Example: 63 Two point charges $+4q$ and $+q$ are placed at a distance L apart. A third charge Q is so placed that all the three charges are in equilibrium. Then location and magnitude of third charge will be

(a) At a distance $\frac{L}{3}$ from $+4q$ charge, $\frac{4q}{9}$

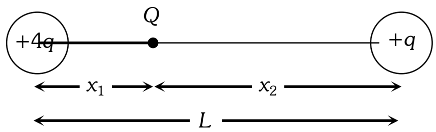
(b) At a distance $\frac{L}{3}$ from $+4q$ charge, $-\frac{4q}{9}$

(c) At a distance $\frac{2L}{3}$ from $+4q$ charge, $-\frac{4q}{9}$

(d) At a distance $\frac{2L}{3}$ from $+q$ charge, $+\frac{4q}{9}$

Solution: (c) Let third charge be placed at a distance x_1 from $+4q$ charge as shown

Now $x_1 = \frac{L}{1+\sqrt{\frac{q}{4q}}} = \frac{2L}{3} \Rightarrow x_2 = \frac{L}{3}$



For equilibrium of q, $Q = +4q\left(\frac{L/3}{L}\right)^2 = \frac{4q}{9} \Rightarrow Q = -\frac{4q}{9}$.

Example: 64 A drop of $10^{-6}\,kg$ water carries $10^{-6}\,C$ charge. What electric field should be applied to balance it's weight (assume $g = 10\ m/sec^2$)

(a) $10\,V/m$, Upward (b) $10\,V/m$, Downward (c) 0.1 V/m Downward (d) $0.1V/m$, Upward

Solution: (a) In equilibrium $QE = mg$

$E = \frac{mg}{Q} = \frac{10^{-6}\times 10}{10^{-6}} = 10\ V/m$; Since charge is positive so electric field will be upward.

Example: 65 A charged water drop of radii 0.1 μm is under equilibrium in some electric field. The charge on the drop is equivalent to electronic charge. The intensity of electric field is **[RPET 1997]**

(a) $1.61\ N/C$ (b) $25.2\ N/C$ (c) $262\ N/C$ (d) $1610\ N/C$

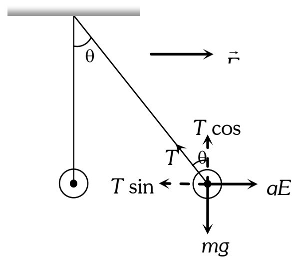
Solution: (c) In equilibrium $QE = mg$; $E = \frac{mg}{Q} = \frac{\left(\frac{4}{3}\pi r^3\rho\right).g}{Q} = \frac{4}{3}\frac{\times(3.14)(0.1\times10^{-6})^3\times10^3\times10}{1.6\times10^{-19}} = 262\ N/C$

Example: 66 The bob of a pendulum of mass $8\,\mu g$ carries an electric charge of 39.2×10^{-10} coulomb in an electric field of $20\times10^3\,volt/meter$ and it is at rest. The angle made by the pendulum with the vertical will be

(a) 27° (b) 45° (c) 87° (d) 127°

Solution: (b) $T\sin\theta = qE$, $T\cos\theta = mg$

$\therefore\ \tan\theta = \frac{qE}{mg}$



$$\tan\theta = \frac{39.2\times10^{-10}\times20\times10^{3}}{8\times10^{-6}\times9.8} = 1$$

$\Rightarrow \quad \theta = 45^{o}$

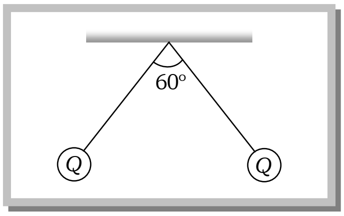
Example: 67 Two small spherical balls each carrying a charge $Q = 10\,\mu C$ (10 micro-coulomb) are suspended by two insulating threads of equal lengths 1 m each, from a point fixed in the ceiling. It is found that in equilibrium threads are separated by an angle 60° between them, as shown in the figure. What is the tension in the threads. (Given : $\frac{1}{(4\pi\varepsilon_0)} = 9\times10^{9}\,Nm/C^2$)

