

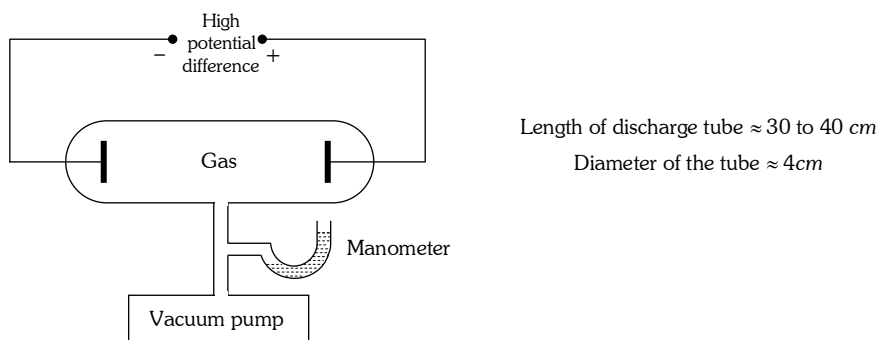
Dual Nature of Radiation and Matter

Electric Discharge Through Gases.

At normal atmospheric pressure, the gases are poor conductor of electricity. If we establish a potential difference (of the order of 30 kV) between two electrodes placed in air at a distance of few cm from each other, electric conduction starts in the form of sparks.

The passage of electric current through air is called electric discharge through the air.

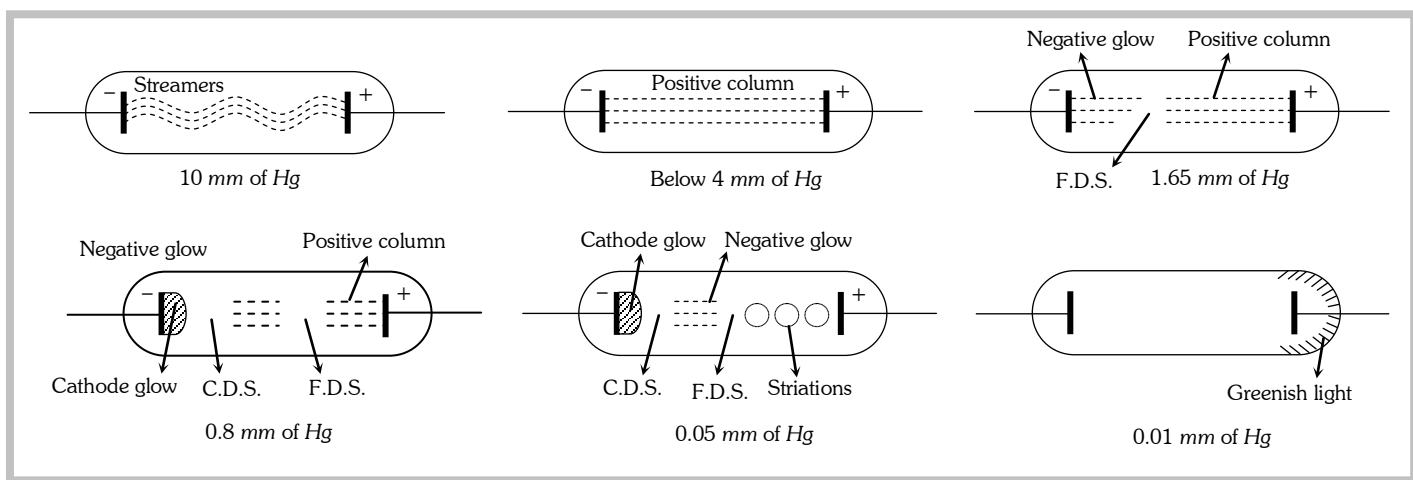
The discharge of electricity through gases can be systematically studied with the help of discharge tube shown below



The discharge tube is filled with the gas through which discharge is to be studied. The pressure of the enclosed gas can be reduced with the help of a vacuum pump and its value is read by manometer.

Sequence of phenomenon

As the pressure inside the discharge tube is gradually reduced, the following is the sequence of phenomenon that are observed.



(1) At normal pressure no discharge takes place.

(2) At the pressure 10 mm of Hg, a zig-zag thin red spark runs from one electrode to other and cracking sound is heard.

(3) At the pressure 4 mm. of Hg, an illumination is observed at the electrodes and the rest of the tube appears dark. This type of discharge is called dark discharge.

(4) When the pressure falls below 4 mm of Hg then the whole tube is filled with bright light called positive column and colour of light depends upon the nature of gas in the tube as shown in the following table.

Dual Nature of Radiation and Matter

Gas	Colour
Air	Purple red
H_2	Blue
N_2	Red
Cl_2	Green
CO_2	Bluish white
Na	Yellow
Neon	Dark red

(5) At a pressure of 1.65 mm of Hg :

(i) Sky colour light is produced at the cathode it is called as negative glow.

(ii) Positive column shrinks towards the anode and the dark space between positive column and negative glow is called Faradays dark space (FDS)

(6) At a pressure of 0.8 mm Hg : At this pressure, negative glow is detached from the cathode and moves towards the anode. The dark space created between cathode and negative glow is called as Crook's dark space length of positive column further reduced. A glow appear at cathode called cathode glow.

(7) At a pressure of 0.05 mm of Hg : The positive column splits into dark and bright disc of light called striations.

(8) At the pressure of 0.01 or 10^{-2} mm of Hg some invisible particle move from cathode which on striking with the glass tube of the opposite side of cathode cause the tube to glow. These invisible rays emerging from cathode are called cathode rays.

(9) Finally when pressure drops to nearly 10^{-4} mm of Hg , there is no discharge in tube.

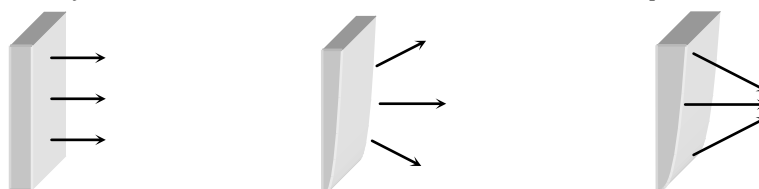
Cathode Rays.

Cathode rays, discovered by sir William Crooke are the stream of electrons. They can be produced by using a discharge tube containing gas at a low pressure of the order of 10^{-2} mm of Hg . At this pressure the gas molecules ionise and the emitted electrons travel towards positive potential of anode. The positive ions hit the cathode to cause emission of electrons from cathode. These electrons also move towards anode. Thus the cathode rays in the discharge tube are the electrons produced due to ionisation of gas and that emitted by cathode due to collision of positive ions.

(1) Properties of cathode rays

(i) Cathode rays travel in straight lines (cast shadows of objects placed in their path)

(ii) Cathode rays emit normally from the cathode surface. Their direction is independent of the position of the anode.



Dual Nature of Radiation and Matter

(iii) Cathode rays exert mechanical force on the objects they strike.

(iv) Cathode rays produce heat when they strike a material surface.

(v) Cathode rays produce fluorescence.

(vi) When cathode rays strike a solid object, specially a metal of high atomic weight and high melting point X-rays are emitted from the objects.

(vii) Cathode rays are deflected by an electric field and also by a magnetic field.

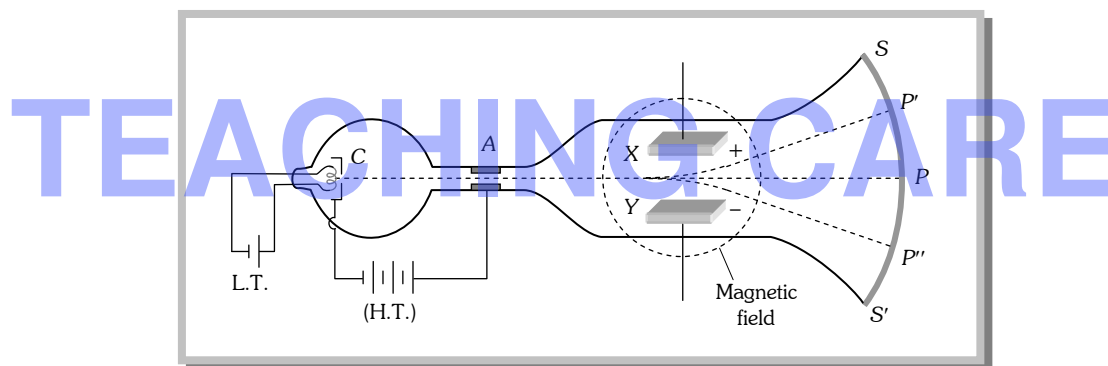
(viii) Cathode rays ionise the gases through which they are passed.

(ix) Cathode rays can penetrate through thin foils of metal.

(x) Cathode rays are found to have velocity ranging $\frac{1}{30}$ th to $\frac{1}{10}$ th of velocity of light.

(2) J.J. Thomson's method to determine specific charge of electron

Its working is based on the fact that if a beam of electron is subjected to the crossed electric field \vec{E} and magnetic field \vec{B} , it experiences a force due to each field. In case the forces on the electrons in the electron beam due to these fields are equal and opposite, the beam remains undeflected.



C = Cathode, A = Anode, F = Filament, LT = Battery to heat the filament, V = potential difference to accelerate the electrons, SS' = ZnS coated screen, XY = metallic plates (Electric field produced between them)

(i) When no field is applied, the electron beam produces illuminations at point P .

(ii) In the presence of any field (electric and magnetic) electron beam deflected up or down (illumination at P' or P'')

(iii) If both the fields are applied simultaneously and adjusted such that electron beam passes undeflected and produces illumination at point P .

In this case; Electric force = Magnetic force $\Rightarrow eE = evB \Rightarrow v = \frac{E}{B}$; v = velocity of electron

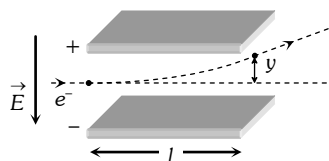
As electron beam accelerated from cathode to anode its potential energy at the cathode appears as gain in the K.E. at the anode. If suppose V is the potential difference between cathode and anode then, potential energy = eV

And gain in kinetic energy at anode will be K.E. = $\frac{1}{2}mv^2$ i.e. $eV = \frac{1}{2}mv^2 \Rightarrow \frac{e}{m} = \frac{v^2}{2V} \Rightarrow \frac{e}{m} = \frac{E^2}{2VB^2}$

Thomson found, $\frac{e}{m} = 1.77 \times 10^{11} \text{ C/kg}$.

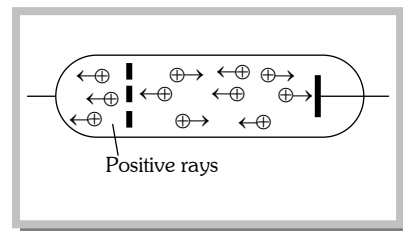
Dual Nature of Radiation and Matter

Note : ≡ The deflection of an electron in a purely electric field is given by $y = \frac{1}{2} \left(\frac{eE}{m} \right) \cdot \frac{l^2}{v^2}$; where l length of each plate, y = deflection of electron in the field region, v = speed of the electron.



