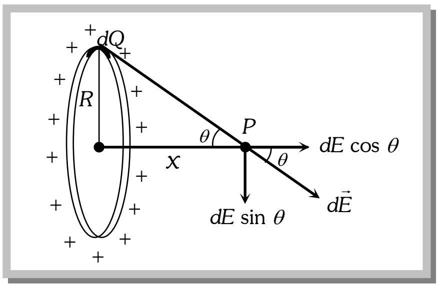
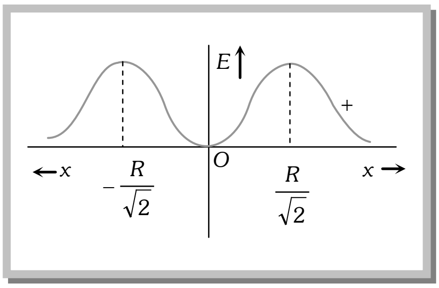
(ii) **Charged circular ring :** Suppose we have a charged circular ring of radius R and charge Q. On it's axis electric field and potential is to be determined, at a point 'x' away from the centre of the ring.





(a) **Electric field :** Consider an element carrying charge dQ. It's electric field $dE = \frac{KdQ}{(R^2 + x^2)}$ directed as shown. It's component along the axis is $dE\cos\theta$ and perpendicular to the axis is $dE\sin\theta$. By symmetry $\int dE\sin\theta = 0$, hence $E = \int dE\cos\theta = \int \frac{kdQ}{(R^2+x^2)} \cdot \frac{x}{(R^2+x^2)^{1/2}}$

$$\boldsymbol{E} = \frac{\boldsymbol{kQx}}{(\boldsymbol{R^2} + \boldsymbol{x^2})^{3/2}}$$ directed away from the centre if Q is positive

(b) **Potential :** $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{\sqrt{x^2 + R^2}}$

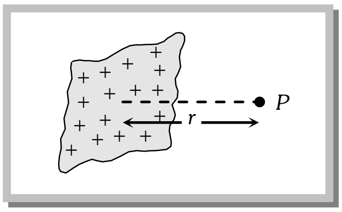
Note : ☐ At centre $x = 0$ so $E_{centre} = 0$ and $V_{centre} = \frac{kQ}{R}$

☐ At a point on the axis such that $x >> R$ $E = \frac{kQ}{x^2}$ and $V = \frac{kQ}{x}$

☐ At a point on the axis if $x = \pm\frac{R}{\sqrt{2}}$, $E_{max} = \frac{Q}{6\sqrt{3}\pi\varepsilon_0 a^2}$

(3) **Surface charge :**

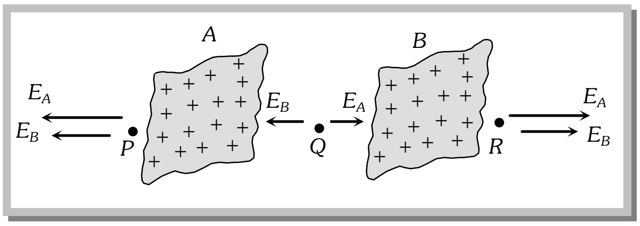
(i) **Infinite sheet of charge :** Electric field and potential at a point P as shown



$$E = \frac{\sigma}{2\varepsilon_0} \quad (E \propto r^o)$$

and $$V = -\frac{\sigma r}{2\varepsilon_0} + C$$

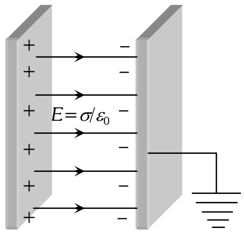
(ii) **Electric field due to two parallel plane sheet of charge :** Consider two large, uniformly charged parallel. Plates A and B, having surface charge densities are σ_A and σ_B respectively. Suppose net electric field at points P, Q and R is to be calculated.



At P, $E_P = (E_A + E_B) = \frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$

At Q, $E_Q = (E_A - E_B) = \frac{1}{2\varepsilon_0}(\sigma_A - \sigma_B)$; At R, $E_R = -(E_A + E_B) = -\frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$

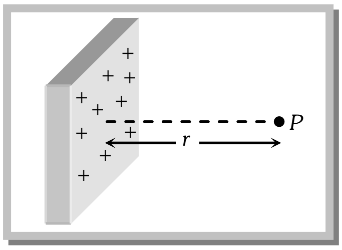
Note : ☐ If $\sigma_A = +\sigma$ and $\sigma_B = -\sigma$ then $E_p = 0, E_Q = \dfrac{\sigma}{\varepsilon_0}, E_R = 0$. Thus in case of two infinite plane sheets of charges having equal and opposite surface charge densities, the field is non-zero only in the space between the two sheets and is independent of the distance between them *i.e.*, field is uniform in this region. It should be noted that this result will hold good for finite plane sheet also, if they are held at a distance much smaller then the dimensions of sheets *i.e.*, parallel plate capacitor.



(iii) **Conducting sheet of charge :**

$$E = \frac{\sigma}{\varepsilon_0}$$

$$V = -\frac{\sigma r}{\varepsilon_0} + C$$



(iv) **Charged conducting sphere :** If charge on a conducting sphere of radius R is Q as shown in figure then electric field and potential in different situation are –

(a) **Out side the sphere :** P is a point outside the sphere at a distance r from the centre at which electric field and potential is to be determined.

Electric field at P

$$E_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2} \quad \text{and} \quad V_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r} = \frac{\sigma R^2}{\varepsilon_0 r} \quad \begin{cases} Q = \sigma \times A \\ \quad = \sigma \times 4\pi R^2 \end{cases}$$

(b) **At the surface of sphere :** At surface $r = R$

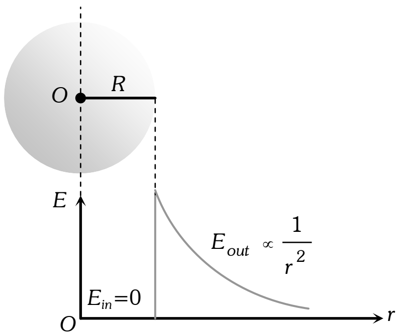
So, $$E_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^2} = \frac{\sigma}{\varepsilon_0} \quad \text{and} \quad V_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{\sigma R}{\varepsilon_0}$$

(c) **Inside the sphere :** Inside the conducting charge sphere electric field is zero and potential remains constant every where and equals to the potential at the surface.

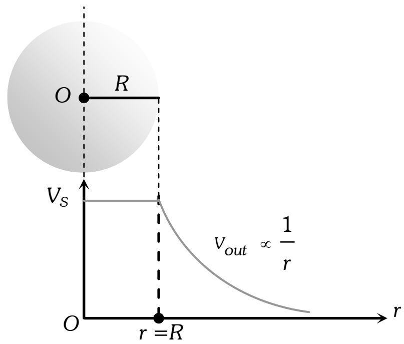
$$E_{in} = 0 \text{ and } V_{in} = \text{constant} = V_s$$

Note : ≅Graphical variation of electric field and potential of a charged spherical conductor with distance

E - *r* graph



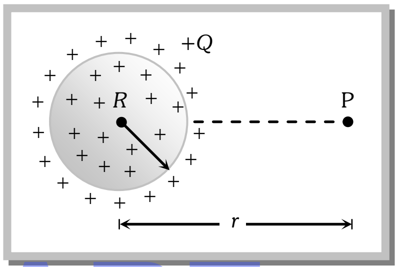
V - *r* graph



(4) **Volume charge (charged non-conducting sphere) :**

Charge given to a non conducting spheres spreads uniformly throughout it's volume.

(i) **Outside the sphere at *P***



$$E_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2} \text{ and } V_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r} \text{ by using } \rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$E_{out} = \frac{\rho R^3}{3\varepsilon_0 r^2} \text{ and } V_{out} = \frac{\rho R^3}{3\varepsilon_0 r}$$

(ii) **At the surface of sphere :** At surface $r = R$

$$E_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^2} = \frac{\rho R}{3\varepsilon_0} \quad \text{and} \quad V_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{\rho R^2}{3\varepsilon_0}$$

(iii) **Inside the sphere :** At a distance *r* from the centre

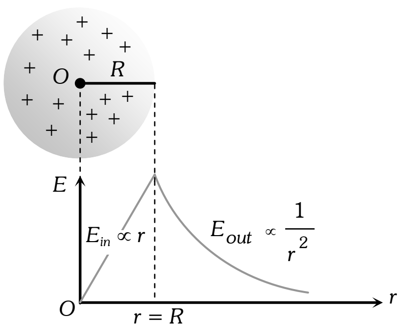
$$E_{in} = \frac{1}{4\pi\varepsilon_0}.\frac{Qr}{R^3} = \frac{\rho r}{3\varepsilon_0} \ \{E_{in} \propto r\} \quad \text{and} \quad V_{in} = \frac{1}{4\pi\varepsilon_0}\frac{Q[3R^2 - r^2]}{2R^3} = \frac{\rho(3R^2 - r^2)}{6\varepsilon_0}$$

Note : ≅ At centre $r = 0$ So, $V_{centre} = \frac{3}{2} \times \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{3}{2} V_s$ *i.e.*,

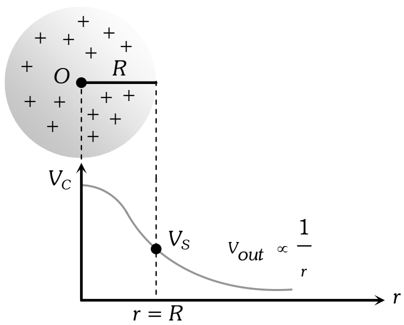
$\boldsymbol{V_{centre} > V_{surface} > V_{out}}$

≅ Graphical variation of electric field and potential with distance

E-*r* graph



V-*r* graph



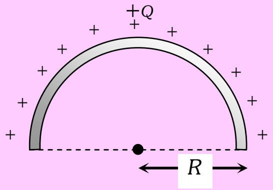
(5) **Electric field and potential in some other cases**

(i) **Uniformly charged semicircular ring :** $\lambda = \dfrac{\text{charge}}{\text{length}}$

At centre :

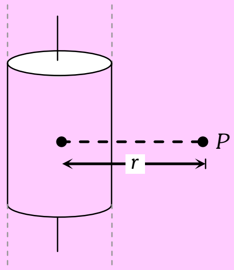
$$E = \frac{2K\lambda}{R} = \frac{Q}{2\pi^2\varepsilon_0 R^2}$$

$$V = \frac{KQ}{R} = \frac{Q}{4\pi\varepsilon_0 R}$$

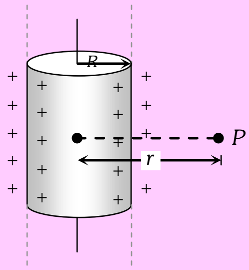


(iii) **Charged cylinder of infinite length**

(a) Conducting



(b) Non-conducting

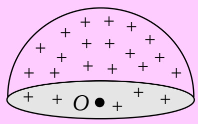


For both type of cylindrical charge distribution $E_{out} = \dfrac{\lambda}{2\pi\varepsilon_0 r}$, and $E_{suface} = \dfrac{\lambda}{2\pi\varepsilon_0 R}$ but for conducting $E_{in} = 0$ and for non-conducting $E_{in} = \dfrac{\lambda r}{2\pi\varepsilon_0 R^2}$. (we can also write formulae in form of ρ *i.e.*, $E_{out} = \dfrac{\rho R^2}{2\varepsilon_0 r}$ *etc.*)