(a) 18 N

(b) 1.8 N

(c) 0.18 N

(d) None of these



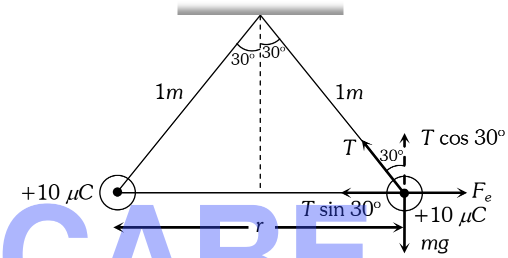
Solution: (b) From the geometry of figure

$r = 1m$

In the condition of equilibrium $T\sin30^{o} = F_e$

$$T\times\frac{1}{2} = 9\times10^{9}.\frac{(10\times10^{-6})^2}{1^2}$$

$\Rightarrow T = 1.8\,N$



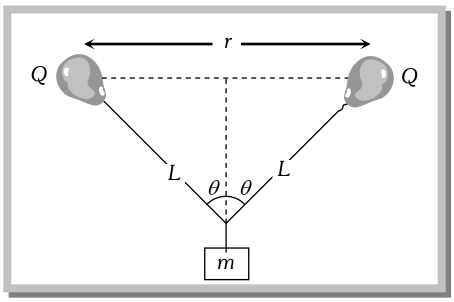
Example: 68 Two similar balloons filled with helium gas are tied to L m long strings. A body of mass m is tied to another ends of the strings. The balloons float on air at distance r. If the amount of charge on the balloons is same then the magnitude of charge on each balloon will be

(a) $\left[\frac{mgr^2}{2k}\tan\theta\right]^{1/2}$

(b) $\left[\frac{2k}{mgr^2}\tan\theta\right]^{1/2}$

(c) $\left[\frac{mgr}{2k}\cot\theta\right]^{1/2}$

(d) $\left[\frac{2k}{mgr}\tan\theta\right]^{1/2}$



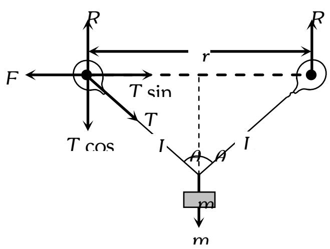
Solution: (a) In equilibrium

$2R = mg$ .... (i) $F_e = T\sin\theta$ .... (ii) $R = T\cos\theta$ .... (iii)

From equation (i) and (iii)

$2T\cos\theta = mg$ .... (iv)

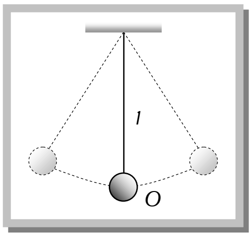
Dividing equation (ii) by equation (iv)



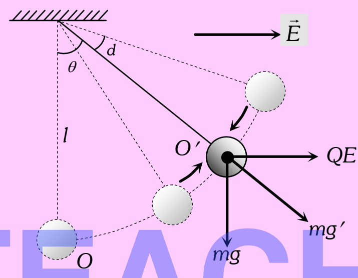
$$\frac{1}{2}\tan\theta = \frac{F_e}{mg} \Rightarrow \frac{1}{2}\tan\theta = \frac{k\frac{Q^2}{r^2}}{mg} \Rightarrow \theta = \left(\frac{mgr^2}{2k}\tan\theta\right)^{1/2}$$

Time Period of Oscillation of a Charged Body.

(1) **Simple pendulum based :** If a simple pendulum having length l and mass of bob m oscillates about it's mean position than it's time period of oscillation $T = 2\pi\sqrt{\frac{l}{g}}$



Case – 1 : If some charge say $+Q$ is given to bob and an electric field E is applied in the direction as shown in figure then equilibrium position of charged bob (point charge) changes from O to O'.



On displacing the bob from it's equilibrium position 0′. It will oscillate under the effective acceleration g', where

$$mg' = \sqrt{(mg)^2 + (QE)^2}$$

$$\Rightarrow g' = \sqrt{g^2 + (QE/m)^2}$$

Hence the new time period is $T_1 = 2\pi\sqrt{\frac{l}{g'}}$

$$T_1 = 2\pi\sqrt{\frac{l}{\left(g^2 + (QE/m)^2\right)^{\frac{1}{2}}}}$$

Since $g' > g$, hence $T_1 < T$

i.e. time period of pendulum will decrease.