Positive Rays.

Positive rays are sometimes known as the canal rays. These were discovered by Goldstein. If the cathode of a discharge tube has holes in it and the pressure of the gas is around 10^{-3} mm of Hg then faint luminous glow comes out from each hole on the backside of the cathode. It is said positive rays which are coming out from the holes.



(1) Origin of positive rays

When potential difference is applied across the electrodes, electrons are emitted from the cathode. As they move towards anode, they gain energy. These energetic electrons when collide with the atoms of the gas in the discharge tube, they ionize the atoms. The positive ions so formed at various places between cathode and anode, travel towards the cathode. Since during their motion, the positive ions when reach the cathode, some pass through the holes in the cathode. These streams are the positive rays.

(2) Properties of positive rays

(i) These are positive ions having same mass if the experimental gas does not have isotopes. However if the gas has isotopes then positive rays are group of positive ions having different masses.

(ii) They travel in straight lines and cast shadows of objects placed in their path. But the speed of the positive rays is much smaller than that of cathode rays.

(iii) They are deflected by electric and magnetic fields but the deflections are small as compared to that for cathode rays.

(iv) They show a spectrum of velocities. Different positive ions move with different velocities. Being heavy, their velocity is much less than that of cathode rays.

(v) q/m ratio of these rays depends on the nature of the gas in the tube (while in case of the cathode rays q/m is constant and doesn't depend on the gas in the tube). q/m for hydrogen is maximum.

(vi) They carry energy and momentum. The kinetic energy of positive rays is more than that of cathode rays.

(vii) The value of charge on positive rays is an integral multiple of electronic charge.

(viii) They cause ionisation (which is much more than that produced by cathode rays).

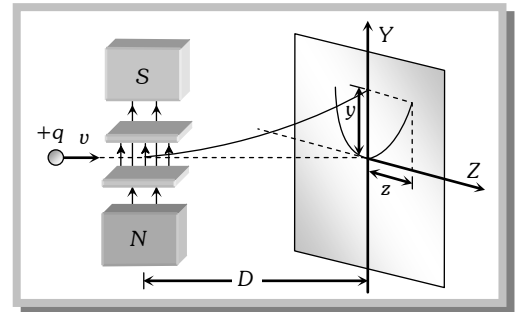
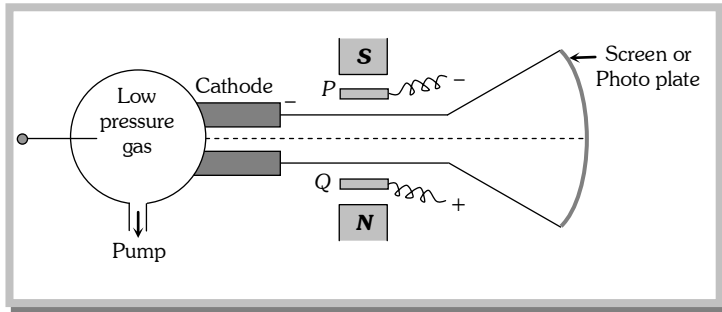
Mass Spectrograph.

It is a device used to determine the mass or (q/m) of positive ions.

(1) Thomson mass spectrograph

Dual Nature of Radiation and Matter

It is used to measure atomic masses of various isotopes in gas. This is done by measuring q/m of singly ionised positive ions of the gas.



The positive ions are produced in the bulb at the left hand side. These ions are accelerated towards cathode. Some of the positive ions pass through the fine hole in the cathode. This fine ray of positive ions is subjected to electric field E and magnetic field B and then allowed to strike a fluorescent screen ($\vec{E} \parallel \vec{B}$ but \vec{E} or $\vec{B} \perp \vec{v}$).

If the initial motion of the ions is in $+x$ direction and electric and magnetic fields are applied along $+y$ axis then force due to electric field is in the direction of y -axis and due to magnetic field it is along z -direction.

The deflection due to electric field alone $y = \frac{qELD}{mv^2}$ (i)

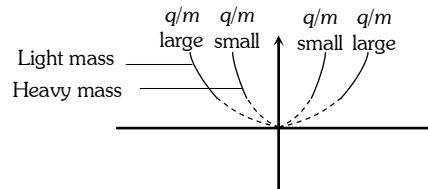
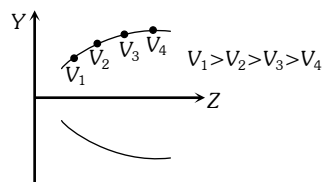
The deflection due to magnetic field alone $z = \frac{qBLD}{mv}$ (ii)

From equation (i) and (ii)

$z^2 = k\left(\frac{q}{m}\right)y$, where $k = \frac{B^2LD}{E}$; This is the equation of parabola. It means all the charged particles moving

with different velocities but of same q/m value will strike the screen placed in yz plane on a parabolic track as shown in the above figure.

Note : \cong All the positive ions of same q/m moving with different velocity lie on the same parabola. Higher is the velocity lower is the value of y and z . The ions of different specific charge will lie on different parabola.



\cong The number of parabola tells the number of isotopes present in the given ionic beam.

(2) Bainbridge mass spectrograph

In Bainbridge mass spectrograph, field particles of same velocity are selected by using a velocity selector and then they are subjected to a uniform magnetic field perpendicular to the velocity of the particles. The particles corresponding to different isotopes follow different circular paths as shown in the figure.

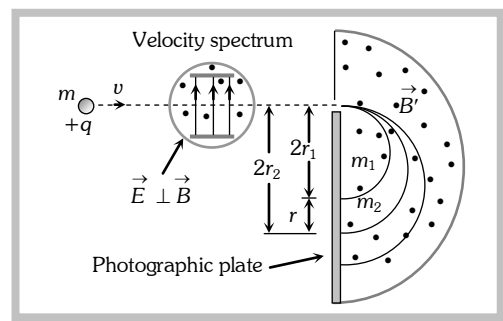
Dual Nature of Radiation and Matter

(i) **Velocity selector** : The positive ions having a certain velocity v gets isolated from all other velocity particles. In this chamber the electric and magnetic fields are so balanced that the particle moves undeflected. For this the necessary condition is $v = \frac{E}{B}$.

(ii) **Analysing chamber** : In this chamber magnetic field B is applied perpendicular to the direction of motion of the particle. As a result the particles move along a circular path of radius

$$r = \frac{mE}{qBB'} \Rightarrow \frac{q}{m} = \frac{E}{BB'r} \text{ also } \frac{r_1}{r_2} = \frac{m_1}{m_2}$$

In this way the particles of different masses gets deflected on circles of different radii and reach on different points on the photo plate.



Note : \cong Separation between two traces $= d = 2r_2 - 2r_1 \Rightarrow d = \frac{2v(m_2 - m_1)}{qB'}$.

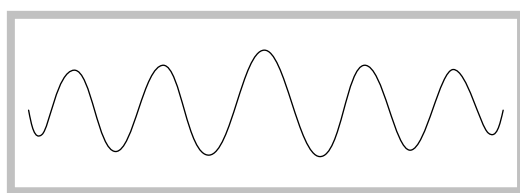
Matter waves (de-Broglie Waves).

According to de-Broglie a moving material particle sometimes acts as a wave and sometimes as a particle.

or

A wave is associated with moving material particle which control the particle in every respect.

The wave associated with moving particle is called matter wave or de-Broglie wave and it propagates in the form of wave packets with group velocity.



(1) de-Broglie wavelength

According to de-Broglie theory, the wavelength of de-Broglie wave is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \quad \Rightarrow \quad \lambda \propto \frac{1}{p} \propto \frac{1}{v} \propto \frac{1}{\sqrt{E}}$$

Where h = Plank's constant, m = Mass of the particle, v = Speed of the particle, E = Energy of the particle.

The smallest wavelength whose measurement is possible is that of γ -rays.

The wavelength of matter waves associated with the microscopic particles like electron, proton, neutron, α -particle etc. is of the order of 10^{-10} m.

(i) de-Broglie wavelength associated with the charged particles.

The energy of a charged particle accelerated through potential difference V is $E = \frac{1}{2}mv^2 = qV$

Dual Nature of Radiation and Matter

Hence de-Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$

$$\lambda_{\text{electron}} = \frac{12.27}{\sqrt{V}} \text{ \AA}, \quad \lambda_{\text{proton}} = \frac{0.286}{\sqrt{V}} \text{ \AA}, \quad \lambda_{\text{deuteron}} = \frac{0.202 \times 10^{-10}}{\sqrt{V}} \text{ \AA}, \quad \lambda_{\alpha\text{-particle}} = \frac{0.101}{\sqrt{V}} \text{ \AA}$$

(ii) **de-Broglie wavelength associated with uncharged particles.**

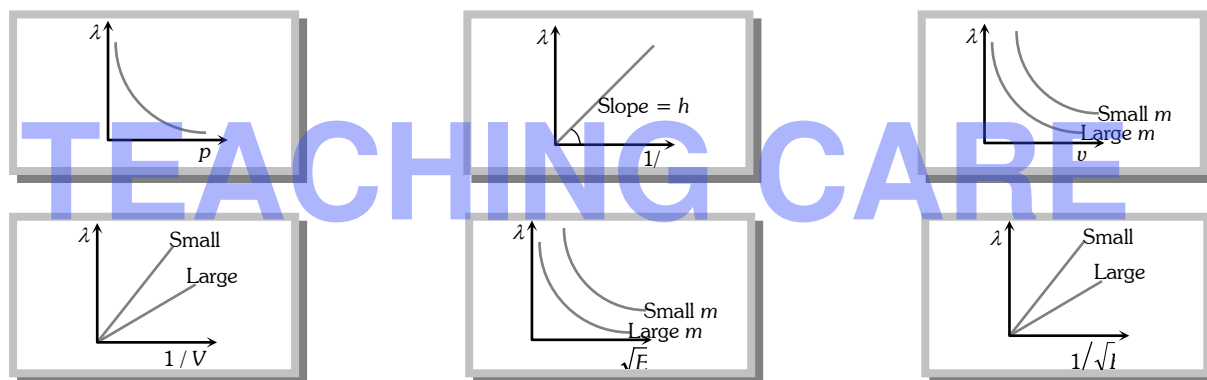
For Neutron de-Broglie wavelength is given as $\lambda_{\text{Neutron}} = \frac{0.286 \times 10^{-10}}{\sqrt{E \text{ (in eV)}}} m = \frac{0.286}{\sqrt{E \text{ (in eV)}}} \text{ \AA}$

Energy of thermal neutrons at ordinary temperature

$\therefore E = kT \Rightarrow \lambda = \frac{h}{\sqrt{2mkT}}$; where $k = \text{Boltzman's constant} = 1.38 \times 10^{-23} \text{ Joules/kelvin}$, $T = \text{Absolute temp.}$

$$\text{So } \lambda_{\text{Thermal Neutron}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.38 \times 10^{-23} T}} = \frac{30.83}{\sqrt{T}} \text{ \AA}$$

(2) **Some graphs**



Note : \equiv A photon is not a material particle. It is a quanta of energy.