(ii) **Hemispherical charged body :**

At centre O, $E = \dfrac{\sigma}{4\varepsilon_0}$

$$V = \frac{\sigma R}{2\varepsilon_0}$$

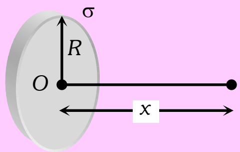


(iv) **Uniformly charged disc**

At a distance x from centre O on it's axis

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2 + R^2} - x\right]$$

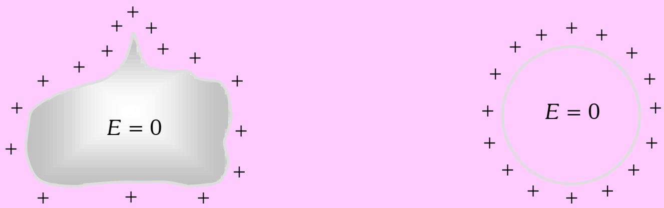


Note : ≅ Total charge on disc $Q = \sigma\pi R^2$

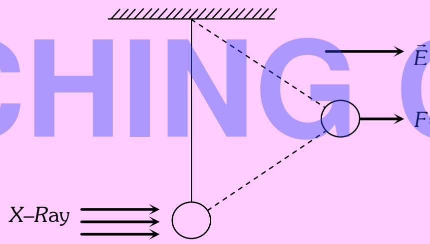
≅ If $x \to 0$, $E \simeq \dfrac{\sigma}{2\varepsilon_0}$ *i.e.* for points situated near the disc, it behaves as an infinite sheet of charge.

Concepts

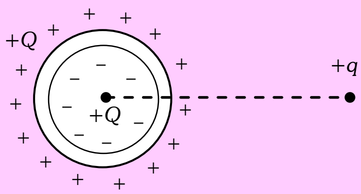
- *No point charge produces electric field at it's own position.*
- *Since charge given to a conductor resides on it's surface hence electric field inside it is zero.*



- *The electric field on the surface of a conductor is directly proportional to the surface charge density at that point i.e,* $E \propto \sigma$
- *Two charged spheres having radii* r_1 *and* r_2 *charge densities* σ_1 *and* σ_2 *respectively, then the ratio of electric field on their surfaces will be* $\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{r_2^2}{r_1^2}$ $\left\{ \sigma = \frac{Q}{4\pi r^2} \right.$
- *In air if intensity of electric field exceeds the value* $3 \times 10^6 N/C$ *air ionizes.*
- *A small ball is suspended in a uniform electric field with the help of an insulated thread. If a high energy x–ray bean falls on the ball, x-rays knock out electrons from the ball so the ball is positively charged and therefore the ball is deflected in the direction of electric field.*



- *Electric field is always directed from higher potential to lower potential.*
- *A positive charge if left free in electric field always moves from higher potential to lower potential while a negative charge moves from lower potential to higher potential.*
- *The practical zero of electric potential is taken as the potential of earth and theoretical zero is taken at infinity.*
- *An electric potential exists at a point in a region where the electric field is zero and it's vice versa.*
- *A point charge +Q lying inside a closed conducting shell does not exert force another point charge q placed outside the shell as shown in figure*



Actually the point charge +Q is unable to exert force on the charge +q because it can not produce electric field at the position of +q. All the field lines emerging from the point charge +Q terminate inside as these lines cannot penetrate the conducting medium (properties of lines of force).

The charge q however experiences a force not because of charge +Q but due to charge induced on the outer surface of the shell.

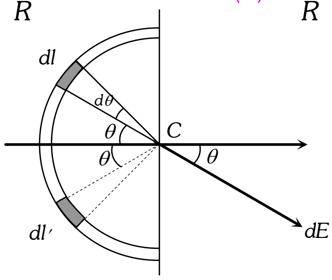
Examples based on electric field and electric potential

Example: 21 A half ring of radius R has a charge of λ per unit length. The electric field at the centre is $\left(k = \dfrac{1}{4\pi\varepsilon_0}\right)$

[CPMT 2000; CBSE PMT 2000; REE 1999]

(a) Zero (b) $\dfrac{k\lambda}{R}$ (c) $\dfrac{2k\lambda}{R}$ (d) $\dfrac{k\pi\lambda}{R}$

Solution: (c) $dl = Rd\theta$



Charge on $dl = \lambda R d\theta$.

Field at C due to $dl = k\dfrac{\lambda R d\theta.}{R^2} = dE$

We need to consider only the component $dE\cos\theta$, as the component $dE\sin\theta$ will cancel out because of the field at C due to the symmetrical element dl',

The total field at C is $= 2\int_0^{\pi/2} dE\cos\theta = 2\dfrac{k\lambda}{R}\int_0^{\pi/2}\cos\theta d\theta = 2k\dfrac{\lambda}{R}$ $\left\{= \dfrac{Q}{2\pi\varepsilon_0 R^2}\right\}$

Example: 22 What is the magnitude of a point charge due to which the electric field 30 cm away has the magnitude 2 *newton/coulomb* $[1/4\pi\varepsilon_0 = 9\times10^9\ Nm^2]$ **[MP PMT 1996]**

(a) 2×10^{-11} *coulomb* (b) 3×10^{-11} *coulomb* (c) 5×10^{-11} *coulomb* (d) 9×10^{-11} *coulomb*

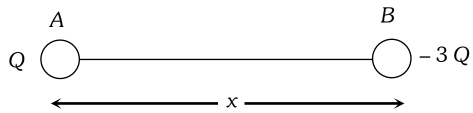
Solution: (a) By using $E = \dfrac{1}{4\pi\varepsilon_0}.\dfrac{Q}{r^2}$; $2 = 9\times10^9\times\dfrac{Q}{(30\times10^{-2})^2} \Rightarrow Q = 2\times10^{-11}C$

Example: 23 Two point charges Q and $-3Q$ are placed at some distance apart. If the electric field at the location of Q is E, then at the locality of $-3Q$, it is

(a) $-E$ (b) $E/3$ (c) $-3E$ (d) $-E/3$

Solution: (b) Let the charge Q and $-3Q$ be placed respectively at A and B at a distance x

Now we will determine the magnitude and direction to the field produced by charge $-3Q$ at A, this is E as mentioned in the Example.



$\therefore E = \dfrac{3Q}{x^2}$ (along AB directed towards negative charge)

Now field at location of $-3Q$ i.e. field at B due to charge Q will be $E' = \dfrac{Q}{x^2} = \dfrac{E}{3}$ (along AB directed away from positive charge)

Example: 24 Two charged spheres of radius R_1 and R_2 respectively are charged and joined by a wire. The ratio of electric field of the spheres is **[CPMT 1999 Similar to CBSE 2003]**

(a) $\dfrac{R_1}{R_2}$ (b) $\dfrac{R_2}{R_1}$ (c) $\dfrac{R_1^2}{R_2^2}$ (d) $\dfrac{R_2^2}{R_1^2}$

Solution: (b) After connection their potential becomes equal *i.e.*, $k.\dfrac{Q_1}{R_1} = \dfrac{k.Q_2}{R_2}$; $\Rightarrow \dfrac{Q_1}{Q_2} = \dfrac{R_1}{R_2}$

Ratio of electric field $\frac{E_1}{E_2} = \frac{Q_1}{Q_2} \times \left(\frac{R_2}{R_1}\right)^2 = \frac{R_2}{R_1}$.

Example: 25 The number of electrons to be put on a spherical conductor of radius 0.1m to produce an electric field of 0.036 N/C just above its surface is **[MNR 1994]**

(a) 2.7×10^5 (b) 2.6×10^5 (c) 2.5×10^5 (d) 2.4×10^5

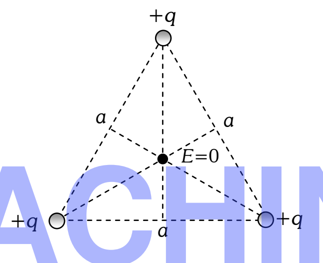
Solution: (c) By using $E = k\frac{Q}{R^2}$, where R = radius of sphere so $0.036 = 9 \times 10^9 \times \frac{ne}{(0.1)^2} \Rightarrow n = 2.5 \times 10^5$

Example: 26 Eight equal charges each $+Q$ are kept at the corners of a cube. Net electric field at the centre will be $\left(k = \frac{1}{4\pi\varepsilon_0}\right)$

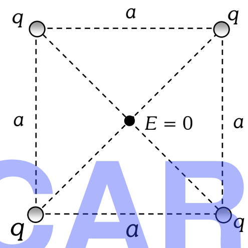
(a) $\frac{kQ}{r^2}$ (b) $\frac{8kQ}{r^2}$ (c) $\frac{2kQ}{r^2}$ (d) Zero

Solution: (d) Due to the symmetry of charge. Net Electric field at centre is zero.

Note : ❑

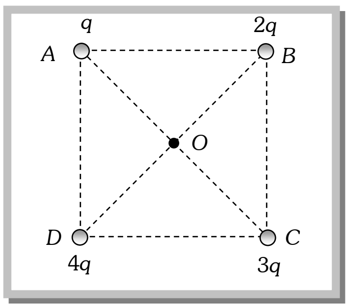


Equilateral triangle



Square

Example: 27 q, $2q$, $3q$ and $4q$ charges are placed at the four corners A, B, C and D of a square. The field at the centre O of the square has the direction along. **[CPMT 1989]**



(a) AB (b) CB (c) AC (d) BD

Solution: (b) By making the direction of electric field due to all charges at centre. Net electric field has the direction along CB

Example: 28 Equal charges Q are placed at the vertices A and B of an equilateral triangle ABC of side a. The magnitude of electric field at the point A is

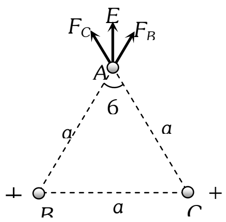
(a) $\frac{Q}{4\pi\varepsilon_0 a^2}$ (b) $\frac{\sqrt{2}Q}{4\pi\varepsilon_0 a^2}$ (c) $\frac{\sqrt{3}Q}{4\pi\varepsilon_0 a^2}$ (d) $\frac{Q}{2\pi\varepsilon_0 a^2}$

Solution: (c) As shown in figure Net electric field at A

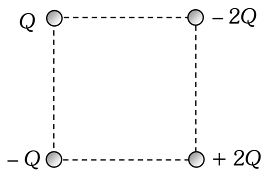
$$E = \sqrt{E_B^2 + E_C^2 + 2E_B E_C \cos 60}$$

$$E_B = E_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{a^2}$$

So, $E = \dfrac{\sqrt{3}Q}{4\pi\varepsilon_0 a^2}$



Example: 29 Four charges are placed on corners of a square as shown in figure having side of 5 cm. If Q is one micro coulomb, then electric field intensity at centre will be **[RPET 1999]**



(a) $1.02 \times 10^7 N/C$ upwards

(b) $2.04 \times 10^7 N/C$ downwards

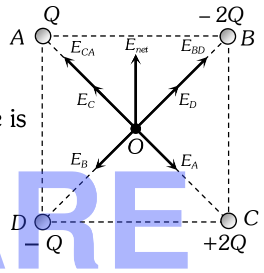
(c) $2.04 \times 10^7 N/C$ upwards

(d) $1.02 \times 10^7 N/C$ downwards

Solution: (a) $|E_C| > |E_A|$ so resultant of E_C & E_A is $E_{CA} = E_C - E_A$ directed toward Q

Also $|E_B| > |E_D|$ so resultant of E_B and E_D i.e.