Case – 2 : If electric field is applied in the downward direction then.

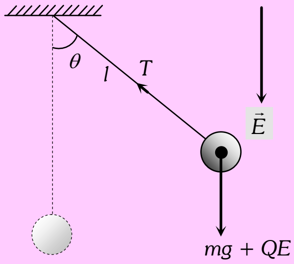
Effective acceleration

$g' = g + QE/m$

So new time period

$$T_2 = 2\pi\sqrt{\frac{l}{g + (QE/m)}}$$

$T_2 < T$



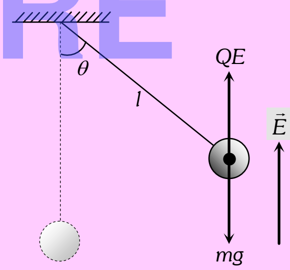
Case – 3 : In case 2 if electric field is applied in upward direction then, effective acceleration.

$g' = g - QE/m$

So new time period

$$T_3 = 2\pi\sqrt{\frac{l}{g - (QE/m)}}$$

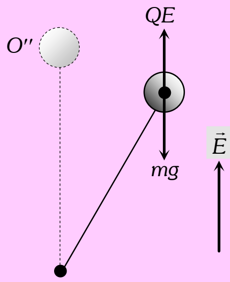
$T_3 > T$



Case – 4 : In the case 3,

if $T_3 = \frac{T}{2}$ i.e., $2\pi\sqrt{\frac{l}{(g - QE/m)}}$

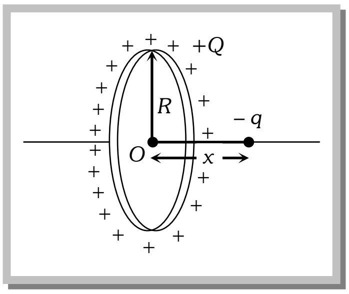
$= \frac{1}{2} 2\pi\sqrt{\frac{l}{g}}$ $\Rightarrow QE = 3\ mg$



i.e., effective vertical force (gravity + electric) on the bob $= mg - 3\ mg = -2\ mg$, hence the equilibrium position O'' of the bob will be above the point of suspension and bob will oscillate under on effective acceleration $2g$ directed upward.

Hence new time period $T_4 = 2\pi\sqrt{\frac{l}{2g}}$, $T_4 < T$

(2) **Charged circular ring :** A thin stationary ring of radius R has a positive charge $+Q$ unit. If a negative charge $-q$ (mass m) is placed at a small distance x from the centre. Then motion of the particle will be simple harmonic motion.



Electric field at the location of $-q$ charge $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qx}{(x^2+R^2)^{\frac{3}{2}}}$

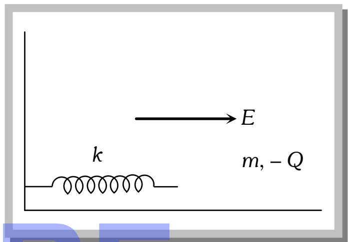
Since $x << R$, So x^2 neglected hence $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qx}{R^3}$

Force experienced by charge $-q$ is $F = -q\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qx}{R^3}$

$\Rightarrow F \propto -x$ hence motion is simple harmonic

Having time period $\boldsymbol{T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mR^3}{Qq}}}$

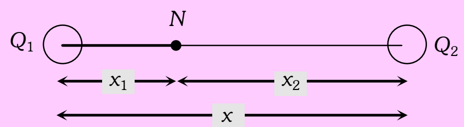
(3) **Spring mass system :** A block of mass m containing a negative charge $-Q$ is placed on a frictionless horizontal table and is connected to a wall through an unstretched spring of spring constant k as shown. If electric field E applied as shown in figure the block experiences an electric force, hence spring compress and block comes in new position. This is called the equilibrium position of block under the influence of electric field. If block compressed further or stretched, it execute oscillation having time period $\boldsymbol{T = 2\pi\sqrt{\frac{m}{k}}}$. Maximum compression in the spring due to electric field $= \frac{QE}{k}$



Neutral Point.