\equiv When a particle exhibits wave nature, it is associated with a wave packet, rather than a wave.

(3) **Characteristics of matter waves**

(i) Matter wave represents the probability of finding a particle in space.

(ii) Matter waves are not electromagnetic in nature.

(iii) de-Broglie or matter wave is independent of the charge on the material particle. It means, matter wave of de-Broglie wave is associated with every moving particle (whether charged or uncharged).

(iv) Practical observation of matter waves is possible only when the de-Broglie wavelength is of the order of the size of the particles is nature.

(v) Electron microscope works on the basis of de-Broglie waves.

(vi) The electric charge has no effect on the matter waves or their wavelength.

(vii) The phase velocity of the matter waves can be greater than the speed of the light.

(viii) Matter waves can propagate in vacuum, hence they are not mechanical waves.

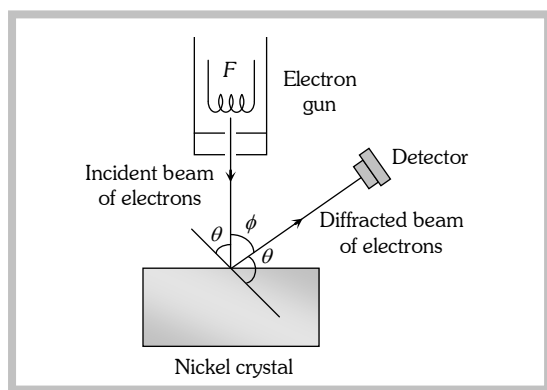
Dual Nature of Radiation and Matter

(ix) The number of de-Broglie waves associated with n^{th} orbital electron is n .

(x) Only those circular orbits around the nucleus are stable whose circumference is integral multiple of de-Broglie wavelength associated with the orbital electron.

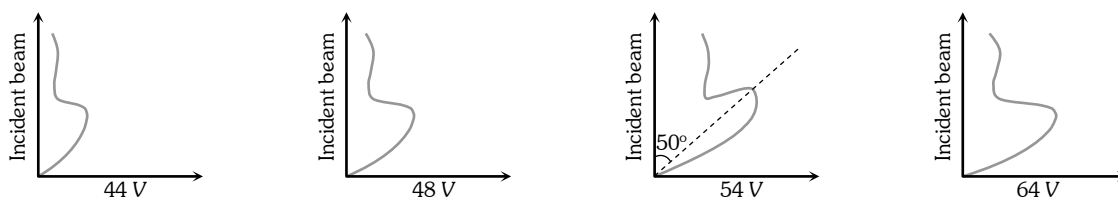
(4) Davison and Germer experiment

It is used to study the scattering of electron from a solid or to verify the wave nature of electron. A beam of electrons emitted by electron gun is made to fall on nickel crystal cut along cubical axis at a particular angle. Ni crystal behaves like a three dimensional diffraction grating and it diffracts the electron beam obtained from electron gun.



The diffracted beam of electrons is received by the detector which can be positioned at any angle by rotating it about the point of incidence. The energy of the incident beam of electrons can also be varied by changing the applied voltage to the electron gun.

According to classical physics, the intensity of scattered beam of electrons at all scattering angle will be same but Davisson and Germer, found that the intensity of scattered beam of electrons was not the same but different at different angles of scattering.

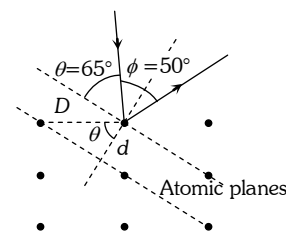


Intensity is maximum at 54 V potential difference and 50° diffraction angle.

If the de-Broglie waves exist for electrons then these should be diffracted as X-rays. Using the Bragg's formula $2d \sin \theta = n\lambda$, we can determine the wavelength of these waves.

Where d = distance between diffracting planes, $\theta = \frac{(180 - \phi)}{2}$ = glancing angle for incident beam = Bragg's angle.

The distance between diffraction planes in Ni-crystal for this experiment is $d = 0.91 \text{ \AA}$ and the Bragg's angle = 65°. This gives for $n = 1$, $\lambda = 2 \times 0.91 \times 10^{-10} \sin 65^\circ = 1.65 \text{ \AA}$



Now the de-Broglie wavelength can also be determined by using the formula $\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{54}} = 1.67 \text{ \AA}$.

Thus the de-Broglie hypothesis is verified.

Dual Nature of Radiation and Matter

Heisenberg Uncertainty Principle.

According to Heisenberg's uncertainty principle, it is impossible to measure simultaneously both the position and the momentum of the particle.

Let Δx and Δp be the uncertainty in the simultaneous measurement of the position and momentum of the particle, then $\Delta x \Delta p = \hbar$; where $\hbar = \frac{h}{2\pi}$ and $h = 6.63 \times 10^{-34}$ J-s is the Planck's constant.

If $\Delta x = 0$ then $\Delta p = \infty$

and if $\Delta p = 0$ then $\Delta x = \infty$ i.e., if we are able to measure the exact position of the particle (say an electron) then the uncertainty in the measurement of the linear momentum of the particle is infinite. Similarly, if we are able to measure the exact linear momentum of the particle i.e., $\Delta p = 0$, then we can not measure the exact position of the particle at that time.

Photon.

According to Einstein's quantum theory light propagates in the bundles (packets or quanta) of energy, each bundle being called a photon and possessing energy.

(1) Energy of photon

Energy of each photon is given by $E = h\nu = \frac{hc}{\lambda}$; where c = Speed of light, h = Planck's constant = 6.6×10^{-34} J-sec, ν = Frequency in Hz, λ = Wavelength of light

$$\text{Energy of photon in electron volt } E(\text{eV}) = \frac{hc}{e\lambda} = \frac{12375}{\lambda(\text{\AA})} \approx \frac{12400}{\lambda(\text{\AA})}$$

(2) Mass of photon

Actually rest mass of the photon is zero. But its effective mass is given as

$$E = mc^2 = h\nu \Rightarrow m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{c\lambda}. \text{ This mass is also known as kinetic mass of the photon}$$

(3) Momentum of the photon

$$\text{Momentum } p = m \times c = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

(4) Number of emitted photons

The number of photons emitted per second from a source of monochromatic radiation of wavelength λ and power P is given as $(n) = \frac{P}{E} = \frac{P}{h\nu} = \frac{P\lambda}{hc}$; where E = energy of each photon

(5) Intensity of light (I)

Energy crossing per unit area normally per second is called intensity or energy flux

Dual Nature of Radiation and Matter

i.e. $I = \frac{E}{At} = \frac{P}{A} \quad \left(\frac{E}{t} = P = \text{radiation power} \right)$

At a distance r from a point source of power P intensity is given by $I = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$

Concepts

- Discovery of positive rays helps in discovering of isotopes.
- The de-Broglie wavelength of electrons in first Bohr orbit of an atom is equal to circumference of orbit.
- A particle having zero rest mass and non zero energy and momentum must travel with a speed equal to speed of light.
- **de-Broglie wave length associates with gas molecules** is given as $\lambda = \frac{h}{mv_{rms}} = \frac{h}{\sqrt{3mkT}}$ (Energy of gas molecules at temperature T is $E = \frac{3}{2}kT$)

Example

Example: 1 The ratio of specific charge of an α -particle to that of a proton is [BCECE 2003]

- (a) 2 : 1 (b) 1 : 1 (c) 1 : 2 (d) 1 : 3

Solution : (c) Specific charge = $\frac{q}{m}$; Ratio = $\frac{(q/m)_\alpha}{(q/m)_p} = \frac{q_\alpha}{q_p} \times \frac{m_p}{m_\alpha} = \frac{1}{2}$.

Example: 2 The speed of an electron having a wavelength of $10^{-10}m$ is [AIIMS 2002]

- (a) $7.25 \times 10^6 m/s$ (b) $6.26 \times 10^6 m/s$ (c) $5.25 \times 10^6 m/s$ (d) $4.24 \times 10^6 m/s$

Solution : (a) By using $\lambda_{electron} = \frac{h}{m_e v} \Rightarrow v = \frac{h}{m_e \lambda_e} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-10}} = 7.25 \times 10^6 m/s$.

Example: 3 In Thomson experiment of finding e/m for electrons, beam of electron is replaced by that of muons (particle with same charge as of electrons but mass 208 times that of electrons). No deflection condition in this case satisfied if [Orissa (Engg.) 2002]

- (a) B is increased 208 times (b) E is increased 208 times
(c) B is increased 14.4 times (d) None of these

Solution : (c) In the condition of no deflection $\frac{e}{m} = \frac{E^2}{2VB^2}$. If m is increased to 208 times then B should be increased by $\sqrt{208} = 14.4$ times.

Example: 4 In a Thomson set-up for the determination of e/m , electrons accelerated by 2.5 kV enter the region of crossed electric and magnetic fields of strengths $3.6 \times 10^4 Vm^{-1}$ and $1.2 \times 10^{-3} T$ respectively and go through undeflected. The measured value of e/m of the electron is equal to [AMU 2002]

- (a) $1.0 \times 10^{11} C\text{-kg}^{-1}$ (b) $1.76 \times 10^{11} C\text{-kg}^{-1}$ (c) $1.80 \times 10^{11} C\text{-kg}^{-1}$ (d) $1.85 \times 10^{11} C\text{-kg}^{-1}$

Solution : (c) By using $\frac{e}{m} = \frac{E^2}{2VB^2} \Rightarrow \frac{e}{m} = \frac{(3.6 \times 10^4)^2}{2 \times 2.5 \times 10^3 \times (1.2 \times 10^{-3})^2} = 1.8 \times 10^{11} C/kg$.

Dual Nature of Radiation and Matter

Example: 5 In Bainbridge mass spectrograph a potential difference of 1000 V is applied between two plates distant 1 cm apart and magnetic field in $B = 1T$. The velocity of undeflected positive ions in m/s from the velocity selector is [RPMT 1998]

- (a) 10^7 m/s (b) 10^4 m/s (c) 10^5 m/s (d) 10^2 m/s

Solution : (c) By using $v = \frac{E}{B}$; where $E = \frac{V}{d} = \frac{1000}{1 \times 10^{-2}} = 10^5$ V/m $\Rightarrow v = \frac{10^5}{1} = 10^5$ m/s.