$E_{BD} = E_B - E_D$ directed toward $-2Q$ charge hence Net electric field at centre is



$E = \sqrt{(E_{CA})^2 + (E_{BD})^2}$ .... (i)

By proper calculations $|E_A| = 9 \times 10^9 \times \dfrac{10^{-6}}{\left(\dfrac{5}{\sqrt{2}} \times 10^{-2}\right)^2} = 0.72 \times 10^7 N/C$

$|E_B| = 9 \times 10^9 \times \dfrac{2 \times 10^{-6}}{\left(\dfrac{5}{\sqrt{2}} \times 10^{-2}\right)^2} = 1.44 \times 10^7 N/C$; $|E_C| = 9 \times 10^9 \times \dfrac{2 \times 10^{-6}}{\left(\dfrac{5}{\sqrt{2}} \times 10^{-2}\right)^2} = 1.44 \times 10^7 N/C$

$|E_D| = 9 \times 10^9 \times \dfrac{10^{-6}}{\left(\dfrac{5}{\sqrt{2}} \times 10^{-2}\right)^2} = 0.72 \times 10^7 N/C$; So, $|E_{CA}| = |E_C| - |E_A| = 0.72 \times 10^7 N/C$

and $|E_{BD}| = |E_B| - |E_D| = 0.72 \times 10^7 N/C$. Hence from equation – (i) $E = 1.02 \times 10^7 N/C$ upwards

Example: 30 Infinite charges are lying at x = 1, 2, 4, 8…*meter* on X-axis and the value of each charge is Q. The value of intensity of electric field and potential at point x = 0 due to these charges will be respectively

(a) $12 \times 10^9 Q$ N/C, 1.8×10^4 V

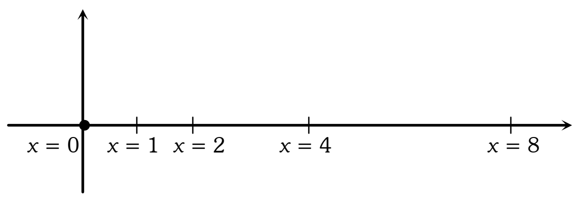
(b) Zero, $1.2 \times 10^4 V$

(c) $6 \times 10^9 Q$ N/C, 9×10^3 V

(d) $4 \times 10^9 Q$ N/C, 6×10^3 V

Solution: (a) By the superposition, Net electric field at origin

$$E = kQ\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \ldots\infty\right]$$



$$E = kQ\left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\infty\right]$$

$1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\infty$ is an infinite geometrical progression it's sum can be obtained by using the formula $S_\infty = \frac{a}{1-r}$; Where a = First term, r = Common ratio.

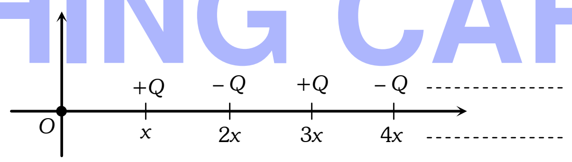
Here $a = 1$ and $r = \frac{1}{4}$ so, $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\ldots\infty = \frac{1}{1-1/4} = \frac{4}{3}$.

Hence $E = 9 \times 10^9 \times Q \times \frac{4}{3} = 12 \times 10^9 Q\, N/C$

Electric potential at origin $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{1\times 10^{-6}}{1} + \frac{1\times 10^{-6}}{2} + \frac{1\times 10^{-6}}{4} + \frac{1\times 10^{-6}}{8} + \ldots\ldots\infty\right]$

$$= 9\times 10^9 \times 10^{-6}\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots\ldots\ldots\infty\right] = 9\times 10^3\left[\frac{1}{1-\frac{1}{2}}\right] = 1.8\times 10^4 \; volt$$

Note : ≅ In the arrangement shown in figure $+Q$ and $-Q$ are alternatively and equally spaced from each other, the net potential at the origin O is $V = \frac{1}{4\pi\varepsilon_0}.\frac{Q\log_e 2}{x}$



Example: 31 Potential at a point x-distance from the centre inside the conducting sphere of radius R and charged with charge Q is **[MP PMT 2001]**

(a) $\frac{Q}{R}$ (b) $\frac{Q}{x}$ (c) $\frac{Q}{x^2}$ (d) xQ

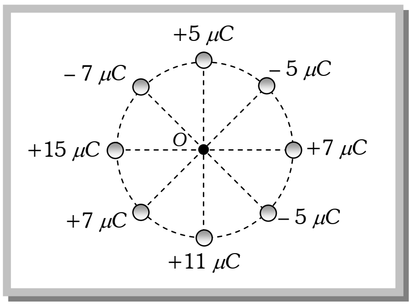
Solution: (a) Potential inside the conductor is constant.

Example: 32 The electric potential at the surface of an atomic nucleus (Z = 50) of radius $9\times 10^5\,V$ is

(a) 80 V (b) $8\times 10^6\ V$ (c) 9 V (d) $9\times 10^5\ V$

Solution: (b) $V = 9\times 10^9 \times \frac{ne}{r} = 9\times 10^9 \times \frac{50\times 1.6\times 10^{-19}}{9\times 10^{-15}} = 8\times 10^6 V$

Example: 33 Eight charges having the valves as shown are arranged symmetrically on a circle of radius 0.4m in air. Potential at centre O will be

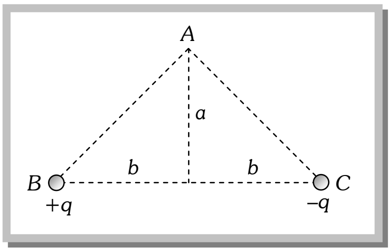


(a) 63×10^4 *volt* (b) 63×10^{10} *volt* (c) 63×10^6 *volt* (d) Zero

Solution: (a) Due to the principle of superposition potential at O

$$V=\frac{1}{4\pi\varepsilon_0}\times\frac{28\times10^{-6}}{0.4}=9\times10^9\times\frac{28\times10^{-6}}{0.4}=63\times10^4\ volt$$

Example: 34 As shown in the figure, charges $+q$ and $-q$ are placed at the vertices B and C of an isosceles triangle. The potential at the vertex A is **[MP PET 2000]**



(a) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{2q}{\sqrt{a^2+b^2}}$ (b) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{a^2+b^2}}$ (c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{\sqrt{a^2+b^2}}$ (d) Zero

Solution: (d) Potential at A = Potential due to $(+q)$ charge + Potential due to $(-q)$ charge

$$=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{a^2+b^2}}+\frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{\sqrt{a^2+b^2}}=0$$

Example: 35 A conducting sphere of radius R is given a charge Q. consider three points B at the surface, A at centre and C at a distance $R/2$ from the centre. The electric potential at these points are such that

(a) $V_A=V_B=V_C$ (b) $V_A=V_B\neq V_C$ (c) $V_A\neq V_B\neq V_C$ (d) $V_A\neq V_B=V_C$

Solution: (a) Potential inside a conductor is always constant and equal to the potential at the surface.

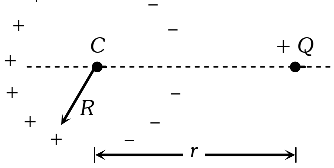
Example: 36 Equal charges of $\frac{10}{3}\times10^{-9}$ coulomb are lying on the corners of a square of side 8 *cm*. The electric potential at the point of intersection of the diagonals will be

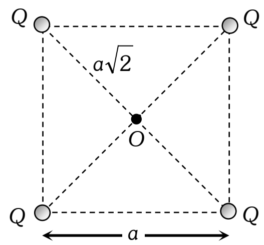
(a) 900 *V* (b) $900\sqrt{2}\,V$ (c) $150\sqrt{2}\,V$ (d) $1500\sqrt{2}\,V$

Solution: (d) Potential at the centre O

$$V=4\times\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{a/\sqrt{2}}\ \text{given}\ Q=\frac{10}{3}\times10^{-9}C\Rightarrow a=8\,cm=8\times10^{-2}m$$

$$V=5\times9\times10^9\times\frac{\frac{10}{3}\times10^{-9}}{\frac{8\times10^{-2}}{\sqrt{2}}}=1500\sqrt{2}\ volt$$





Tricky example: 3

A point charge Q is placed outside a hollow spherical conductor of radius R, at a distance $(r>R)$ from its centre C. The field at C due to the induced charges on the conductor is $\left(K=\frac{1}{4\pi\varepsilon_0}\right)$

(a) Zero (b) $K\frac{Q}{(r-R)^2}$

(c) $K\frac{Q}{r^2}$ directed towards Q (d) $K\frac{Q}{r^2}$ directed away from Q

Solution: (c) A according to the figure shown below. The total field at C must be zero. The field at C due to the point charge is $E=K\frac{Q}{r^2}$ towards left. The field at C due to the induced charges must be $\frac{KQ}{r^2}$ towards right i.e. directed towards Q.

Tricky example: 4

A point charge q is placed at a distance of r from the centre of an uncharged conducting sphere of radius R ($< r$). The potential at any point on the sphere is

(a) Zero (b) $\frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$ (c) $\frac{1}{4\pi\varepsilon_0}.\frac{qR}{r^2}$ (d) $\frac{1}{4\pi\varepsilon_0}.\frac{qr^2}{R}$

Solution: (c) Since, potential V is same for all points of the sphere. Therefore, we can calculate its value at the centre of the sphere.

$\therefore\ V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r} + V'$; where V' = potential at centre due to induced charge = 0 (because net induced charge will be zero) $\therefore\ V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$.

Potential Due to Concentric Spheres.

To find potential at a point due to concentric sphere following guideline are to be considered

Guideline 1: Identity the point (P) at which potential is to be determined.

Guideline 2: Start from inner most sphere, you should know where point (P) lies *w.r.t.* concerning sphere/shell (*i.e.* outside, at surface or inside)

Guideline 3: Then find the potential at the point (P) due to inner most sphere and then due to next and so on.

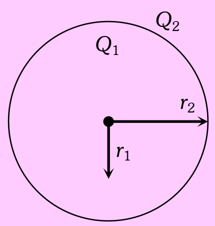
Guideline 4: Using the principle of superposition find net potential at required shell/sphere.

Standard cases

Case (i) : If two concentric conducting shells of radii r_1 and $r_2 (r_2 > r_1)$ carrying uniformly distributed charges Q_1 and Q_2 respectively. What will be the potential of each shell

To find the solution following guidelines are to be taken.