A neutral point is a point where resultant electrical field is zero. It is obtained where two electrical field are equal and opposite. Thus neutral points can be obtained only at those points where the resultant field is subtractive. Thus it can be obtained.

(1) **At an internal point along the line joining two like charges (Due to a system of two like point charge) :** Suppose two like charges. Q_1 and Q_2 are separated by a distance x from each other along a line as shown in following figure.



If N is the neutral point at a distance x_1 from Q_1 and at a distance $x_2 (= x - x_1)$ from Q_2 then –

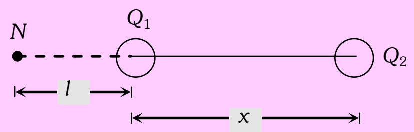
At N $|\textbf{E.F. due to } Q_1| = |\textbf{E.F. due to } Q_2|$

i.e., $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_1}{x_1^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_2}{x_2^2} \Rightarrow \frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2$

Short trick : $\boldsymbol{x_1 = \frac{x}{1+\sqrt{Q_2/Q_1}}}$ and $\boldsymbol{x_2 = \frac{x}{1+\sqrt{Q_1/Q_2}}}$

Note : ≅ In the above formula if $Q_1 = Q_2$, neutral point lies at the centre so remember that resultant field at the midpoint of two equal and like charges is zero.

(2) **At an external point along the line joining two like charges (Due to a system of two unlike point charge) :** Suppose two unlike charge Q_1 and Q_2 separated by a distance x from each other.



Here neutral point lies outside the line joining two unlike charges and also it lies nearer to charge which is smaller in magnitude.

If $|\boldsymbol{Q_1}| < |\boldsymbol{Q_2}|$ then neutral point will be obtained on the side of Q_1, suppose it is at a distance l from Q_1

Hence at neutral point ; $\frac{kQ_1}{l^2} = \frac{kQ_2}{(x+l)^2} \Rightarrow \frac{Q_1}{Q_2} = \left(\frac{l}{x+l}\right)^2$

Short trick : $\boldsymbol{l = \frac{x}{(\sqrt{Q_2/Q_1} - 1)}}$

Note : ≅ In the above discussion if $|Q_1| = |Q_2|$ neutral point will be at infinity.

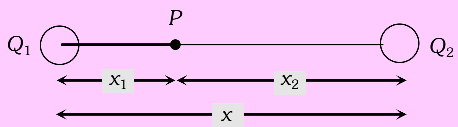
Zero Potential Due to a System of Two Point Charge.

If both charges are like then resultant potential is not zero at any finite point because potentials due to like charges will have same sign and can therefore never add up to zero. Such a point can be therefore obtained only at infinity.

If the charges are unequal and unlike then all such points where resultant potential is zero lies on a closed curve, but we are interested only in those points where potential is zero along the line joining the two charges.

Two such points exist, one lies inside and one lies outside the charges on the line joining the charges. Both the above points lie nearer the smaller charge, as potential created by the charge larger in magnitude will become equal to the potential created by smaller charge at the desired point at larger distance from it.

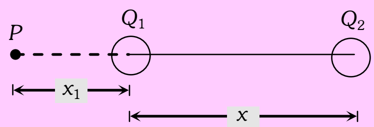
I. For internal point :



(It is assumed that $|Q_1| < |Q_2|$).

$$\frac{Q_1}{x_1} = \frac{Q_2}{(x - x_1)} \Rightarrow \boldsymbol{x_1 = \frac{x}{(Q_2/Q_1 + 1)}}$$

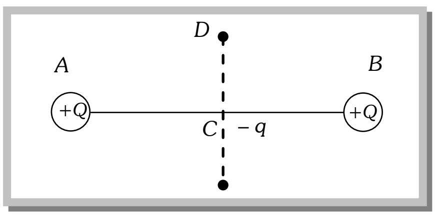
II. For External point :



$$\frac{Q_1}{x_1} = \frac{Q_2}{(x + x_1)} \Rightarrow \boldsymbol{x_1 = \frac{x}{(Q_2/Q_1 - 1)}}$$