Example: 6 An electron and a photon have same wavelength. If p is the momentum of electron and E the energy of photon. The magnitude of p/E in S.I. unit is

- (a) 3.0×10^8 (b) 3.33×10^{-9} (c) 9.1×10^{-31} (d) 6.64×10^{-34}

Solution : (b) $\lambda = \frac{h}{p}$ (for electron) or $p = \frac{h}{\lambda}$ and $E = \frac{hc}{\lambda}$ (for photon)

$$\therefore \frac{p}{E} = \frac{1}{c} = \frac{1}{3 \times 10^8 \text{ m/s}} = 3.33 \times 10^{-9} \text{ s/m}$$

Example: 7 The energy of a photon is equal to the kinetic energy of a proton. The energy of the photon is E . Let λ_1 be the de-Broglie wavelength of the proton and λ_2 be the wavelength of the photon. The ratio λ_1/λ_2 is proportional to [UPSEAT 2003; IIT-JEE (Screening) 2004]

- (a) E^0 (b) $E^{1/2}$ (c) E^{-1} (d) E^{-2}

Solution : (b) For photon $\lambda_2 = \frac{hc}{E}$ (i) and For proton $\lambda_1 = \frac{h}{\sqrt{2mE}}$ (ii)

Therefore $\frac{\lambda_1}{\lambda_2} = \frac{E^{1/2}}{\sqrt{2m}c} \Rightarrow \frac{\lambda_1}{\lambda_2} \propto E^{1/2}$.

TEACHING CARE

Example: 8 The de-Broglie wavelength of an electron having 80eV of energy is nearly ($1\text{eV} = 1.6 \times 10^{-19} \text{ J}$, Mass of electron $9 \times 10^{-31} \text{ kg}$ and Plank's constant $6.6 \times 10^{-34} \text{ J-sec}$) [EAMCET (Engg.) 2001]

- (a) 140 \AA (b) 0.14 \AA (c) 14 \AA (d) 1.4 \AA

Solution : (d) By using $\lambda = \frac{h}{\sqrt{2mE}} = \frac{12.27}{\sqrt{V}}$. If energy is 80 eV then accelerating potential difference will be 80 V. So $\lambda = \frac{12.27}{\sqrt{80}} = 1.37 \approx 1.4 \text{ \AA}$.

Example: 9 The kinetic energy of electron and proton is 10^{-32} J . Then the relation between their de-Broglie wavelengths is [CPMT 1999]

- (a) $\lambda_p < \lambda_e$ (b) $\lambda_p > \lambda_e$ (c) $\lambda_p = \lambda_e$ (d) $\lambda_p = 2\lambda_e$

Solution : (a) By using $\lambda = \frac{h}{\sqrt{2mE}}$ $E = 10^{-32} \text{ J} = \text{Constant}$ for both particles. Hence $\lambda \propto \frac{1}{\sqrt{m}}$

Since $m_p > m_e$ so $\lambda_p < \lambda_e$.

Example: 10 The energy of a proton and an α particle is the same. Then the ratio of the de-Broglie wavelengths of the proton and the α is [RPET 1991]

- (a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

Solution : (b) By using $\lambda = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$ (E - same) $\Rightarrow \frac{\lambda_{\text{proton}}}{\lambda_{\alpha\text{-particle}}} = \sqrt{\frac{m_\alpha}{m_p}} = \frac{2}{1}$.

Example: 11 The de-Broglie wavelength of a particle accelerated with 150 volt potential is 10^{-10} m . If it is accelerated by 600 volts p.d., its wavelength will be [RPET 1988]

Dual Nature of Radiation and Matter

- (a) 0.25 \AA (b) 0.5 \AA (c) 1.5 \AA (d) 2 \AA

Solution : (b) By using $\lambda \propto \frac{1}{\sqrt{V}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{V_2}{V_1}} \Rightarrow \frac{10^{-10}}{\lambda_2} = \sqrt{\frac{600}{150}} = 2 \Rightarrow \lambda_2 = 0.5 \text{ \AA}$.

Example: 12 The de-Broglie wavelength of an electron in an orbit of circumference $2\pi r$ is [MP PET 1987]

- (a) $2\pi r$ (b) πr (c) $1/2\pi r$ (d) $1/4\pi r$

Solution : (a) According to Bohr's theory $mvr = n \frac{h}{2\pi} \Rightarrow 2\pi r = n \left(\frac{h}{mv} \right) = n\lambda$

For $n = 1$ $\lambda = 2\pi r$

Example: 13 The number of photons of wavelength 540 nm emitted per second by an electric bulb of power 100 W is (taking $h = 6.6 \times 10^{-34} \text{ J-sec}$) [Kerala (Engg.) 2002]

- (a) 100 (b) 1000 (c) 3×10^{20} (d) 3×10^{18}

Solution : (c) By using $n = \frac{P\lambda}{hc} = \frac{100 \times 540 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 3 \times 10^{20}$

Example: 14 A steel ball of mass 1 kg is moving with a velocity 1 m/s . Then its de-Broglie waves length is equal to

- (a) h (b) $h/2$ (c) Zero (d) $1/h$

Solution : (a) By using $\lambda = \frac{h}{mv} \Rightarrow \lambda = \frac{h}{1 \times 1} = h$.

Example: 15 The de-Broglie wavelength associated with a hydrogen atom moving with a thermal velocity of 3 km/s will be (a) 1 \AA (b) 0.66 \AA (c) 6.6 \AA (d) 66 \AA

Solution : (b) By using $\lambda = \frac{h}{mv_{rms}} \Rightarrow \lambda = \frac{6.6 \times 10^{-34}}{2 \times 1.67 \times 10^{-27} \times 3 \times 10^3} = 0.66 \text{ \AA}$

Example: 16 When the momentum of a proton is changed by an amount P_0 , the corresponding change in the de-Broglie wavelength is found to be 0.25% . Then, the original momentum of the proton was [CPMT 2002]

- (a) p_0 (b) $100 p_0$ (c) $400 p_0$ (d) $4 p_0$

Solution : (c) $\lambda \propto \frac{1}{p} \Rightarrow \frac{\Delta p}{p} = -\frac{\Delta \lambda}{\lambda} \Rightarrow \left| \frac{\Delta p}{p} \right| = \left| \frac{\Delta \lambda}{\lambda} \right| \Rightarrow \frac{p_0}{p} = \frac{0.25}{100} = \frac{1}{400} \Rightarrow p = 400 p_0$.

Example: 17 If the electron has same momentum as that of a photon of wavelength 5200 \AA , then the velocity of electron in m/sec is given by

- (a) 10^3 (b) 1.4×10^3 (c) 7×10^{-5} (d) 7.2×10^6

Solution : (b) $\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 5200 \times 10^{-10}} \Rightarrow v = 1.4 \times 10^3 \text{ m/s}$.

Example: 18 The de-Broglie wavelength of a neutron at 27°C is λ . What will be its wavelength at 927°C

- (a) $\lambda/2$ (b) $\lambda/3$ (c) $\lambda/4$ (d) $\lambda/9$

Solution : (a) $\lambda_{neutron} \propto \frac{1}{\sqrt{T}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_2}{T_1}} \Rightarrow \frac{\lambda}{\lambda_2} = \sqrt{\frac{(273+927)}{(273+27)}} = \sqrt{\frac{1200}{300}} = 2 \Rightarrow \lambda_2 = \frac{\lambda}{2}$.

Example: 19 The de-Broglie wavelength of a vehicle is λ . Its load is changed such that its velocity and energy both are doubled. Its new wavelength will be

Dual Nature of Radiation and Matter

- (a) λ (b) $\frac{\lambda}{2}$ (c) $\frac{\lambda}{4}$ (d) 2λ

Solution : (a) $\lambda = \frac{h}{mv}$ and $E = \frac{1}{2}mv^2 \Rightarrow \lambda = \frac{hv}{2E}$ when v and E both are doubled, λ remains unchanged i.e. $\lambda' = \lambda$.

Example: 20 In Thomson mass spectrograph when only electric field of strength 20 kV/m is applied, then the displacement of the beam on the screen is 2 cm . If length of plates = 5 cm , distance from centre of plate to the screen = 20 cm and velocity of ions = 10^6 m/s , then q/m of the ions is

- (a) 10^6 C/kg (b) 10^7 C/kg (c) 10^8 C/kg (d) 10^{11} C/kg

Solution : (c) By using $y = \frac{qELD}{mv^2}$; where y = deflection on screen due to electric field only

$$\Rightarrow \frac{q}{m} = \frac{yv^2}{ELD} = \frac{2 \times 10^{-2} \times (10^6)^2}{20 \times 10^3 \times 5 \times 10^{-2} \times 0.2} = 10^8 \text{ C/kg.}$$

Example: 21 The minimum intensity of light to be detected by human eye is 10^{-10} W/m^2 . The number of photons of wavelength $5.6 \times 10^{-7} \text{ m}$ entering the eye, with pupil area 10^{-6} m^2 , per second for vision will be nearly

- (a) 100 (b) 200 (c) 300 (d) 400

Solution : (c) By using $I = \frac{P}{A}$; where P = radiation power

$$\Rightarrow P = I \times A \Rightarrow \frac{nhc}{t\lambda} = IA \Rightarrow \frac{n}{t} = \frac{IA\lambda}{hc}$$

Hence number of photons entering per sec the eye $\left(\frac{n}{t}\right) = \frac{10^{-10} \times 10^{-6} \times 5.6 \times 10^{-7}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 300$.

TEACHING CARE

Tricky example: 1

A particle of mass M at rest decays into two particles of masses m_1 and m_2 , having non-zero velocities.

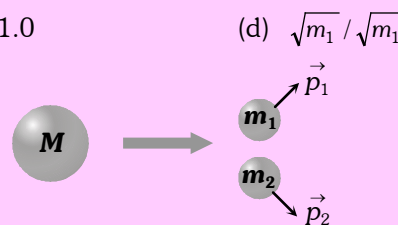
The ratio of the de-Broglie wavelengths of the particles, λ_1 / λ_2 is

[IIT-JEE 1999]

- (a) m_1 / m_2 (b) m_2 / m_1 (c) 1.0 (d) $\sqrt{m_1} / \sqrt{m_2}$

Solution : (c) According to conservation of momentum i.e. $p_1 = p_2$

Hence from $\lambda = \frac{h}{p} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{p_2}{p_1} = 1$



The curve drawn between velocity and frequency of photon in vacuum will be a

[MP PET 2000]

- (a) Straight line parallel to frequency axis
 (b) Straight line parallel to velocity axis
 (c) Straight line passing through origin and making an angle of 45° with frequency axis
 (d) Hyperbola

Solution : (a) Velocity of photon (i.e. light) doesn't depend upon frequency. Hence the graph between velocity of

Dual Nature of Radiation and Matter

photon and frequency will be as follows.

Photo-electric Effect.

It is the phenomenon of emission of electrons from the surface of metals, when light radiations (Electromagnetic radiations) of suitable frequency fall on them. The emitted electrons are called photoelectrons and the current so produced is called photoelectric current.

This effect is based on the principle of conservation of energy.