Here after following the above guideline potential at the surface of inner shell is



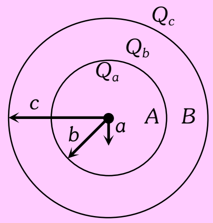
$$V_1 = \frac{1}{4\pi\varepsilon_0}.\frac{Q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}.\frac{Q_2}{r_2}$$

and potential at the surface of outer shell

$$V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{Q_1}{r_2} + \frac{1}{4\pi\varepsilon_0}.\frac{Q_2}{r_2}$$

Case (ii) : The figure shows three conducting concentric shell of radii a, b and c ($a < b < c$) having charges Q_a, Q_b and Q_c respectively what will be the potential of each shell

After following the guidelines discussed above



Potential at A; $V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_a}{a} + \frac{Q_b}{b} + \frac{Q_c}{c}\right]$

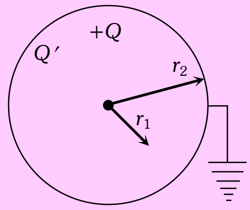
Potential at B; $V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_a}{b} + \frac{Q_b}{b} + \frac{Q_c}{c}\right]$

Potential at C; $V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_a}{c} + \frac{Q_b}{c} + \frac{Q_c}{c}\right]$

Case (iii) : The figure shows two concentric spheres having radii r_1 and r_2 respectively ($r_2 > r_1$). If charge on inner sphere is $+Q$ and outer sphere is earthed then determine.

(a) **The charge on the outer sphere**

(b) **Potential of the inner sphere**



(*i*) Potential at the surface of outer sphere $V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r_2} + \frac{1}{4\pi\varepsilon_0}.\frac{Q'}{r_2} = 0$

$\Rightarrow Q' = -Q$

(*ii*) Potential of the inner sphere $V_1 = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{r_2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

Case (iv) : In the case III if outer sphere is given a charge $+Q$ and inner sphere is earthed then

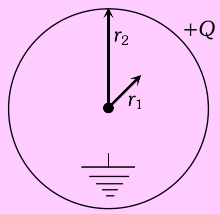
(a) **What will be the charge on the inner sphere**

(b) **What will be the potential of the outer sphere**

(*i*) In this case potential at the surface of inner sphere is zero, so if Q' is the charge induced on inner sphere

then $V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q'}{r_1} + \frac{Q}{r_2}\right] = 0$ *i.e.*, $Q' = -\frac{r_1}{r_2}Q$

(Charge on inner sphere is less than that of the outer sphere.)



(*ii*) Potential at the surface of outer sphere

$$V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{Q'}{r_2} + \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r_2}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0 r_2}\left[-Q\frac{r_1}{r_2} + Q\right] = \frac{Q}{4\pi\varepsilon_0 r_2}\left[1 - \frac{r_1}{r_2}\right]$$

Examples based on concentric spheres

Example: 37 A hollow metal sphere of radius 5 *cm* is charged such that the potential on its surface is 10 *volts*. The potential at the centre of the sphere is

(a) Zero (b) 10 *V*

(c) Same as at a point 5 *cm* away from the surface (d) Same as at a point 25 *cm* away from the surface

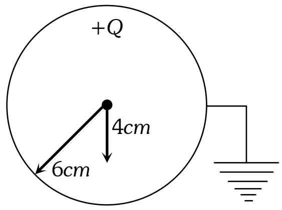
Solution: (b) Inside the conductors potential remains same and it is equal to the potential of surface, so here potential at the centre of sphere will be 10 *V*

Example: 38 A sphere of 4 *cm* radius is suspended within a hollow sphere of 6 *cm* radius. The inner sphere is charged to a potential 3 *e.s.u.* When the outer sphere is earthed. The charge on the inner sphere is

(a) 54 *e.s.u.* (b) $\frac{1}{4}$ *e.s.u.* (c) 30 *e.s.u.* (d) 36 *e.s.u.*

Solution: (d) Let charge on inner sphere be $+Q$. charge induced on the inner surface of outer sphere will be $-Q$.

So potential at the surface of inner sphere (in CGS)



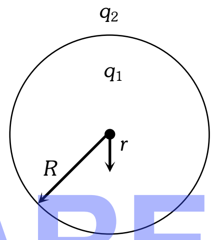
$$3 = \frac{Q}{4} - \frac{Q}{6}$$

$$\Rightarrow \quad Q = 36 \ \ e.s.u.$$

Example: 39 A charge Q is distributed over two concentric hollow spheres of radii r and $(R > r)$ such that the surface densities are equal. The potential at the common centre is **[IIT-JEE 1981]**

(a) $\frac{Q(R^2 + r^2)}{4\pi\varepsilon_0(R + r)}$ (b) $\frac{Q}{R + r}$ (c) Zero (d) $\frac{Q(R + r)}{4\pi\varepsilon_0(R^2 + r^2)}$

Solution: (d) If q_1 and q_2 are the charges on spheres of radius r and R respectively, in accordance with conservation of charge



$$Q = q_1 + q_2 \qquad \text{....(i)}$$

and according to the given problem $\sigma_1 = \sigma_2$

$$i.e., \ \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2} \Rightarrow \quad \frac{q_1}{q_2} = \frac{r^2}{R^2} \qquad \text{.... (ii)}$$

So equation (i) and (ii) gives $q_1 = \frac{Qr^2}{(R^2 + r^2)}$ and $q_2 = \frac{QR^2}{(R^2 + r^2)}$

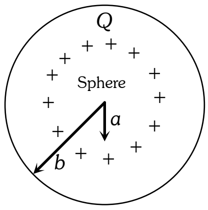
Potential at common centre $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r} + \frac{q_2}{R}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qr}{(R^2 + r^2)} + \frac{QR}{(R^2 + r^2)}\right] = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q(R + r)}{(R^2 + r^2)}$

Example: 40 A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-3Q$, the new potential difference between the two surfaces is

(a) V (b) $2V$ (c) $4V$ (d) $-2V$

Solution: (a) If a and b are radii of spheres and spherical shell respectively, potential at their surfaces will be

$$V_{\text{sphere}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{a} \quad \text{and} \quad V_{\text{shell}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{b}$$



and so according to the given problem.

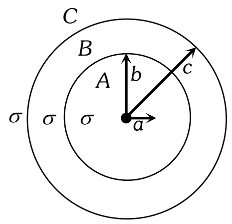
$$V = V_{\text{sphere}} - V_{\text{shell}} \quad = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] \qquad \text{.... (i)}$$

Now when the shell is given a charge $-3Q$ the potential at its surface and also inside will change by $V_0 = \frac{1}{4\pi\varepsilon_0}\left[-\frac{3Q}{b}\right]$

So that now $V_{\text{sphere}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{a} - \frac{3Q}{b}\right]$ and $V_{\text{shell}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{b} - \frac{3Q}{b}\right]$ hence

$$V_{\text{sphere}} - V_{\text{shell}} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] = V$$

Example: 41 Three concentric metallic spheres A, B and C have radii a, b and c $(a < b < c)$ and surface charge densities on them are $\sigma, -\sigma$ and σ respectively. The valves of V_A and V_B will be



(a) $\frac{\sigma}{\varepsilon_0}(a-b-c), \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{b}-b+c\right)$

(b) $(a-b-c), \frac{a^2}{c}$

(c) $\frac{\varepsilon_0}{\sigma}(a-b-c), \frac{\varepsilon_0}{\sigma}\left(\frac{a^2}{c}-b+c\right)$

(d) $\frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c}-\frac{b^2}{c}+c\right)$ and $\frac{\sigma}{\varepsilon_0}(a-b+c)$

Solution: (a) Suppose charges on A, B and C are q_a, q_b and q_c

Respectively, so $\sigma_A = \sigma = \frac{q_a}{4\pi a^2} \Rightarrow q_a = \sigma \times 4\pi a^2$, $\sigma_B = -\sigma = \frac{q_b}{4\pi b^2} \Rightarrow q_b = -\sigma \times 4\pi b^2$

and $\sigma_C = \sigma = \frac{q_c}{4\pi c^2} \Rightarrow q_c = \sigma \times 4\pi c^2$

Potential at the surface of A

$$V_A = (V_A)_{\text{surface}} + (V_B)_{\text{in}} + (V_C)_{\text{in}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_a}{a}+\frac{q_b}{b}+\frac{q_c}{c}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma\times 4\pi a^2}{a}+\frac{(-\sigma)\times 4\pi b^2}{b}+\frac{\sigma\times 4\pi c^2}{c}\right]$$

$$V_A = \frac{\sigma}{\varepsilon_0}[a-b-c]$$

Potential at the surface of B

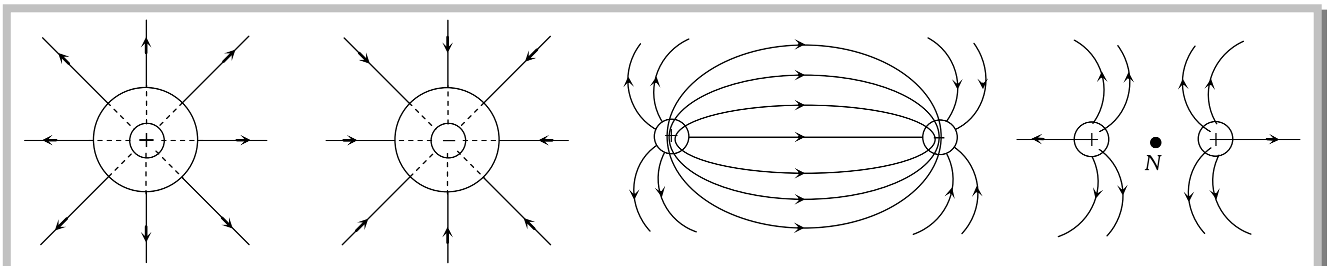
$$V_B = (V_A)_{\text{out}} + (V_B)_{\text{surface}} + (V_C)_{\text{in}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_a}{b}+\frac{q_b}{b}+\frac{q_c}{c}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma\times 4\pi a^2}{b}-\frac{\sigma\times 4\pi b^2}{b}+\frac{\sigma\times 4\pi c^2}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b}-b+c\right]$$

Electric Lines of Force.

(1) **Definition :** The electric field in a region is represented by continuous lines (also called lines of force). Field line is an imaginary line along which a positive test charge will move if left free.

Electric lines of force due to an isolated positive charge, isolated negative charge and due to a pair of charge are shown below

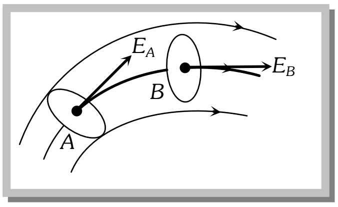


(Radially outward) (Radially inward)

(2) **Properties of electric lines of force**

(i) Electric field lines come out of positive charge and go into the negative charge.

(ii) Tangent to the field line at any point gives the direction of the field at that point.

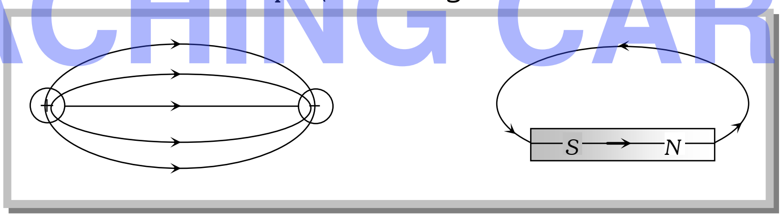


(iii) Field lines never cross each other.

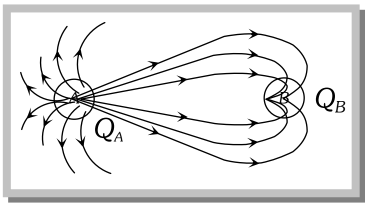
(iv) Field lines are always normal to conducting surface.