Examples based on oscillation of charge and neutral

Example: 69 Two similar charges of $+Q$ as shown in figure are placed at points A and B. $-q$ charge is placed at point C midway between A and B. $-q$ charge will oscillate if



(a) It is moved towards A

(b) It is moved towards B

(c) It is moved along CD

(d) Distance between A and B is reduced

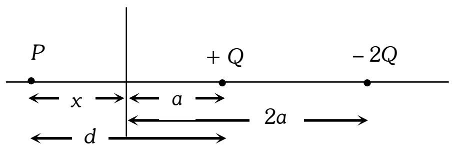
Solution: (c) When $-q$ charge displaced along CD, a restoring force act on it which causes oscillation.

Example: 70 Two point charges $(+Q)$ and $(-2Q)$ are fixed on the X-axis at positions a and $2a$ from origin respectively. At what position on the axis, the resultant electric field is zero

(a) Only $x = \sqrt{2}\,a$ (b) Only $x = -\sqrt{2}\,a$ (c) Both $x = \pm\sqrt{2}\,a$ (d) $x = \frac{3a}{2}$ only

Solution: (b) Let the electric field is zero at a point P distance d from the charge $+Q$ so at P. $\frac{k.Q}{d^2} + \frac{k(-2Q)}{(a+d)^2} = 0$

$$\Rightarrow \frac{1}{d^2} = \frac{2}{(a+d)^2} \Rightarrow d = \frac{a}{(\sqrt{2}-1)}$$



Since $d > a$ *i.e.* point P must lies on negative x-axis as shown at a distance x from origin hence $x = d - a = \frac{a}{\sqrt{2}-1} - a = \sqrt{2}a$. Actually P lies on negative x-axis so $x = -\sqrt{2}\,a$.

Example: 71 Two charges $9e$ and $3e$ are placed at a distance r. The distance of the point where the electric field intensity will be zero is **[MP PMT 1989]**

(a) $\frac{r}{(\sqrt{3}+1)}$ from 9e charge

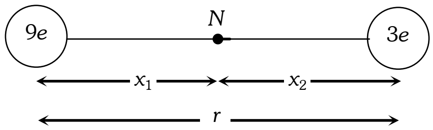
(b) $\frac{r}{1+\sqrt{1/3}}$ from 9e charge

(c) $\frac{r}{(1-\sqrt{3})}$ from 3e charge

(d) $\frac{r}{1+\sqrt{1/3}}$ from 3e charge

Solution: (b) Suppose neutral point is obtained at a distance x_1 from charge 9e and x_2 from charge 3e

By using $x_1 = \frac{x}{1+\sqrt{\frac{Q_2}{Q_1}}} = \frac{r}{1+\sqrt{\frac{3e}{9e}}} = \frac{r}{\left(1+\frac{1}{\sqrt{3}}\right)}$



Example: 72 Two point charges $-Q$ and $2Q$ are separated by a distance R, neutral point will be obtained at

(a) A distance of $\frac{R}{(\sqrt{2}-1)}$ from $-Q$ charge and lies between the charges.

(b) A distance of $\frac{R}{(\sqrt{2}-1)}$ from $-Q$ charge on the left side of it

(c) A distance of $\frac{R}{(\sqrt{2}-1)}$ from $2Q$ charge on the right side of it

(d) A point on the line which passes perpendicularly through the centre of the line joining $-Q$ and $2Q$ charge.

Solution: (b) As already we discussed neutral point will be obtained on the side of charge which is smaller in magnitude *i.e.* it will obtained on the left side of $-Q$ charge and at a distance.

$$l = \frac{R}{\sqrt{\frac{2Q}{Q}}-1} \Rightarrow l = \frac{R}{(\sqrt{2}-1)}$$

Example: 73 A charge of $+4\mu C$ is kept at a distance of 50 *cm* from a charge of $-6\mu C$. Find the two points where the potential is zero

(a) Internal point lies at a distance of 20 *cm* from $4\mu C$ charge and external point lies at a distance of 100 *cm* from $4\mu C$ charge.