(1) Terms related to photoelectric effect

(i) **Work function (or threshold energy) (W_0)** : The minimum energy of incident radiation, required to eject the electrons from metallic surface is defined as work function of that surface.

$$W_0 = h\nu_0 = \frac{hc}{\lambda_0} \text{ Joules}; \quad \nu_0 = \text{Threshold frequency}; \quad \lambda_0 = \text{Threshold wavelength}$$

$$\text{Work function in electron volt } W_0(\text{eV}) = \frac{hc}{e\lambda_0} = \frac{12375}{\lambda_0(\text{\AA})}$$

Note : \cong By coating the metal surface with a layer of barium oxide or strontium oxide its work function is lowered.

(ii) **Threshold frequency (ν_0)** : The minimum frequency of incident radiations required to eject the electron from metal surface is defined as threshold frequency.

If incident frequency $\nu < \nu_0 \Rightarrow$ No photoelectron emission

(iii) **Threshold wavelength (λ_0)** : The maximum wavelength of incident radiations required to eject the electrons from a metallic surface is defined as threshold wavelength.

If incident wavelength $\lambda > \lambda_0 \Rightarrow$ No photoelectron emission

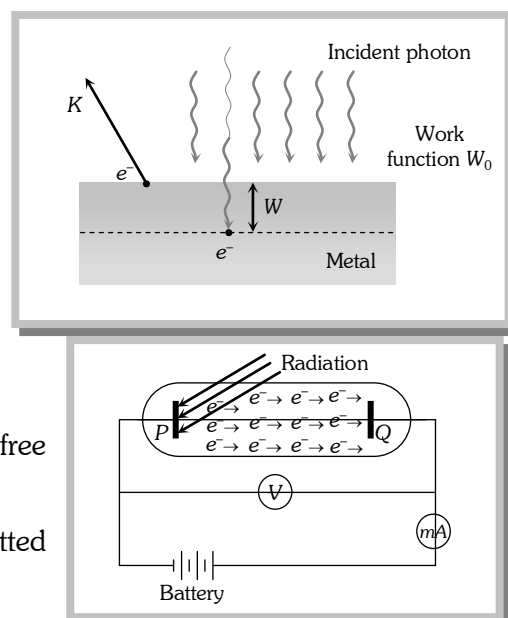
(2) Einstein's photoelectric equation

According to Einstein, photoelectric effect is the result of one to one inelastic collision between photon and electron in which photon is completely absorbed. So if an electron in a metal absorbs a photon of energy $E (= h\nu)$, it uses the energy in three following ways.

(i) Some energy (say W) is used in shifting the electron from interior to the surface of the metal.

(ii) Some energy (say W_0) is used in making the surface electron free from the metal.

(iii) Rest energy will appear as kinetic energy (K) of the emitted photoelectrons.



Dual Nature of Radiation and Matter

Hence $E = W + W_0 + K$

For the electrons emitting from surface $W = 0$ so kinetic energy of emitted electron will be max.

Hence $E = W_0 + K_{max}$; This is the Einstein's photoelectric equation

(3) Experimental arrangement to observe photoelectric effect

When light radiations of suitable frequency (or suitable wavelength and suitable energy) falls on plate P , photoelectrons are emitted from P .

(i) If plate Q is at zero potential *w.r.t.* P , very small current flows in the circuit because of some electrons of high kinetic energy are reaching to plate Q , but this current has no practical utility.

(ii) If plate Q is kept at positive potential *w.r.t.* P current starts flowing through the circuit because more electrons are able to reach upto plate Q .

(iii) As the positive potential of plate Q increases, current through the circuit increases but after some time constant current flows through the circuit even positive potential of plate Q is still increasing, because at this condition all the electrons emitted from plate P are already reached up to plate Q . This constant current is called **saturation current**.

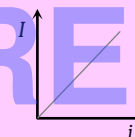
(iv) To increase the photoelectric current further we will have to increase the intensity of incident light.

Photoelectric current (i) depends upon

(a) Potential difference between electrodes (till saturation)

(b) Intensity of incident light (I)

(c) Nature of surface of metal



(v) To decrease the photoelectric current plate Q is maintained at negative potential *w.r.t.* P , as the anode Q is made more and more negative, fewer and fewer electrons will reach the cathode and the photoelectric current decreases.

(vi) At a particular negative potential of plate Q no electron will reach the plate Q and the current will become zero, this negative potential is called **stopping potential** denoted by V_0 .

(vii) If we increase further the energy of incident light, kinetic energy of photoelectrons increases and more negative potential should be applied to stop the electrons to reach upto plate Q . Hence $eV_0 = K_{max}$.

Note : \equiv Stopping potential depends only upon frequency or wavelength or energy of incident radiation. It doesn't depend upon intensity of light.

We must remember that intensity of incident light radiation is inversely proportional to the square of distance between source of light and photosensitive plate P i.e., $I \propto \frac{1}{d^2}$ so $I \propto i \propto \frac{1}{d^2}$

Important formulae

$$\Rightarrow h\nu = h\nu_0 + K_{max}$$

$$\Rightarrow K_{max} = eV_0 = h(\nu - \nu_0) \Rightarrow \frac{1}{2}mv_{max}^2 = h(\nu - \nu_0) \Rightarrow v_{max} = \sqrt{\frac{2h(\nu - \nu_0)}{m}}$$

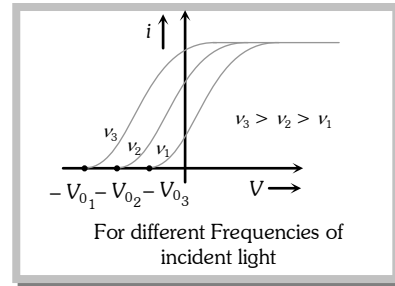
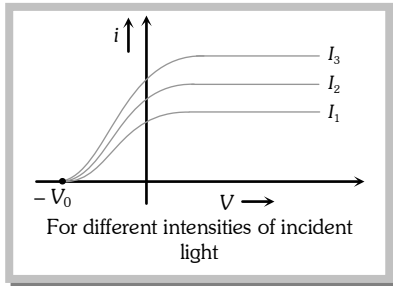
Dual Nature of Radiation and Matter

$$\Rightarrow K_{\max} = \frac{1}{2} m v_{\max}^2 = eV_0 = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) = hc \left(\frac{\lambda_0 - \lambda}{\lambda \lambda_0} \right) \Rightarrow v_{\max} = \sqrt{\frac{2hc}{m} \frac{(\lambda - \lambda_0)}{\lambda \lambda_0}}$$

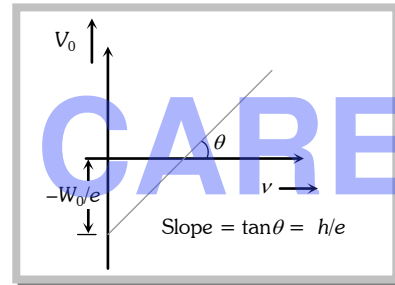
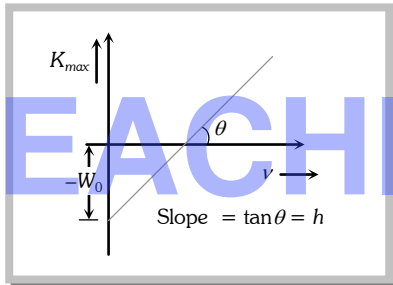
$$\Rightarrow V_0 = \frac{h}{e} (\nu - \nu_0) = \frac{hc}{e} \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) = 12375 \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)$$

(4) Different graphs

(i) Graph between potential difference between the plates P and Q and photoelectric current



(ii) Graph between maximum kinetic energy / stopping potential of photoelectrons and frequency of incident light



Photoelectric Cell.

A device which converts light energy into electrical energy is called photoelectric cell. It is also known as photocell or electric eye.

Photoelectric cell are mainly of three types

Photo-emissive cell	Photo-conductive cell	Photo-voltaic cell
<p>It consists of an evacuated glass or quartz bulb containing anode A and cathode C. The cathode is semi-cylindrical metal on which a layer of photo-sensitive material is coated.</p>	<p>It is based on the principle that conductivity of a semiconductor increases with increase in the intensity of incident light.</p>	<p>It consists of a Cu plate coated with a thin layer of cuprous oxide (Cu_2O). On this plate is laid a semi-transparent thin film of silver.</p>

Dual Nature of Radiation and Matter

When light incident on the cathode, it emits photo-electrons which are attracted by the anode. The photoelectrons constitute a small current which flows through the external circuit.

In this, a thin layer of some semiconductor (as selenium) is placed below a transparent foil of some metal. This combination is fixed over an iron plate. When light is incident on the transparent foil, the electrical resistance of the semiconductor layer is reduced. Hence a current starts flowing in the battery circuit connected.

When light fall, the electrons emitted from the layer of Cu_2O and move towards the silver film. Then the silver film becomes negatively charged and copper plate becomes positively charged. A potential difference is set up between these two and current is set up in the external resistance.

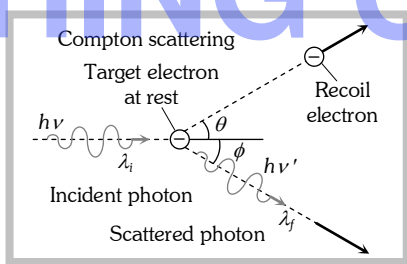
Note : ≡ The photoelectric current can be increased by filling some inert gas like Argon into the bulb. The photoelectrons emitted by cathode ionise the gas by collision and hence the current is increased.

Compton effect

The scattering of a photon by an electron is called Compton effect. The energy and momentum is conserved. Scattered photon will have less energy (more wavelength) as compare to incident photon (less wavelength). The energy lost by the photon is taken by electron as kinetic energy.

The change in wavelength due to Compton effect is called Compton shift. Compton shift

$$\lambda_f - \lambda_i = \frac{h}{m_0c} (1 - \cos \theta)$$



Note : ≡ Compton effect shows that photon have momentum.

X-rays.

X-rays was discovered by scientist Rontgen that's why they are also called Rontgen rays.

Rontgen discovered that when pressure inside a discharge tube kept 10^{-3} mm of Hg and potential difference is 25 kV then some unknown radiations (X-rays) are emitted by anode.

(1) Production of X-rays

There are three essential requirements for the production of X-rays

- A source of electron
- An arrangement to accelerate the electrons
- A target of suitable material of high atomic weight and high melting point on which these high speed electrons strike.