(v) Field lines do not exist inside a conductor.

(vi) The electric field lines never form closed loops. (While magnetic lines of forces form closed loop)

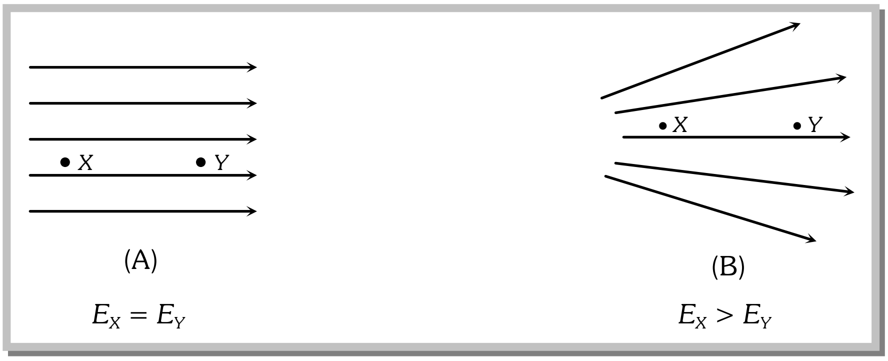


(vii) The number of lines originating or terminating on a charge is proportional to the magnitude of charge. In the following figure electric lines of force are originating from A and terminating at B hence Q_A is positive while Q_B is negative, also number of electric lines at force linked with Q_A are more than those linked with Q_B hence $|Q_A| > |Q_B|$

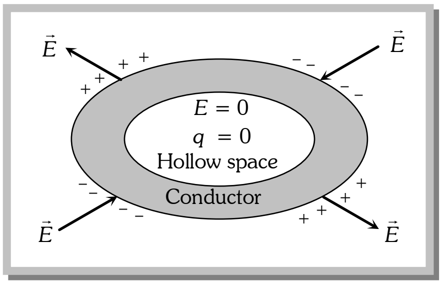


(viii) Number of lines of force per unit area normal to the area at a point represents magnitude of intensity (concept of electric flux *i.e.*, $\phi = EA$)

(ix) If the lines of forces are equidistant and parallel straight lines the field is uniform and if either lines of force are not equidistant or straight line or both the field will be non uniform, also the density of field lines is proportional to the strength of the electric field. For example see the following figures.



(3) **Electrostatic shielding :** Electrostatic shielding/screening is the phenomenon of protecting a certain region of space from external electric field. Sensitive instruments and appliances are affected seriously with strong external electrostatic fields. Their working suffers and they may start misbehaving under the effect of unwanted fields.

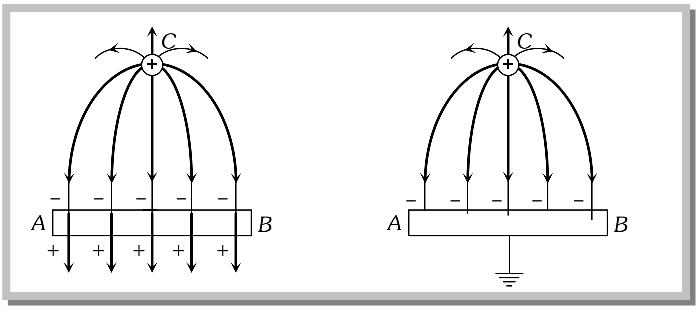


The electrostatic shielding can be achieved by protecting and enclosing the sensitive instruments inside a hollow conductor because inside hollow conductors, electric fields is zero.

(i) It is for this reason that it is safer to sit in a car or a bus during lightening rather than to stand under a tree or on the open ground.

(ii) A high voltage generator is usually enclosed in such a cage which is earthen. This would prevent the electrostatic field of the generator from spreading out of the cage.

(iii) An earthed conductor also acts as a screen against the electric field. When conductor is not earthed field of the charged body C due to electrostatic induction continues beyond AB. If AB is earthed, induced positive charge neutralizes and the field in the region beyond AB disappears.



Equipotential Surface or Lines.

If every point of a surface is at same potential, then it is said to be an equipotential surface

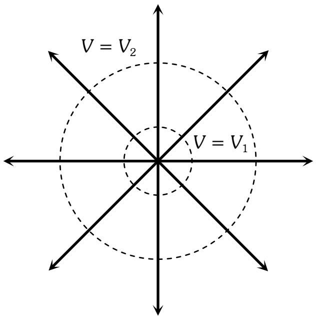
or

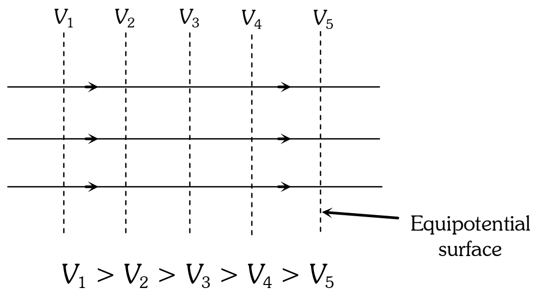
for a given charge distribution, locus of all points having same potential is called "equipotential surface" regarding equipotential surface following points should keep in mind :

(1) The density of the equipotential lines gives an idea about the magnitude of electric field. Higher the density larger the field strength.

(2) The direction of electric field is perpendicular to the equipotential surfaces or lines.

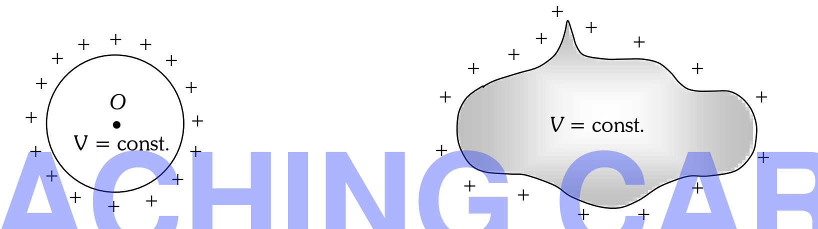
(3) The equipotential surfaces produced by a point charge or a spherically charge distribution are a family of concentric spheres.





(4) For a uniform electric field, the equipotential surfaces are a family of plane perpendicular to the field lines.

(5) A metallic surface of any shape is an equipotential surface *e.g.* When a charge is given to a metallic surface, it distributes itself in a manner such that its every point comes at same potential even if the object is of irregular shape and has sharp points on it.



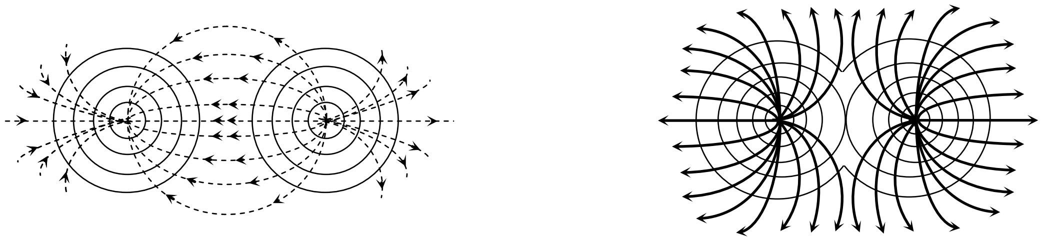
Metallic charged sphere

Charged metallic body of irregular shape

If it is not so, that is say if the sharp points are at higher potential then due to potential difference between these points connected through metallic portion, charge will flow from points of higher potential to points of lower potential until the potential of all points become same.

(6) Equipotential surfaces can never cross each other

(7) Equipotential surface for pair of charges



Pair of two equal and opposite charges

Pair of two equal and similar charges

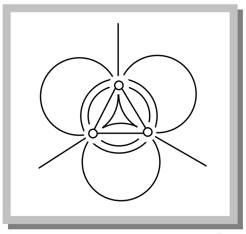
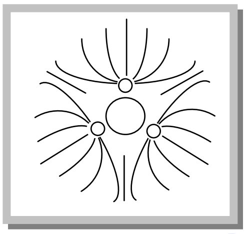
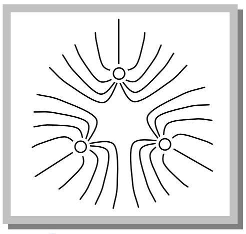
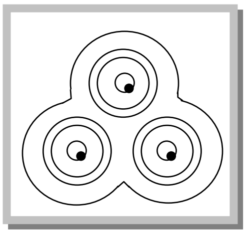
Concepts

- *Unit field i.e. 1N/C is defined arbitrarily as corresponding to unit density of lines of force.*

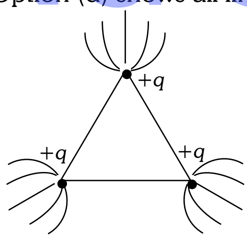
- *Number of lines originating from a unit charge is* $\frac{1}{\varepsilon_0}$
- *It is a common misconception that the path traced by a positive test charge is a field line but actually the path traced by a unit positive test charge represents a field full line only when it moves along a straight line.*
- *Both the equipotential surfaces and the lines of force can be used to depict electric field in a certain region of space. The advantage of using equipotential surfaces over the lines of force is that they give a visual picture of both the magnitude and direction of the electric field.*

Examples based on electric lines of force

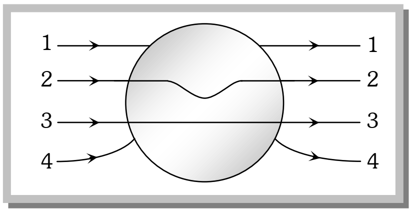
Example: 42 Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in

(a) 
(b) 
(c) 
(d) 

Solution (c) Option (a) shows lines of force starting from one positive charge and terminating at another. Option (b) has one line of force making closed loop. Option (d) shows all lines making closed loops. All these are not correct. Hence option (c) is correct



Example: 43 A metallic sphere is placed in a uniform electric field. The lines of force follow the path (s) shown in the figure as

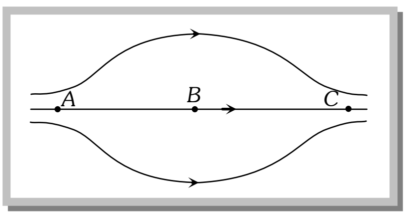


(a) 1 (b) 2 (c) 3 (d) 4

Solution: (d) The field is zero inside a conductor and hence lines of force cannot exist inside it. Also, due to induced charges on its surface the field is distorted close to its surface and a line of force must deviate near the surface outside the sphere.

Example: 44 The figure shows some of the electric field lines corresponding to an electric field. The figure suggests

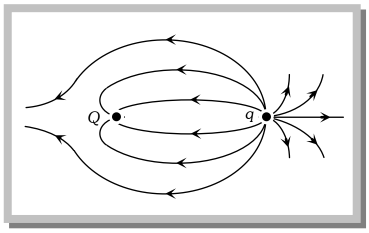
[MP PMT 1999]



(a) $E_A > E_B > E_C$ (b) $E_A = E_B = E_C$ (c) $E_A = E_C > E_B$ (d) $E_A = E_C < E_B$

Solution: (c)

Example: 45 The lines of force of the electric field due to two charges q and Q are sketched in the figure. State if



(a) Q is positive and $|Q| > |q|$

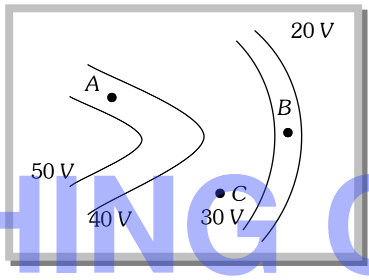
(b) Q is negative and $|Q| > |q|$

(c) q is positive and $|Q| < |q|$

(d) q is negative and $|Q| < |q|$

Solution: (c) q is $+ve$ because lines of force emerge from it and $|Q| < |q|$ because more lines emerge from q and less lines terminate at Q.

