Ω

Solution : (b) By using $r = \left(\frac{l_1 - l_2}{l_2}\right) R'$ $\Rightarrow r = \left(\frac{l_1 - 2}{2}\right) \times 5$ …… (i)

and $r = \left(\frac{l_1 - 3}{3}\right) \times 10$ ……. (ii)

On solving (i) and (ii) $r = 10\,\Omega$

***Example*: 92** A resistance of 4 Ω and a wire of length 5 *metres* and resistance 5 Ω are joined in series and connected to a cell of emf 10 *V* and internal resistance 1 Ω. A parallel combination of two identical cells is balanced across 300 *cm* of the wire. The emf *E* of each cell is **[RPET 2001; MP PMT 1997]**



(a) 1.5 *V* (b) 3.0 *V* (c) 0.67 *V* (d) 1.33 *V*

Solution : (b) By using $E_{eq} = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L} \times l \Rightarrow E = \frac{10}{(5+4+1)} \times \frac{5}{5} \times 3 \Rightarrow E = 3\ volt$

***Example*: 93** A potentiometer has uniform potential gradient across it. Two cells connected in series (i) to support each other and (ii) to oppose each other are balanced over 6 *m* and 2 *m* respectively on the potentiometer wire. The emf's of the cells are in the ratio of **[MP PMT 2002; RPMT 2000]**

(a) 1 : 2 (b) 1 : 1 (c) 3 : 1 (d) 2 : 1

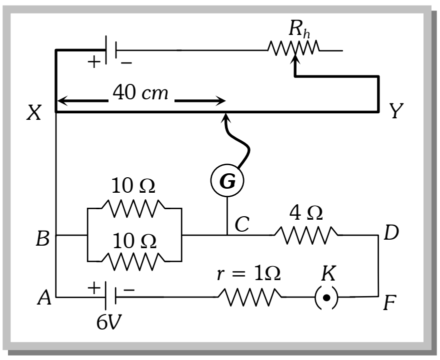
Solution : (d) If suppose emf's of the cells are E_1 and E_2 respectively then

$E_1 + E_2 = x \times 6$ .... (i) [x = potential gradient)]

and $E_1 - E_2 = x \times 2$ ....(ii)

$\Rightarrow \frac{E_1 + E_2}{E_1 - E_2} = \frac{3}{1} \Rightarrow \frac{E_1}{E_2} = \frac{2}{1}$

***Example*: 94** In the following circuit the potential difference between the points *B* and *C* is balanced against 40 *cm* length of potentiometer wire. In order to balance the potential difference between the points *C* and *D*, where should jockey be pressed



(a) 32 *cm* (b) 16 *cm* (c) 8 *cm* (d) 4 *cm*

Solution : (a) $\frac{1}{R} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$ or $R_1 = 5\ \Omega$

$R_2 = 4\Omega,\ l_1 = 40\ cm,\ l_2 = ?$

$l_2 = l_1 \frac{R_2}{R_1}$ or $l_2 = \frac{40 \times 4}{5} = 32\,cm$

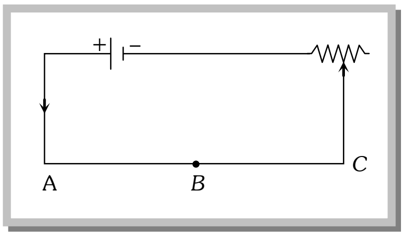
***Example*: 95** In the following circuit diagram fig. the lengths of the wires *AB* and *BC* are same but the radius of *AB* is three times that of *BC*. The ratio of potential gradients at *AB* and *BC* will be

(a) 1 : 9

(b) 9 : 1

(c) 3 : 1

(d) 1 : 3



Solution : (a) $x \propto R_p \propto \frac{1}{r^2} \Rightarrow \frac{x_1}{x_2} = \frac{r_2^2}{r_1^2} = \left(\frac{r}{3r}\right)^2 = \frac{1}{9}$

***Example*: 96** With a certain cell the balance point is obtained at 0.60 *m* from one end of the potentiometer. With another cell whose emf differs from that of the first by 0.1 *V*, the balance point is obtained at 0.55 *m*. Then, the two emf's are

(a) 1.2 *V*, 1.1 *V* (b) 1.2 *V*, 1.3 *V* (c) – 1.1 *V*, – 1.0 *V* (d) None of the above

Solution : (a) $E_1 = x\,(0.6)$ and $E_2 = E_1 - 0.1 = x\,(0.55) \Rightarrow \frac{E_1}{E_1 - 0.1} = \frac{0.6}{0.55}$

or $55\,E_1 = 60\,E_1 - 6 \Rightarrow E_1 = \frac{6}{5} = 1.2\,V$ thus $E_2 = 1.1\,V$

Tricky Example: 11

A cell of internal resistance 1.5Ω and of emf 1.5 *volt* balances 500 *cm* on a potentiometer wire. If a wire of 15Ω is connected between the balance point and the cell, then the balance point will shift

[MP PMT 1985]

(a) To zero (b) By 500 *cm*

(c) By 750 *cm* (d) None of the above

Solution : (d) In balance condition no current flows in the galvanometer circuit. Hence there will be no shift in balance point after connecting a resistance between balance point and cell.