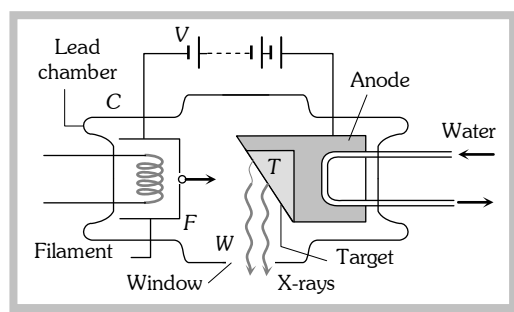
(2) Coolidge X-ray tube

Dual Nature of Radiation and Matter

It consists of a highly evacuated glass tube containing cathode and target. The cathode consists of a tungsten filament. The filament is coated with oxides of barium or strontium to have an emission of electrons even at low temperature. The filament is surrounded by a molybdenum cylinder kept at negative potential *w.r.t.* the target.

The target (its material of high atomic weight, high melting point and high thermal conductivity) made of tungsten or molybdenum is embedded in a copper block.

The face of the target is set at 45° to the incident electron stream.



The filament is heated by passing the current through it. A high potential difference ($\approx 10 \text{ kV}$ to 80 kV) is applied between the target and cathode to accelerate the electrons which are emitted by filament. The stream of highly energetic electrons are focussed on the target.

Most of the energy of the electrons is converted into heat (above 98%) and only a fraction of the energy of the electrons (about 2%) is used to produce X-rays.

During the operation of the tube, a huge quantity of heat is produced in this target, this heat is conducted through the copper anode to the cooling fins from where it is dissipated by radiation and convection.

(i) Control of intensity of X-rays : Intensity implies the number of X-ray photons produced from the target. The intensity of X-rays emitted is directly proportional to the electrons emitted per second from the filament and this can be increased by increasing the filament current. So *intensity of X-rays* \propto *Filament current*

(ii) Control of quality or penetration power of X-rays : Quality of X-rays implies the penetrating power of X-rays, which can be controlled by varying the potential difference between the cathode and the target.

For large potential difference, energy of bombarding electrons will be large and hence larger is the penetration power of X-rays.

Depending upon the penetration power, X-rays are of two types

Hard X-rays	Soft X-rays
More penetration power	Less penetration power
More frequency of the order of $\approx 10^{19} \text{ Hz}$	Less frequency of the order of $\approx 10^{16} \text{ Hz}$
Lesser wavelength range ($0.1\text{\AA} - 4\text{\AA}$)	More wavelength range ($4\text{\AA} - 100\text{\AA}$)

Note : \equiv Production of X-ray is the reverse phenomenon of photoelectric effect.

(3) Properties of X-rays

(i) X-rays are electromagnetic waves with wavelength range $0.1\text{\AA} - 100\text{\AA}$.

(ii) The wavelength of X-rays is very small in comparison to the wavelength of light. Hence they carry much more energy (This is the only difference between X-rays and light)

Dual Nature of Radiation and Matter

- (iii) X-rays are invisible.
- (iv) They travel in a straight line with speed of light.
- (v) X-rays are measured in Rontgen (measure of ionization power).
- (vi) X-rays carry no charge so they are not deflected in magnetic field and electric field.
- (vii) $\lambda_{\text{Gamma rays}} < \lambda_{\text{X-rays}} < \lambda_{\text{UV rays}}$
- (viii) They used in the study of crystal structure.
- (ix) They ionise the gases
- (x) X-rays do not pass through heavy metals and bones.
- (xi) They affect photographic plates.
- (xii) Long exposure to X-rays is injurious for human body.
- (xiii) Lead is the best absorber of X-rays.
- (xiv) For X-ray photography of human body parts, BaSO_4 is the best absorber.
- (xv) They produce photoelectric effect and Compton effect
- (xvi) X-rays are not emitted by hydrogen atom.
- (xvii) These cannot be used in Radar because they are not reflected by the target.
- (xviii) They show all the important properties of light rays like; reflection, refraction, interference, diffraction and polarization etc.

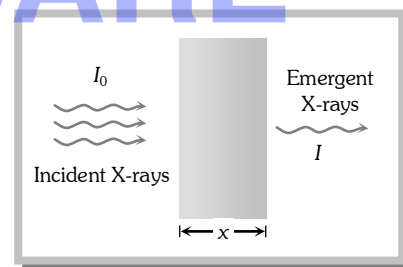
(4) Absorption of X-rays

X-rays are absorbed when they incident on substance.

Intensity of emergent X-rays $I = I_0 e^{-\mu x}$

So intensity of absorbed X-rays $I' = I_0 - I = I_0(1 - e^{-\mu x})$

where x = thickness of absorbing medium, μ = absorption coefficient



Note : \cong The thickness of medium at which intensity of emergent X-rays becomes half i.e. $I' = \frac{I_0}{2}$ is called

half value thickness ($x_{1/2}$) and it is given as $x_{1/2} = \frac{0.693}{\mu}$.

Classification of X-rays.

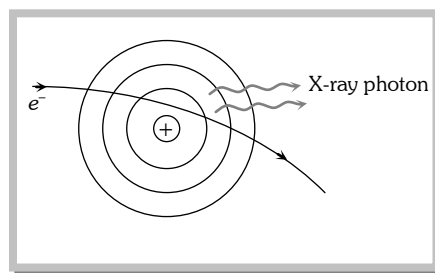
In X-ray tube, when high speed electrons strikes the target, they penetrate the target. They loses their kinetic energy and comes to rest inside the metal. The electron before finally being stopped makes several collisions with the atoms in the target. At each collision one of the following two types of X-rays may get form.

(1) Continuous X-rays

As an electron passes close to the positive nucleus of atom, the electron is deflected from it's path as shown in figure. This results in deceleration of the electron. The loss in energy of the electron during deceleration is emitted in the form of X-rays.

The X-ray photons emitted so form the continuous X-ray spectrum.

Dual Nature of Radiation and Matter



Note : \cong Continuous X-rays are produced due to the phenomenon called "Bremsstrahlung". It means slowing down or braking radiation.

Minimum wavelength

When the electron loses whole of its energy in a single collision with the atom, an X-ray photon of maximum energy $h\nu_{\max}$ is emitted i.e. $\frac{1}{2}mv^2 = eV = h\nu_{\max} = \frac{hc}{\lambda_{\min}}$

where v = velocity of electron before collision with target atom, V = potential difference through which electron is accelerated, c = speed of light = 3×10^8 m/s

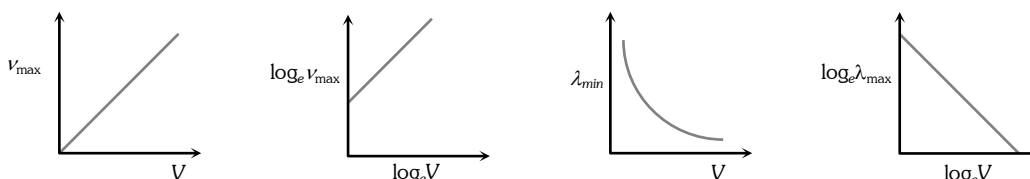
Maximum frequency of radiations (X-rays)

$$\nu_{\max} = \frac{eV}{h}$$

Minimum wavelength = cut off wavelength of X-ray

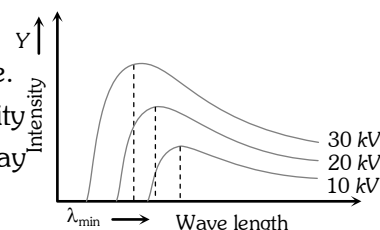
$$\lambda_{\min} = \frac{hc}{eV} = \frac{12375}{V} \text{ \AA}$$

Note : \cong Wavelength of continuous X-ray photon ranges from certain minimum (λ_{\min}) to infinity.



Intensity wavelength graph

The continuous X-ray spectra consist of all the wavelengths over a given range. These wavelengths are of different intensities. Following figure shows the intensity variation of different wavelengths for various accelerating voltages applied to X-ray tube.



For each voltage, the intensity curve starts at a particular minimum wavelength (λ_{\min}). Rises rapidly to a maximum and then drops gradually.

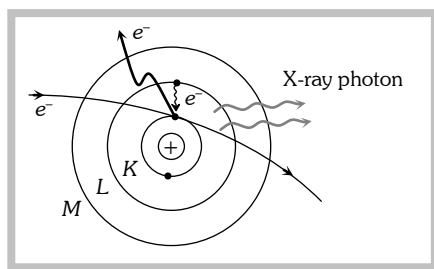
The wavelength at which the intensity is maximum depends on the accelerating voltage, being shorter for higher voltage and vice-versa.

(2) Characteristic X-rays

Few of the fast moving electrons having high velocity penetrate the surface atoms of the target material and knock out the tightly bound electrons even from the inner most shells of the atom. Now when the electron is knocked out, a vacancy is created at that place. To fill this vacancy electrons from higher shells jump to fill the

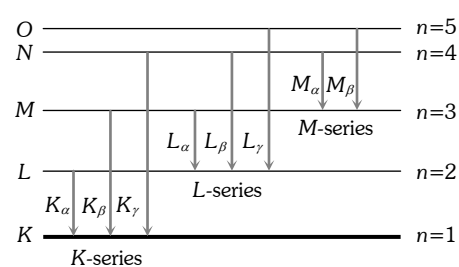
Dual Nature of Radiation and Matter

created vacancies, we know that when an electron jumps from a higher energy orbit E_1 to lower energy orbit E_2 , it radiates energy $(E_1 - E_2)$. Thus this energy difference is radiated in the form of X-rays of very small but definite wavelength which depends upon the target material. The X-ray spectrum consist of sharp lines and is called characteristic X-ray spectrum.



K, L, M, series

If the electron striking the target eject an electron from the K -shell of the atom, a vacancy is crated in the K -shell. Immediately an electron from one of the outer shell, say L -shell jumps to the K -shell, emitting an X-ray photon of energy equal to the energy difference between the two shells. Similarly, if an electron from the M -shell jumps to the K -shell, X-ray photon of higher energy is emitted. The X-ray photons emitted due to the jump of electron from the L, M, N shells to the K -shells gives $K_\alpha, K_\beta, K_\gamma$ lines of the K -series of the spectrum.



If the electron striking the target ejects an electron from the L -shell of the target atom, an electron from the M, N shells jumps to the L -shell so that X-rays photons of lesser energy are emitted. These photons form the lesser energy emission. These photons form the L -series of the spectrum. In a similar way the formation of M series, N series etc. may be explained.