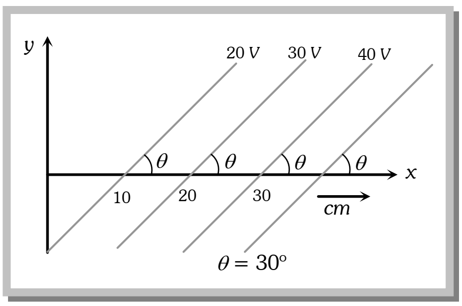
Example: 46 The figure shows the lines of constant potential in a region in which an electric field is present. The magnitude of electric field is maximum at



(a) A (b) B (c) C (d) Equal at A, B and C

Solution: (b) Since lines of force are denser at B hence electric field is maximum at B

Example: 47 Some equipotential surface are shown in the figure. The magnitude and direction of the electric field is

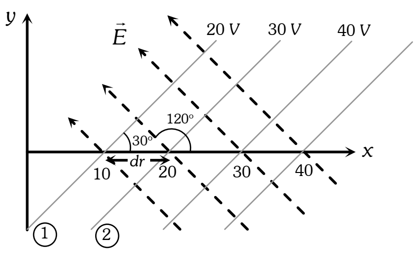


(a) 100 V/m making angle 120° with the x-axis (b) 100 V/m making angle 60° with the x-axis

(c) 200 V/m making angle 120° with the x-axis (d) None of the above

Solution: (c) By using $dV = E\,dr\cos\theta$ suppose we consider line 1 and line 2 then

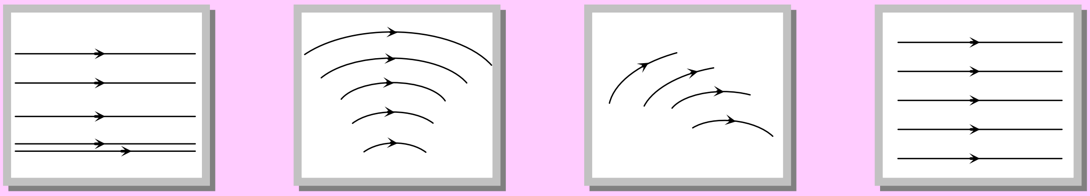
$(30 - 20) = E\cos 60^\circ\,(20 - 10) \times 10^{-2}$ So $E = 200\,volt/m$ making in angle 120° with x-axis



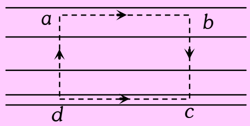
Tricky example: 5

Which of the following maps cannot represent an electric field

(a) (b) (c) (d)

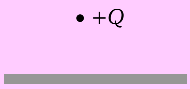


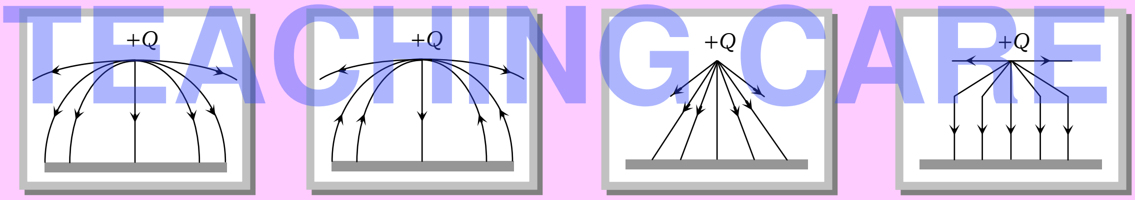
Solution: (a) If we consider a rectangular closed path, two parallel sides of it considering with lines of force as shown, then we find that work done along the closed path *abcd* is $abE_1 - cdE_2 \neq 0$. Hence the field cannot represent a conservative field. But electric field is a conservative field. Hence a field represented by these lines cannot be an electric field.



Tricky example: 6

A charge *Q* is fixed at a distance *d* in front of an infinite metal plate. The lines of force are represented by



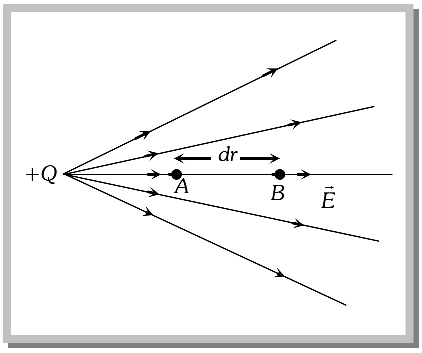


(a) (b) (c) (d)

Solution: (a) Metal plate acts as an equipotential surface, therefore the field lines should act normal to the surface of the metal plate.

Relation Between Electric Field and Potential.

In an electric field rate of change of potential with distance is known as **potential gradient.** It is a vector quantity and it's direction is opposite to that of electric field. Potential gradient relates with electric field according to the following relation $\boldsymbol{E} = -\frac{\boldsymbol{dV}}{\boldsymbol{dr}}$**;** This relation gives another unit of electric field is $\frac{\textbf{volt}}{\textbf{meter}}$.



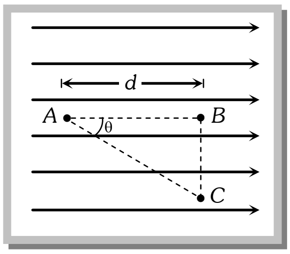
In the above relation negative sign indicates that in the direction of electric field potential decreases.

In space around a charge distribution we can also write $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

where $E_x = -\frac{dV}{dx}$, $E_y = -\frac{dV}{dy}$ and $E_z = -\frac{dV}{dz}$

With the help of formula $E=-\frac{dV}{dr}$, potential difference between any two points in an electric field can be determined by knowing the boundary conditions $dV=-\int_{r_1}^{r_2}\vec{E}\,.\,\vec{dr}=-\int_{r_1}^{r_2}\vec{E}\,.\,\vec{dr}\cos\theta$.

For example: Suppose *A*, *B* and *C* are three points in an uniform electric field as shown in figure.



(i) Potential difference between point *A* and *B* is

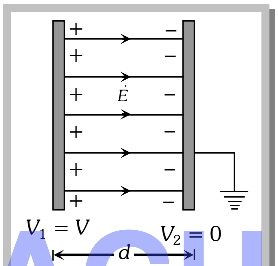
$$\boldsymbol{V_B-V_A=-\int_A^B \vec{E}\cdot\vec{dr}}$$

Since displacement is in the direction of electric field, hence $\theta=0^o$

So, $$V_B-V_A=-\int_A^B E\cdot dr\cos 0=-\int_A^B E\cdot dr=-Ed$$

In general we can say that in an uniform electric field $E=-\frac{V}{d}$ or $|E|=\frac{V}{d}$

Another example



$$\boldsymbol{E=\frac{V}{d}}$$

(ii) Potential difference between points *A* and *C* is :

$$V_C-V_A=-\int_A^C E\,dr\cos\theta=-E(AC)\cos\theta=-E(AB)=-Ed$$

Above relation proves that potential difference between *A* and *B* is equal to the potential difference between *A* and *C* *i.e.* points *B* and *C* are at same potential.

Concept

- *Negative of the slope of the V-r graph denotes intensity of electric field i.e.* $\tan\theta=\frac{V}{r}=-E$

Example based on E = – dV/dr

Example: 48 The electric field, at a distance of 20 *cm* from the centre of a dielectric sphere of radius 10 *cm* is 100 *V/m*. The '*E*' at 3 *cm* distance from the centre of sphere is **[RPMT 2001]**

(a) 100 *V/m* (b) 125 *V/m* (c) 120 *V/m* (d) Zero

Solution: (c) For dielectric sphere *i.e.* for non-conducting sphere $E_{out}=\frac{k.q}{r^2}$ and $E_{in}=\frac{kqr}{R^3}$

$$E_{out}=100\frac{KQ}{(20\times10^{-2})^2}\Rightarrow KQ=100\times(0.2)^2 \text{ so } E_{in}=\frac{100\times(0.2)^2\times(3\times10^{-2})^2}{(10\times10^{-2})^3}=120\ V/m$$

Example: 49 In x-y co-ordinate system if potential at a point $P(x, y)$ is given by $V = axy$; where a is a constant, if r is the distance of point P from origin then electric field at P is proportional to **[RPMT 2000]**

(a) r (b) r^{-1} (c) r^{-2} (d) r^2

Solution: (a) By using $E = -\frac{dV}{dr}$ $E_x = -\frac{dV}{dx} = -ay$, $E_y = -\frac{dV}{dy} = -ax$

Electric field at point P $E = \sqrt{E_x^2 + E_y^2} = a\sqrt{x^2 + y^2} = ar$ i.e., $E \propto r$

Example: 50 The electric potential V at any point x, y, z (all in *metres*) in space is given by $V = 4x^2$ *volt*. The electric field at the point $(1m, 0, 2m)$ in *volt/metre* is **[MP PMT 2001; IIT-JEE 1992; RPET 1999]**

(a) 8 along negative X-axis (b) 8 along positive X-axis

(c) 16 along negative X-axis (d) 16 along positive Z-axis

Solution: (a) By using $E = -\frac{dV}{dx} \Rightarrow E = -\frac{d}{dx}(4x^2) = -8x$. Hence at point $(1m, 0, 2m)$. $E = -8$ *volt/m* *i.e.* 8 along – *ve* x-axis.