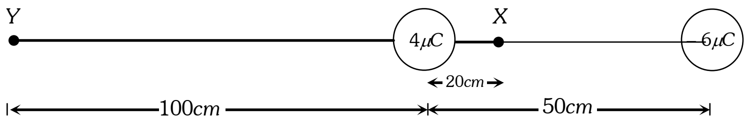
(b) Internal point lies at a distance of 30 *cm* from $4\mu C$ charge and external point lies at a distance of 100 *cm* from $4\mu C$ charge

(c) Potential is zero only at 20 *cm* from $4\mu C$ charge between the two charges

(d) Potential is zero only at 20 *cm* from $-6\mu C$ charge between the two charges

Solution: (a) For internal point X, $x_1 = \frac{x}{\left(\frac{Q_2}{Q_1}+1\right)} = \frac{50}{\frac{6}{4}+1} = 20cm$ and for external point Y,

$$x_1 = \frac{x}{\left(\frac{Q_2}{Q_1}-1\right)} = \frac{50}{\frac{6}{4}-1} = 100cm$$



Tricky example: 9

Two equal negative charges – q are fixed at points (0, a) and (0, – a) on the y-axis. A positive charge Q is released from rest at the point (2a, 0) on the x-axis. The charge Q will

[IIT-JEE 1984, Bihar MEE 1995, MP PMT 1996]

(a) Execute simple harmonic motion about the origin

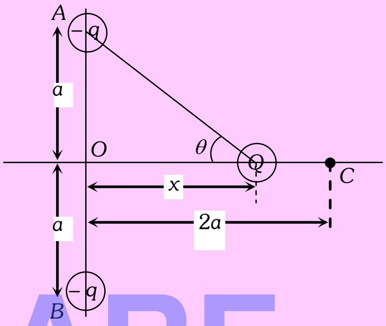
(b) Move to the origin and remains at rest

(c) Move to infinity

(d) Execute oscillatory but not simple harmonic motion.

Solution: (d) By symmetry of problem the components of force on Q due to charges at A and B along y-axis will cancel each other while along x-axis will add up and will be along CO. Under the action of this force charge Q will move towards O. If at any time charge Q is at a distance x from O.

$$F \Rightarrow 2F\cos\theta = 2\frac{1}{4\pi\varepsilon_0}\frac{-qQ}{(a^2+x^2)} \times \frac{x}{(a^2+x^2)^{1/2}}$$



$$\text{i.e., } F = -\frac{1}{4\pi\varepsilon_0}.\frac{2qQx}{\left(a^2+x^2\right)^{3/2}}$$

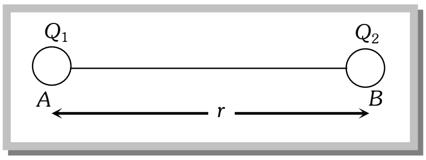
As the restoring force F is not linear, motion will be oscillatory (with amplitude 2a) but not simple harmonic.

Electric Potential Energy.

(1) **Potential energy of a charge :** Work done in bringing the given charge from infinity to a point in the electric field is known as potential energy of the charge. Potential can also be written as potential energy per unit charge. *i.e.* $V = \frac{W}{Q} = \frac{U}{Q}$.

(2) **Potential energy of a system of two charges :** Since work done in bringing charge Q_2 from ∞ to point B is $W = Q_2 V_B$, where V_B is potential of point B due to charge Q_1 i.e. $V_B = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r}$

So, $$W = U_2 = \frac{1}{4\pi\varepsilon_0}.\frac{Q_1Q_2}{r}$$

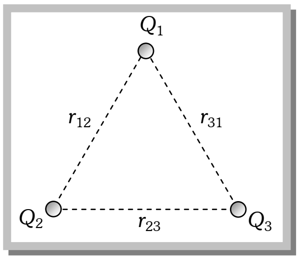


This is the potential energy of charge Q_2, similarly potential energy of charge Q_1 will be $U_1 = \frac{1}{4\pi\varepsilon_0}.\frac{Q_1Q_2}{r}$

Hence potential energy of Q_1 = Potential energy of Q_2 = potential energy of system $U = k\frac{Q_1Q_2}{r}$ (in C.G.S. $U = \frac{Q_1Q_2}{r}$)

Note : ≅Electric potential energy is a scalar quantity so in the above formula take sign of Q_1 and Q_2.