Energy and wavelength of different lines

Series	Transition	Energy	Wavelength
K_α	$L_{(2)} \rightarrow K_{(1)}$	$E_L - E_K = h\nu_{K\alpha}$	$\lambda_{K\alpha} = \frac{hc}{E_L - E_K} = \frac{12375}{(E_L - E_K)eV} \text{ \AA}$
K_β	$M_{(3)} \rightarrow K_{(1)}$	$E_M - E_K = h\nu_{K\beta}$	$\lambda_{K\beta} = \frac{hc}{E_M - E_K} = \frac{12375}{(E_M - E_K)eV} \text{ \AA}$
L_α	$M_{(3)} \rightarrow L_{(2)}$	$E_M - E_L = h\nu_{L\alpha}$	$\lambda_{L\alpha} = \frac{hc}{E_M - E_L} = \frac{12375}{(E_M - E_L)eV} \text{ \AA}$

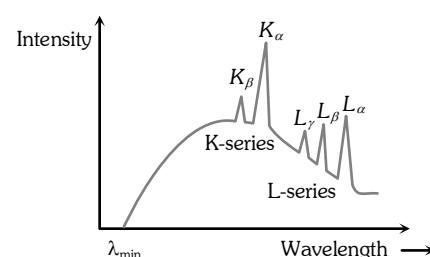
Note : \cong The wavelength of characteristic X-ray doesn't depend on accelerating voltage. It depends on the atomic number (Z) of the target material.

$$\cong \lambda_{K\alpha} < \lambda_{L\alpha} < \lambda_{M\alpha} \text{ and } \nu_{K\alpha} > \nu_{L\alpha} > \nu_{M\alpha}$$

$$\cong \lambda_{K\alpha} > \lambda_{L\beta} < \lambda_{K\gamma}$$

Intensity-wavelength graph

At certain sharply defined wavelengths, the intensity of X-rays is very

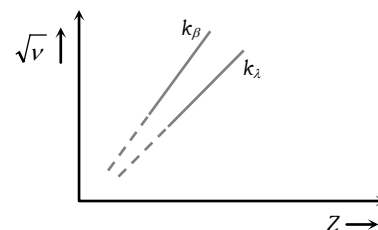


Dual Nature of Radiation and Matter

large as marked $K_\alpha, K_\beta \dots$. As shown in figure. These X-rays are known as characteristic X-rays. At other wavelengths the intensity varies gradually and these X-rays are called continuous X-rays.

Mosley's law

Mosley studied the characteristic X-ray spectrum of a number of a heavy elements and concluded that the spectra of different elements are very similar and with increasing atomic number, the spectral lines merely shift towards higher frequencies.



He also gave the following relation $\sqrt{\nu} = a(Z - b)$

where ν = Frequency of emitted line, Z = Atomic number of target, a = Proportionality constant, b = Screening constant.

Note : \cong a and b doesn't depend on the nature of target. Different values of b are as follows

$b = 1$ for K -series

$b = 7.4$ for L -series

$b = 19.2$ for M -series

TEACHING CARE

$\cong (Z - b)$ is called effective atomic number.

More about Mosley's law

(i) It supported Bohr's theory

(ii) It experimentally determined the atomic number (Z) of elements.

(iii) This law established the importance of ordering of elements in periodic table by atomic number and not by atomic weight.

(iv) Gaps in Moseley's data for $A = 43, 61, 72, 75$ suggested existence of new elements which were later discovered.

(v) The atomic numbers of Cu, Ag and Pt were established to be 29, 47 and 78 respectively.

(vi) When a vacancy occurs in the K -shell, there is still one electron remaining in the K -shell. An electron in the L -shell will feel an effective charge of $(Z - 1)e$ due to $+Ze$ from the nucleus and $-e$ from the remaining K -shell electron, because L -shell orbit is well outside the K -shell orbit.

(vii) Wave length of characteristic spectrum $\frac{1}{\lambda} = R(Z - b)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$ and energy of X-ray radiations.

$$\Delta E = h\nu = \frac{hc}{\lambda} = Rhc(Z - b)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

(viii) If transition takes place from $n_2 = 2$ to $n_1 = 1$ (K_α - line)

Dual Nature of Radiation and Matter

$$(a) a = \sqrt{\frac{3RC}{4}} = 2.47 \times 10^{15} \text{ Hz}$$

$$(b) \nu_{K\alpha} = RC(Z-1)^2 \left(1 - \frac{1}{2^2}\right) = \frac{3RC}{4} (Z-1)^2 = 2.47 \times 10^{15} (Z-1)^2 \text{ Hz}$$

(c) In general the wavelength of all the K-lines are given by $\frac{1}{\lambda_K} = R(Z-1)^2 \left(1 - \frac{1}{n^2}\right)$ where $n = 2, 3, 4, \dots$

While for K_α line $\lambda_{K\alpha} = \frac{1216}{(Z-1)} \text{ \AA}$

$$(d) E_{K\alpha} = 10.2(Z-1)^2 \text{ eV}$$

Uses of X-rays

- (i) In study of crystal structure : Structure of DNA was also determined using X-ray diffraction.
- (ii) In medical science.
- (iii) In radiograph
- (iv) In radio therapy
- (v) In engineering
- (vi) In laboratories
- (vii) In detective department
- (viii) In art the change occurring in old oil paintings can be examined by X-rays.

Concepts

- Nearly all metals emits photoelectrons when exposed to UV light. But alkali metals like lithium, sodium, potassium, rubidium and cesium emit photoelectrons even when exposed to visible light.
- Oxide coated filament in vacuum tubes is used to emit electrons at relatively lower temperature.
- Conduction of electricity in gases at low pressure takes because colliding electrons acquire higher kinetic energy due to increase in mean free path.
- Kinetic energy of cathode rays depends on both voltage and work function of cathode.
- Photoelectric effect is due to the particle nature of light.
- Hydrogen atom does not emit X-rays because it's energy levels are too close to each other.
- The essential difference between X-rays and of γ -rays is that, γ -rays emits from nucleus while X-rays from outer part of atom.
- There is no time delay between emission of electron and incidence of photon i.e. the electrons are emitted out as soon as the light falls on metal surface.
- If light were wave (not photons) it will take about an year to eject a photoelectron out of the metal surface.
- Doze of X-ray are measured in terms of produced ions or free energy via ionisation.
- Safe doze for human body per week is one Rontgen (One Rontgon is the amount of X-rays which emits $2.5 \times 10^4 \text{ J}$ free energy through ionization of 1 gm air at NTP)

Dual Nature of Radiation and Matter

Example

Example: 22 The work function of a substance is 4.0 eV . The longest wavelength of light that can cause photoelectron emission from this substance is approximately [AIEEE 2004]

- (a) 540 nm (b) 400 nm (c) 310 nm (d) 220 nm

Solution : (c) By using $\lambda_0 = \frac{12375}{W_0(\text{eV})} \Rightarrow \lambda_0 = \frac{12375}{4} = 3093.7 \text{ \AA} \approx 310 \text{ nm}$

Example: 23 Photo-energy 6 eV are incident on a surface of work function 2.1 eV . What are the stopping potential [MP PMT 2004]

- (a) -5 V (b) -1.9 V (c) -3.9 V (d) -8.1 V

Solution : (c) By using Einstein's equation $E = W_0 + K_{\text{max}} \Rightarrow 6 = 2.1 + K_{\text{max}} \Rightarrow K_{\text{max}} = 3.9 \text{ eV}$

Also $V_0 = -\frac{K_{\text{max}}}{e} = -3.9 \text{ V}$.

Example: 24 When radiation of wavelength λ is incident on a metallic surface the stopping potential is 4.8 volts . If the same surface is illuminated with radiation of double the wavelength, then the stopping potential becomes 1.6 volts . Then the threshold wavelength for the surface is [EAMCET (Engg.) 2003]

- (a) 2λ (b) 4λ (c) 6λ (d) 8λ

Solution : (b) By using $V_0 = \frac{hc}{e} \left[\frac{1}{\lambda} - \frac{1}{\lambda_0} \right]$
 $4.8 = \frac{hc}{e} \left[\frac{1}{\lambda} - \frac{1}{\lambda_0} \right]$ (i) and $1.6 = \frac{hc}{e} \left[\frac{1}{2\lambda} - \frac{1}{\lambda_0} \right]$ (ii)

From equation (i) and (ii) $\lambda_0 = 4\lambda$.

Example: 25 When radiation is incident on a photoelectron emitter, the stopping potential is found to be 9 volts . If e/m for the electron is $1.8 \times 10^{11} \text{ Ckg}^{-1}$ the maximum velocity of the ejected electrons is [Kerala (Engg.) 2002]

- (a) $6 \times 10^5 \text{ ms}^{-1}$ (b) $8 \times 10^5 \text{ ms}^{-1}$ (c) $1.8 \times 10^6 \text{ ms}^{-1}$ (d) $1.8 \times 10^5 \text{ ms}^{-1}$

Solution : (c) $\frac{1}{2} m v_{\text{max}}^2 = eV_0 \Rightarrow v_{\text{max}} = \sqrt{2 \left(\frac{e}{m} \right) \cdot V_0} = \sqrt{2 \times 1.8 \times 10^{11} \times 9} = 1.8 \times 10^6 \text{ m/s}$.

Example: 26 The lowest frequency of light that will cause the emission of photoelectrons from the surface of a metal (for which work function is 1.65 eV) will be [JIPMER 2002]

- (a) $4 \times 10^{10} \text{ Hz}$ (b) $4 \times 10^{11} \text{ Hz}$ (c) $4 \times 10^{14} \text{ Hz}$ (d) $4 \times 10^{-10} \text{ Hz}$

Solution : (c) Threshold wavelength $\lambda_0 = \frac{12375}{W_0(\text{eV})} = \frac{12375}{1.65} = 7500 \text{ \AA}$.

\therefore so minimum frequency $\nu_0 = \frac{c}{\lambda_0} = \frac{3 \times 10^8}{7500 \times 10^{-10}} = 4 \times 10^{14} \text{ Hz}$.

Example: 27 Light of two different frequencies whose photons have energies 1 eV and 2.5 eV respectively, successively illuminates a metal of work function 0.5 eV . The ratio of maximum kinetic energy of the emitted electron will be [AIEEE 2002]

- (a) $1 : 5$ (b) $1 : 4$ (c) $1 : 2$ (d) $1 : 1$

Dual Nature of Radiation and Matter

Solution : (b) By using $K_{\max} = E - W_0 \Rightarrow \frac{(K_{\max})_1}{(K_{\max})_2} = \frac{1 - 0.5}{2.5 - 0.5} = \frac{0.5}{2} = \frac{1}{4}$.

Example: 28 Photoelectric emission is observed from a metallic surface for frequencies ν_1 and ν_2 of the incident light rays ($\nu_1 > \nu_2$). If the maximum values of kinetic energy of the photoelectrons emitted in the two cases are in the ratio of 1 : k, then the threshold frequency of the metallic surface is [EAMCET (Engg.) 2001]

(a) $\frac{\nu_1 - \nu_2}{k - 1}$ (b) $\frac{k\nu_1 - \nu_2}{k - 1}$ (c) $\frac{k\nu_2 - \nu_1}{k - 1}$ (d) $\frac{\nu_2 - \nu_1}{k - 1}$

Solution : (b) By using $h\nu - h\nu_0 = k_{\max} \Rightarrow h(\nu_1 - \nu_0) = k_1$ and $h(\nu_2 - \nu_0) = k_2$

Hence $\frac{\nu_1 - \nu_0}{\nu_2 - \nu_0} = \frac{k_1}{k_2} = \frac{1}{k} \Rightarrow \nu_0 = \frac{k\nu_1 - \nu_2}{k - 1}$

Example: 29 Light of frequency $8 \times 10^{15} \text{ Hz}$ is incident on a substance of photoelectric work function 6.125 eV. The maximum kinetic energy of the emitted photoelectrons is [AFMC 2001]

(a) 17 eV (b) 22 eV (c) 27 eV (d) 37 eV

Solution : (c) Energy of incident photon $E = h\nu = 6.6 \times 10^{-34} \times 8 \times 10^{15} = 5.28 \times 10^{-18} \text{ J} = 33 \text{ eV}$.