Example: 51 The electric potential V is given as a function of distance x (*metre*) by $V = (5x^2 + 10x - 9)$ *volt*. Value of electric field at $x = 1m$ is **[MP PET 1999]**

(a) $-20\ V/m$ (b) $6\ V/m$ (c) $11\ V/m$ (d) $-23\ V/m$

Solution: (a) By using $E = -\frac{dV}{dx}$; $E = -\frac{d}{dx}(5x^2 + 10x - 9) = (10x + 10)$,

at $x = 1m$ $E = -20\,V/m$

Example: 52 A uniform electric field having a magnitude E_0 and direction along the positive X-axis exists. If the electric potential V, is zero at $X = 0$, then, its value at $X = +x$ will be **[MP PMT 1987]**

(a) $V(x) = +xE_0$ (b) $V(x) = -xE_0$ (c) $V(x) = x^2E_0$ (d) $V(x) = -x^2E_0$

Solution: (b) By using $E = -\frac{\Delta V}{\Delta r} = -\frac{(V_2 - V_1)}{(r_2 - r_1)}$; $E_0 = -\frac{\{V(x) - 0\}}{x - 0} \Rightarrow V(x) = -xE_0$

Example: 53 If the potential function is given by $V = 4x + 3y$, then the magnitude of electric field intensity at the point $(2, 1)$ will be **[MP PMT 1999]**

(a) 11 (b) 5 (c) 7 (d) 1

Solution: (b) By using i.e., $E = \sqrt{E_x^2 + E_y^2}$; $E_x = -\frac{dV}{dx} = -\frac{d}{dx}(4x + 3y) = -4$

and $E_y = -\frac{dV}{dy} = -\frac{d}{dy}(4x + 3y) = -3$

$\therefore$ $E = \sqrt{(-4)^2 + (-3)^2} = 5\,N/C$

Tricky example: 7

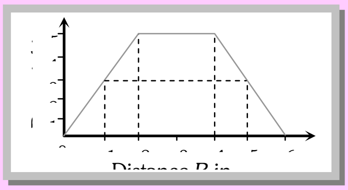
The variation of potential with distance R from a fixed point is as shown below. The electric field at $R = 5m$ is **[NCERT 1975]**

(a) 2.5 *volt/m*

(b) – 2.5 *volt/m*

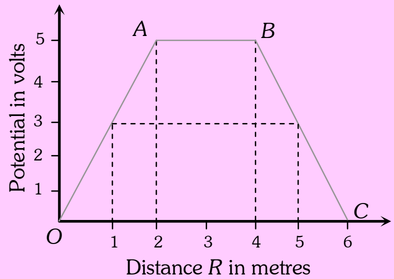
(c) $\frac{2}{5}$ *volt/m*

(d) $-\frac{2}{5}$ *volt/m*



Solution: (a) Intensity at 5 *m* is same as at any point between B and C because the slope of BC is same throughout (i.e. electric field between B and C is uniform). Therefore electric field at $R = 5m$ is equal to the slope of line BC hence by $E = \frac{-dV}{dr}$;

$$E = -\frac{(0-5)}{6-4} = 2.5\frac{V}{m}$$



Note $: \cong$ At $R = 1m$, $E = -\frac{(5-0)}{(2-0)} = -2.5\frac{V}{m}$

and at $R = 3m$ potential is constant so $E = 0$.

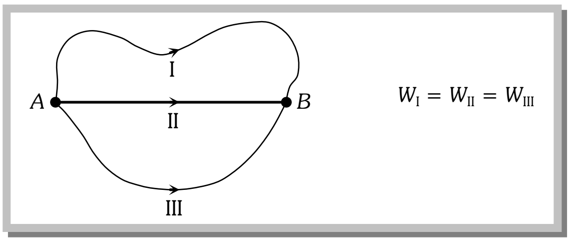
Work Done in Displacing a Charge.

(1) **Definition :** If a charge Q displaced from one point to another point in electric field then work done in this process is $W = Q \times \Delta V$ where ΔV = Potential difference between the two position of charge Q. ($\Delta V = \vec{E}.\Delta\vec{r} = E\Delta r\cos\theta$ where θ is the angle between direction of electric field and direction of motion of charge).

(2) **Work done in terms of rectangular component of $\vec{E}$ and $\vec{r}$:** If charge Q is given a displacement $\vec{r} = (r_1\hat{i} + r_2\hat{j} + r_3\hat{k})$ in an electric field $\vec{E} = (E_1\hat{i} + E_2\hat{j} + E_3\hat{k})$. The work done is $W = Q(\vec{E}.\vec{r}) = Q(E_1r_1 + E_2r_2 + E_3r_3)$.

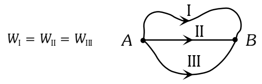
Conservation of Electric Field.

As electric field is conservation, work done and hence potential difference between two point is path independent and depends only on the position of points between. Which the charge is moved.



Concept

- No work is done in moving a charge on an equipotential surface.



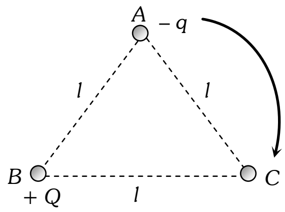
Examples based on work done

Example: 54 A charge (– q) and another charge (+Q) are kept at two points A and B respectively. Keeping the charge (+Q) fixed at B, the charge (– q) at A is moved to another point C such that ABC forms an equilateral triangle of side l. The network done in moving the charge (– q) is

(a) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{l}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{l^2}$ (c) $\frac{1}{4\pi\varepsilon_0}Qql$ (d) Zero

Solution: (d) Since $V_A = V_C = \frac{kQ}{l}$

so $W = q(V_C - V_A) = 0$



Example: 55 The work done in bringing a 20 *coulomb* charge from point A to point B for distance 0.2 m is 2 *Joule*. The potential difference between the two points will be (in *volt*) **[RPET 1999 Similar to MP PET 1999]**

(a) 0.2 (b) 8 (c) 0.1 (d) 0.4

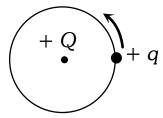
Solution: (c) $W = Q.\Delta V \Rightarrow 2 = 20 \times \Delta V \Rightarrow \Delta V = 0.1$ *volt*

Example: 56 A charge $+q$ is revolving around a stationary $+Q$ in a circle of radius r. If the force between charges is F then the work done of this motion will be

[CPMT 1975, 90, 91, 97; NCERT 1980, 83; EAMCET 1994; MP PET 1993, 95; MNR 1998; AIIMS 1997; DCE 1995; RPET 1998]

(a) $F \times r$ (b) $F \times 2\pi r$ (c) $\frac{F}{2\pi r}$ (d) 0

Solution: (d) Since $+q$ charge is moving on an equipotential surface so work done is zero.



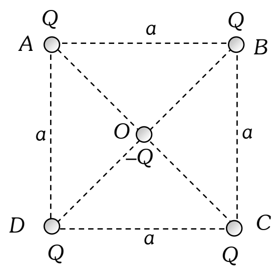
Example: 57 Four equal charge Q are placed at the four corners of a body of side 'a' each. Work done in removing a charge – Q from its centre to infinity is **[AIIMS 1995]**

(a) 0 (b) $\frac{\sqrt{2}Q^2}{4\pi\varepsilon_0 a}$ (c) $\frac{\sqrt{2}Q^2}{\pi\varepsilon_0 a}$ (d) $\frac{Q^2}{2\pi\varepsilon_0 a}$

Solution: (c) We know that work done in moving a charge is $W = Q\Delta V$

Here $W = Q(V_0 - V_\infty)$ $\because$ $V_\infty = 0$ $\therefore W = Q \times V_0$

Also $V_0 = 4 \times \frac{1}{4\pi\varepsilon_0}.\frac{Q}{a/\sqrt{2}} = \frac{4\sqrt{2}Q}{4\pi\varepsilon_0 a} = \frac{\sqrt{2}Q}{\pi\varepsilon_0 a}$

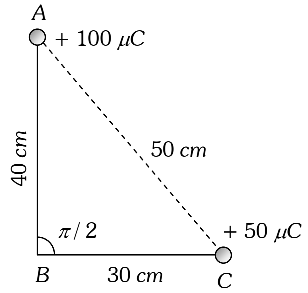


So, $W = \dfrac{\sqrt{2}Q^2}{\pi\varepsilon_0 a}$

Example: 58 Two point charge 100 μC and 5 μC are placed at point A and B respectively with AB = 40 cm. The work done by external force in displacing the charge 5 μC from B to C, where BC = 30 cm, angle $ABC = \dfrac{\pi}{2}$ and $\dfrac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2/C^2$ **[MP PMT 1997]**

(a) 9 J (b) $\dfrac{81}{20}J$ (c) $\dfrac{9}{25}J$ (d) $-\dfrac{9}{4}J$

Solution: (d) Potential at B due to $+100$ μC charge is



$V_B = 9\times10^9 \times \dfrac{100\times10^{-6}}{40\times10^{-2}} = \dfrac{9}{4}\times10^6\ volt$

Potential at C due to $+100$ μC charge is

$V_C = 9\times10^9 \times \dfrac{100\times10^{-6}}{50\times10^{-2}} = \dfrac{9}{5}\times10^6\ volt$

Hence work done in moving charge $+5\mu C$ from B to C

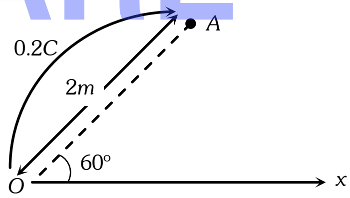
$W = 5\times10^{-6}(V_C - V_B)$

$W = 5\times10^{-6}\left(\dfrac{9}{5}\times10^6 - \dfrac{9}{4}\times10^{+6}\right) = -\dfrac{9}{4}J$

Example: 59 There is an electric field E in x-direction. If the work done in moving a charge 0.2 C through a distance of 2 $metres$ along a line making an angle 60° with the x-axis is 4J, what is the value of E **[CBSE 1995]**

(a) 4 N/C (b) 8 N/C (c) $\sqrt{3}\ N/C$ (d) 20 N/C

Solution: (d) By using $W = q\times\Delta V$ and $\Delta V = E\Delta r\cos\theta$



So, $W = qE\,\Delta r\cos\theta$

$W = 4j = 0.2\times E\times2\times\cos60$

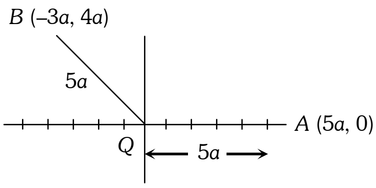
$\Rightarrow$ E = 20 N/C

Example: 60 An electric charge of 20 μC is situated at the origin of X-Y co-ordinate system. The potential difference between the points. (5a, 0) and (– 3a, 4a) will be

(a) a (b) 2a (c) Zero (d) $\dfrac{a}{\sqrt{2}}$

Solution: (c) $V_A = \dfrac{kQ}{5a}$ and $V_B = \dfrac{kQ}{5a}$

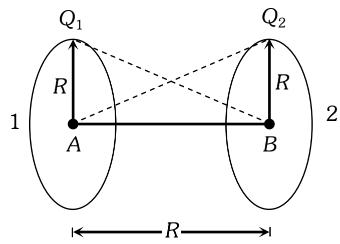
$\therefore$ $V_A - V_B = 0$



Example: 61 Two identical thin rings each of radius R, are coaxially placed a distance R apart. If Q_1 and Q_2 are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of the other is

(a) Zero (b) $\dfrac{q(Q_1 - Q_2)(\sqrt{2}-1)}{4\pi\varepsilon_0 R\sqrt{2}}$ (c) $\dfrac{q(Q_1+Q_2)\sqrt{2}}{4\pi\varepsilon_0 R}$ (d) $\dfrac{q\left(\dfrac{Q_1}{Q_2}\right)(\sqrt{2}-1)}{4\pi\varepsilon_0 R\sqrt{2}}$

Solution: (b) Potential at the centre of first ring $V_A = \dfrac{Q_1}{4\pi\varepsilon_0 R} + \dfrac{Q_2}{4\pi\varepsilon_0\sqrt{R^2+R^2}}$



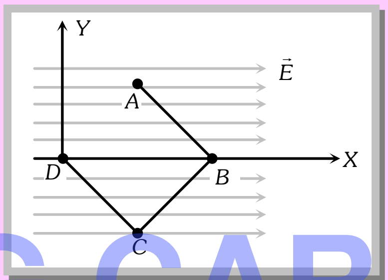
Potential at the centre of second ring $V_B = \dfrac{Q_2}{4\pi\varepsilon_0 R} + \dfrac{Q_1}{4\pi\varepsilon_0\sqrt{R^2+R^2}}$

Potential difference between the two centres $V_A - V_B = \dfrac{(\sqrt{2}-1)(Q_1-Q_2)}{4\pi\varepsilon_0 R\sqrt{2}}$

$\therefore$ Work done $W = \dfrac{q(\sqrt{2}-1)(Q_1-Q_2)}{4\pi\varepsilon_0 R\sqrt{2}}$

Tricky example: 8

A point charge q moves from point A to point D along the path $ABCD$ in a uniform electric field. If the co-ordinates of the points A, B, C and D are $(a, b, 0)$, $(2a, 0, 0)$, $(a, -b, 0)$ and $(0, 0, 0)$ then the work done by the electric field in this process will be **[IIT-JEE 1989]**



(a) $-qEa$

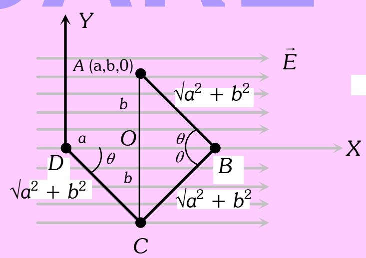
(b) Zero

(c) $2E(a+b)q$

(d) $\dfrac{qEa}{2b}$

Solution: (a) As electric field is a conservative field

Hence the work done does not depend on path



$\therefore \quad W_{ABCD} = W_{AOD} = W_{AO} + W_{OD}$

$= Fb\cos 90^\circ + Fa\cos 180^\circ = 0 + qEa(-1) = -qEa$

Equilibrium of Charge.