(3) **Potential energy of a system of *n* charges :** In a system of *n* charges electric potential energy is calculated for each pair and then all energies so obtained are added algebraically. *i.e.* $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1Q_2}{r_{12}} + \frac{Q_2Q_3}{r_{23}} + ...\right]$ and in case of continuous distribution of charge. As $dU = dQ.V \Rightarrow U = \int V\, dQ$



e.g. Electric potential energy for a system of three charges

Potential energy $= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1Q_2}{r_{12}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_3Q_1}{r_{31}}\right]$

While potential energy of any of the charge say Q_1 is $\frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1Q_2}{r_{12}} + \frac{Q_3Q_1}{r_{31}}\right]$

Note : ≅For the expression of total potential energy of a system of *n* charges consider $\frac{n(n-1)}{2}$ number of pair of charges.

(4) **Electron volt (*eV*) :** It is the smallest practical unit of energy used in atomic and nuclear physics. As electron volt is defined as "the energy acquired by a particle having one quantum of charge 1*e* when accelerated by 1*volt*" *i.e.* $1eV = 1.6\times10^{-19}C\times\frac{1J}{C} = 1.6\times10^{-19}J = 1.6\times10^{-12}\,erg$

Energy acquired by a charged particle in *eV* when it is accelerated by *V volt* is *E* = (charge in quanta) × (p.d. in *volt*)

Commonly asked examples :

S.No.	Charge	Accelerated by p.d.	Gain in K.E.
(i)	Proton	$5\times10^4\,V$	$K = e\times5\times10^4\,V = 5\times10^4\,eV = 8\times10^{-15}\,J$ **[JIPMER 1999]**
(ii)	Electron	$100\,V$	$K = e\times100\,V = 100\,eV = 1.6\times10^{-17}\,J$ **[MP PMT 2000; AFMC 1999]**
(iii)	Proton	$1\,V$	$K = e\times1\,V = 1\,eV = 1.6\times10^{-19}\,J$ **[CBSE 1999]**
(iv)	$0.5\,C$	$2000\,V$	$K = 0.5\times2000 = 1000\,J$ **[JIPMER 2002]**
(v)	α-particle	$10^6\,V$	$K = (2e)\times10^6\,V = 2\,MeV$ **[MP PET/PMT 1998]**

(5) **Electric potential energy of a uniformly charged sphere :** Consider a uniformly charged sphere of radius *R* having a total charge *Q*. The electric potential energy of this sphere is equal to the work done in bringing the charges from infinity to assemble the sphere.

$$U = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

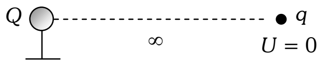
(6) **Electric potential energy of a uniformly charged thin spherical shell :**

$$U = \frac{Q^2}{8\pi\varepsilon_0 R}$$

(7) **Energy density :** The energy stored per unit volume around a point in an electric field is given by $U_e = \frac{U}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$. If in place of vacuum some medium is present then $U_e = \frac{1}{2}\varepsilon_0 \varepsilon_r E^2$

Concepts

- *Electric potential energy is not localised but is distributed all over the field*
- *If a charge moves from one position to another position in an electric field so it's potential energy change and work done in this changing is* $W = U_f - U_i$
- *If two similar charge comes closer potential energy of system increases while if two dissimilar charge comes closer potential energy of system decreases.*



Examples based on electric potential energy

Example: 74 If the distance of separation between two charges is increased, the electrical potential energy of the system **[AMU 1998]**

(a) May increases or decrease
(b) Decreases
(c) Increase
(d) Remain the same

Solution: (a) Since we know potential energy $U = k.\frac{Q_1 Q_2}{r}$

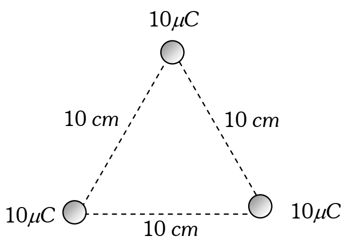
As r increases, U decreases in magnitude. However depending upon the fact whether both charges are similar or disimilar, U may increase or decrease.