From $E = W_0 + K_{\max} \Rightarrow K_{\max} = E - W_0 = 33 - 6.125 = 26.87 \text{ eV} \approx 27 \text{ eV}$.

Example: 30 A photo cell is receiving light from a source placed at a distance of 1 m. If the same source is to be placed at a distance of 2 m, then the ejected electron [MNR 1986; UPSEAT 2000, 2001]

- (a) Moves with one-fourth energy as that of the initial energy
 (b) Moves with one fourth of momentum as that of the initial momentum
 (c) Will be half in number
 (d) Will be one-fourth in number

Solution : (d) Number of photons \propto Intensity $\propto \frac{1}{(\text{distance})^2}$
 $\Rightarrow \frac{N_1}{N_2} = \left(\frac{d_2}{d_1}\right)^2 \Rightarrow \frac{N_1}{N_2} = \left(\frac{2}{1}\right)^2 \Rightarrow N_2 = \frac{N_1}{4}$.

Example: 31 When yellow light incident on a surface no electrons are emitted while green light can emit. If red light is incident on the surface then [MNR 1998; MH CET 2000; MP PET 2000]

- (a) No electrons are emitted (b) Photons are emitted
 (c) Electrons of higher energy are emitted (d) Electrons of lower energy are emitted

Solution : (a) $\lambda_{\text{Green}} < \lambda_{\text{Yellow}} < \lambda_{\text{Red}}$

According to the question λ_{Green} is the maximum wavelength for which photoelectric emission takes place. Hence no emission takes place with red light.

Example: 32 When a metal surface is illuminated by light of wavelengths 400 nm and 250 nm the maximum velocities of the photoelectrons ejected are v and $2v$ respectively. The work function of the metal is (h = Planck's constant, c = velocity of light in air) [EMCET (Engg.) 2000]

(a) $2hc \times 10^6 \text{ J}$ (b) $1.5hc \times 10^6 \text{ J}$ (c) $hc \times 10^6 \text{ J}$ (d) $0.5hc \times 10^6 \text{ J}$

Solution : (a) By using $E = W_0 + K_{\max} \Rightarrow \frac{hc}{\lambda} = W_0 + \frac{1}{2}mv^2$

Dual Nature of Radiation and Matter

$$\frac{hc}{400 \times 10^{-9}} = W_0 + \frac{1}{2}mv^2 \quad \dots\dots(i) \quad \text{and} \quad \frac{hc}{250 \times 10^{-9}} = W_0 + \frac{1}{2}m(2v)^2 \quad \dots\dots(ii)$$

From equation (i) and (ii) $W_0 = 2hc \times 10^6 J$.

Example: 33 The work functions of metals A and B are in the ratio 1 : 2. If light of frequencies f and $2f$ are incident on the surfaces of A and B respectively, the ratio of the maximum kinetic energies of photoelectrons emitted is (f is greater than threshold frequency of A, $2f$ is greater than threshold frequency of B) **[EAMCET (Med.) 2000]**

- (a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4

Solution : (b) By using $E = W_0 + K_{\max} \Rightarrow E_A = hf = W_A + K_A$ and $E_B = h(2f) = W_B + K_B$

So, $\frac{1}{2} = \frac{W_A + K_A}{W_B + K_B} \quad \dots\dots(i)$ also it is given that $\frac{W_A}{W_B} = \frac{1}{2} \quad \dots\dots(ii)$

From equation (i) and (ii) we get $\frac{K_A}{K_B} = \frac{1}{2}$.

Example: 34 When a point source of monochromatic light is at a distance of 0.2m from a photoelectric cell, the cut-off voltage and the saturation current are 0.6 volt and 18 mA respectively. If the same source is placed 0.6 m away from the photoelectric cell, then **[IIT-JEE 1992; MP PMT 1999]**

- (a) The stopping potential will be 0.2 V (b) The stopping potential will be 0.6 V
(c) The saturation current will be 6 mA (d) The saturation current will be 18 mA

Solution : (b) Photoelectric current (i) \propto Intensity $\propto \frac{1}{(\text{distance})^2}$. If distance becomes 0.6 m (i.e. three times) so current becomes $\frac{1}{9}$ times i.e. 2mA.

Also stopping potential is independent of intensity i.e. it remains 0.6 V.

Example: 35 In a photoemissive cell with exciting wavelength λ , the fastest electron has speed v . If the exciting wavelength is changed to $3\lambda/4$, the speed of the fastest emitted electron will be **[CBSE 1998]**

- (a) $v(3/4)^{1/2}$ (b) $v(4/3)^{1/2}$ (c) Less than $v(4/3)^{1/2}$ (d) Greater than $v(4/3)^{1/2}$

Solution : (d) From $E = W_0 + \frac{1}{2}mv_{\max}^2 \Rightarrow v_{\max} = \sqrt{\frac{2E}{m} - \frac{2W_0}{m}}$ (where $E = \frac{hc}{\lambda}$)

If wavelength of incident light changes from λ to $\frac{3\lambda}{4}$ (decreases)

Let energy of incident light changes from E to E' and speed of fastest electron changes from v to v' then

$$v = \sqrt{\frac{2E}{m} - \frac{2W_0}{m}} \quad \dots\dots(i) \quad \text{and} \quad v' = \sqrt{\frac{2E'}{m} - \frac{2W_0}{m}} \quad \dots\dots(ii)$$

$$\text{As } E \propto \frac{1}{\lambda} \Rightarrow E' = \frac{4}{3}E \text{ hence } v' = \sqrt{\frac{2\left(\frac{4}{3}E\right)}{m} - \frac{2W_0}{m}} \Rightarrow v' = \left(\frac{4}{3}\right)^{1/2} \sqrt{\frac{2E}{m} - \frac{2W_0}{m}} \left(\frac{4}{3}\right)^{1/2}$$

Dual Nature of Radiation and Matter

$$\Rightarrow v' = \left(\frac{4}{3}\right)^{1/2} \quad X = \sqrt{\frac{2E}{m} - \frac{2W_0}{m\left(\frac{4}{3}\right)^{1/2}}} > v \text{ so } v' > \left(\frac{4}{3}\right)^{1/2} v.$$

Example: 36 The minimum wavelength of X-rays produced in a coolidge tube operated at potential difference of 40 kV is

[BCECE 2003]

- (a) 0.31Å (b) 3.1Å (c) 31Å (d) 311Å

Solution : (a) $\lambda_{\min} = \frac{12375}{40 \times 10^3} = 0.309\text{Å} \approx 0.31 \text{ Å}$

Example: 37 The X-ray wavelength of L_{α} line of platinum ($Z = 78$) is 1.30Å. The X-ray wavelength of L_{α} line of Molybdenum ($Z = 42$) is

[EAMCET (Engg.) 2000]

- (a) 5.41Å (b) 4.20Å (c) 2.70Å (d) 1.35 Å

Solution : (a) The wave length of L_{α} line is given by $\frac{1}{\lambda} = R(z - 7.4)^2 \left(\frac{1}{2^2} - \frac{1}{3^2} \right) \Rightarrow \lambda \propto \frac{1}{(z - 7.4)^2}$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{(z_2 - 7.4)^2}{(z_1 - 7.4)^2} \Rightarrow \frac{1.30}{\lambda_2} = \frac{(42 - 7.4)^2}{(78 - 7.4)^2} \Rightarrow \lambda_2 = 5.41\text{Å}.$$

Example: 38 The cut off wavelength of continuous X-ray from two coolidge tubes operating at 30 kV but using different target materials (molybdenum $Z = 42$ and tungsten $Z = 74$) are

- (a) 1Å, 3Å (b) 0.3 Å, 0.2 Å (c) 0.414 Å, 0.8 Å (d) 0.414 Å, 0.414 Å

Solution : (d) Cut off wavelength of continuous X-rays depends solely on the voltage applied and does not depend on the material of the target. Hence the two tubes will have the same cut off wavelength.

$$Ve = h\nu = \frac{hc}{\lambda} \quad \text{or} \quad \lambda = \frac{hc}{Ve} = \frac{6.627 \times 10^{-34} \times 3 \times 10^8}{30 \times 10^3 \times 1.6 \times 10^{-19}} \text{m} = 414 \times 10^{-10} \text{m} = 0.414 \text{ Å}.$$

Dual Nature of Radiation and Matter

Tricky example: 3

Two photons, each of energy 2.5eV are simultaneously incident on the metal surface. If the work function of the metal is 4.5 eV , then from the surface of metal

- (a) Two electrons will be emitted (b) Not even a single electron will be emitted
(c) One electron will be emitted (d) More than two electrons will be emitted

Solution : (b) Photoelectric effect is the phenomenon of one to one elastic collision between incident photon and an electron. Here in this question one electron absorbs one photon and gets energy 2.5 eV which is lesser than 4.5 eV . Hence no photoelectron emission takes place.

Tricky example: 4

In X-ray tube when the accelerating voltage V is halved, the difference between the wavelength of K_α line and minimum wavelength of continuous X-ray spectrum

- (a) Remains constant (b) Becomes more than two times
(c) Becomes half (d) Becomes less than two times

Solution : (c) $\Delta\lambda = \lambda_{K_\alpha} - \lambda_{\min}$ when V is halved λ_{\min} becomes two times but λ_{K_α} remains the same.

$$\therefore \Delta\lambda' = \lambda_{K_\alpha} - 2\lambda_{\min} = 2(\Delta\lambda) - \lambda_{K_\alpha}$$

$$\therefore \Delta\lambda' < 2(\Delta\lambda)$$

Tricky example: 5

Molybdenum emits K_α -photons of energy 18.5 keV and iron emits K_α photons of energy 34.7 keV . The times taken by a molybdenum K_α photon and an iron K_α photon to travel 300 m are

- (a) $(3\ \mu\text{s}, 15\ \mu\text{s})$ (b) $(15\ \mu\text{s}, 3\ \mu\text{s})$ (c) $(1\ \mu\text{s}, 1\ \mu\text{s})$ (d) $(1\ \mu\text{s}, 5\ \mu\text{s})$

Solution : (c) Photon have the same speed whatever be their energy, frequency, wavelength, and origin.

$$\therefore \text{time of travel of either photon} = \frac{300}{3 \times 10^8} = 10^{-6}\text{ s} = 1\ \mu\text{ s}$$