(1) **Definition** : A charge is said to be in equilibrium, if net force acting on it is zero. A system of charges is said to be in equilibrium if each charge is separately in equilibrium.

(2) **Type of equilibrium :** Equilibrium can be divided in following type:

(i) **Stable equilibrium :** After displacing a charged particle from it's equilibrium position, if it returns back then it is said to be in stable equilibrium. If U is the potential energy then in case of stable equilibrium $\dfrac{d^2U}{dx^2}$ is positive *i.e.*, U is minimum.

(ii) **Unstable equilibrium :** After displacing a charged particle from it's equilibrium position, if it never returns back then it is said to be in unstable equilibrium and in unstable equilibrium $\dfrac{d^2U}{dx^2}$ is negative *i.e.*, U is maximum.

(iii) **Neutral equilibrium :** After displacing a charged particle from it's equilibrium position if it neither comes back, nor moves away but remains in the position in which it was kept it is said to be in neutral equilibrium and in neutral equilibrium $\frac{d^2U}{dx^2}$ is zero *i.e.*, U is constant

(3) **Guidelines to check the equilibrium**

(i) Identify the charge for which equilibrium is to be analysed.

(ii) Check, how many forces acting on that particular charge.

(iii) There should be atleast two forces acts oppositely on that charge.

(iv) If magnitude of these forces are equal then charge is said to be in equilibrium then identify the nature of equilibrium.

(v) If all the charges of system are in equilibrium then system is said to be in equilibrium

(4) **Different cases of equilibrium of charge**

Case – 1 : Suppose three similar charge Q_1, q and Q_2 are placed along a straight line as shown below

Charge q will be in equilibrium if $|F_1| = |F_2|$

i.e., $\frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2$; This is the condition of equilibrium of charge q. After following the guidelines we can say that charge q is in stable equilibrium and this system is not in equilibrium

Note : ❑ $x_1 = \frac{x}{1+\sqrt{Q_2/Q_1}}$

and $x_2 = \frac{x}{1+\sqrt{Q_1/Q_2}}$

e.g. if two charges $+4\mu C$ and $+16\ \mu C$ are separated by a distance of 30 *cm* from each other then for equilibrium a third charge should be placed between them at a distance $x_1 = \frac{30}{1+\sqrt{16/4}} = 10\,cm$ or $x_2 = 20\,cm$

Case – 2 : Two similar charge Q_1 and Q_2 are placed along a straight line at a distance x from each other and a third dissimilar charge q is placed in between them as shown below

Charge q will be in equilibrium if $|F_1| = |F_2|$

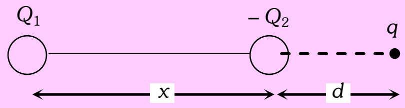
i.e., $\frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2$.

Note : ❑ Same short trick can be used here to find the position of charge q as we discussed in Case–1 *i.e.*,

$x_1 = \frac{x}{1+\sqrt{Q_2/Q_1}}$ and $x_2 = \frac{x}{1+\sqrt{Q_1/Q_2}}$

❑ It is very important to know that magnitude of charge q can be determined if one of the extreme charge (either Q_1 or Q_2) is in equilibrium *i.e.* if Q_2 is in equilibrium then $|q| = Q_1\left(\frac{x_2}{x}\right)^2$ and if Q_1 is in equilibrium then $|q| = Q_2\left(\frac{x_1}{x}\right)^2$ (It should be remember that sign of q is opposite to that of Q_1 (or Q_2))

Case – 3 : Two dissimilar charge Q_1 and Q_2 are placed along a straight line at a distance x from each other, a third charge q should be placed out side the line joining Q_1 and Q_2 for it to experience zero net force.



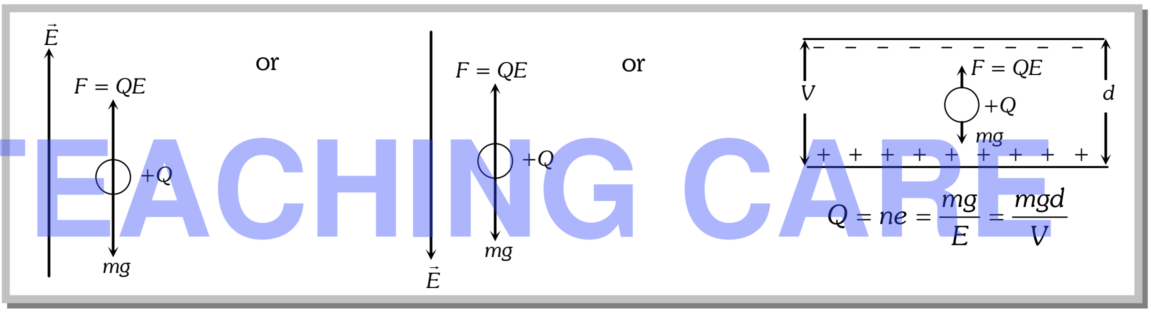
(Let $|Q_2| < |Q_1|$)

Short Trick :

For it's equilibrium. Charge q lies on the side of chare which is smallest in magnitude and $\boldsymbol{d = \dfrac{x}{\sqrt{Q_1/Q_2} - 1}}$

(5) **Equilibrium of suspended charge in an electric field**

(i) **Freely suspended charged particle :** To suspend a charged a particle freely in air under the influence of electric field it's downward weight should be balanced by upward electric force for example if a positive charge is suspended freely in an electric field as shown then



In equilibrium $QE = mg \Rightarrow \boldsymbol{E = \dfrac{mg}{Q}}$

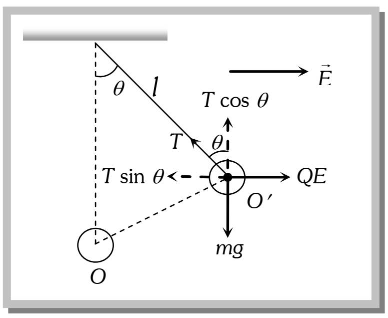
Note : ≅In the above case if direction of electric field is suddenly reversed in any figure then acceleration of charge particle at that instant will be $a = 2g$.

(ii) **Charged particle suspended by a massless insulated string** (like simple pendulum) : Consider a charged particle (like Bob) of mass m, having charge Q is suspended in an electric field as shown under the influence of electric field. It turned through an angle (say θ) and comes in equilibrium.

So, in the position of equilibrium (O' position)

$T\sin\theta = QE$ ....(i)

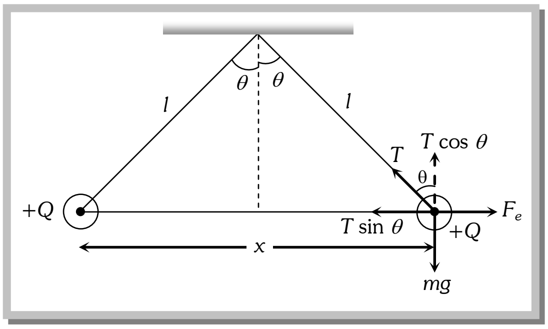
$T\cos\theta = mg$ ....(ii)



By squaring and adding equation (i) and (ii) $T = \sqrt{(QE)^2 + (mg)^2}$

Dividing equation (i) by (ii) $\tan\theta = \dfrac{QE}{mg} \Rightarrow \theta = \tan^{-1}\dfrac{QE}{mg}$

(iii) **Equilibrium of suspended point charge system :** Suppose two small balls having charge $+Q$ on each are suspended by two strings of equal length l. Then for equilibrium position as shown in figure.



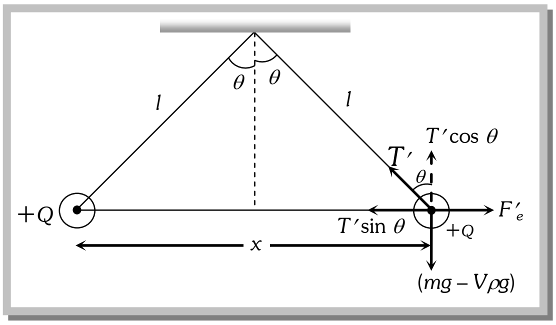
$T\sin\theta = F_e$ ....(i)

$T\cos\theta = mg$ ....(ii)

$T^2 = (F_e)^2 + (mg)^2$

and $\tan\theta = \frac{F_e}{mg}$; here $F_e = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{x^2}$ and $\frac{x}{2} = l\sin\theta$

(iv) **Equilibrium of suspended point charge system in a liquid :** In the previous discussion if point charge system is taken into a liquid of density ρ such that θ remain same then



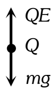
In equilibrium $Fe' = T'\sin\theta$ and $(mg - V\rho g) = T'\cos\theta$

$\therefore \tan\theta = \frac{Fe'}{(mg - V\rho g)} = \frac{Q^2}{4\pi\varepsilon_0 K(mg - V\rho g)x^2}$

When this system was in air $\tan\theta = \frac{Fe}{mg} = \frac{Q^2}{4\pi\varepsilon_0 mgx^2}$

$\therefore$ So equating these two gives us $\frac{1}{m} = \frac{1}{k(m - V\rho)} \Rightarrow K = \frac{m}{m - V\rho} = \frac{1}{\left(1 - \frac{V}{m}\rho\right)}$

If σ is the density of material of ball then $K = \frac{1}{\left(1 - \frac{□}{□}\right)}$



Examples based on equilibrium of charge

Example: 62 A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium. If q is equal to

[CPMT 1999; MP PET 1999, MP PMT 1999; CBSE 1995; Bihar MEE 1995; IIT 1987]

(a) $-\frac{Q}{2}$ (b) $-\frac{Q}{4}$ (c) $+\frac{Q}{4}$ (d) $+\frac{Q}{2}$

Solution: (b) By using Tricky formula $q = Q\left(\frac{x/2}{x}\right)^2$

$\Rightarrow q = \frac{Q}{4}$ since q should be negative so $q = -\frac{Q}{4}$.