Example: 75 Three particles, each having a charge of $10\mu C$ are placed at the corners of an equilateral triangle of side $10cm$. The electrostatic potential energy of the system is (Given $\frac{1}{4\pi\varepsilon_0} = 9\times 10^9 N - m^2/C^2$) **[AMU 1998]**

(a) Zero
(b) Infinite
(c) $27\ J$
(d) $100\ J$

Solution: (c) Potential energy of the system,

$$U = 9\times 10^9 \left[\frac{(10\times 10^{-6})^2}{0.1}\times 3\right] = 27\ J$$

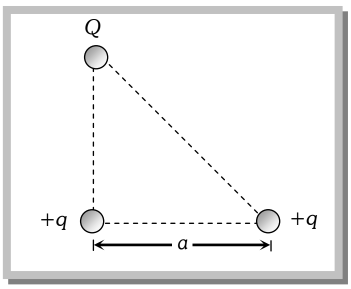


Example: 76 Three charges Q, $+q$ and $+q$ are placed at the vertices of a right-angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero if Q is equal to **[IIT (Screening) 2000]**

(a) $\frac{-q}{1+\sqrt{2}}$

(b) $\frac{-\sqrt{2}q}{1+\sqrt{2}}$

(c) $-2q$



(d) $+q$

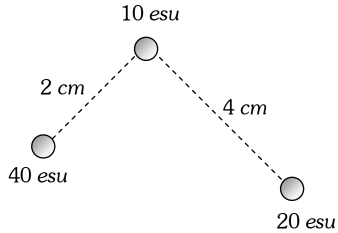
Solution: (b) Potential energy of the configuration $U = k.\frac{Qq}{a} + \frac{k.q^2}{a} + k.\frac{Qq}{a\sqrt{2}} = 0 \Rightarrow Q = \frac{-\sqrt{2}\,q}{\sqrt{2}+1}$

Example: 77 A charge 10 *e.s.u.* is placed at a distance of 2 *cm* from a charge 40 *e.s.u.* and 4 *cm* from another charge of 20 *e.s.u.* The potential energy of the charge 10 *e.s.u.* is (in *ergs*)

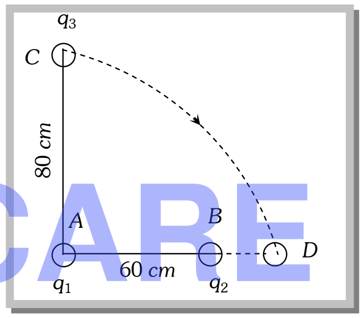
(a) 87.5 (b) 112.5 (c) 150 (d) 250

Solution: (d) Potential energy of 10 *e.s.u.* charge is

$U = \frac{10 \times 40}{2} + \frac{10 \times 20}{4} = 250\, erg.$



Example: 78 In figure are shown charges $q_1 = +2 \times 10^{-8}$ *C* and $q_2 = -0.4 \times 10^{-8}$ *C*. A charge $q_3 = 0.2 \times 10^{-8}$ *C* in moved along the arc of a circle from *C* to *D*. The potential energy of q_3 **[CPMT 1986]**



(a) Will increase approximately by 76%

(b) Will decreases approximately by 76%

(c) Will remain same

(d) Will increases approximately by 12%

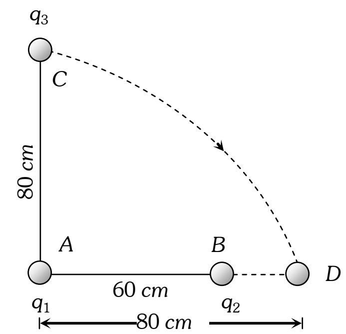
Solution: (b) Initial potential energy of q_3 $U_i = \left(\frac{q_1 q_3}{0.8} + \frac{q_2 q_3}{1}\right) \times 9 \times 10^9$

Final potential energy of q_3 $U_f = \left(\frac{q_1 q_3}{0.8} + \frac{q_2 q_3}{0.2}\right) \times 9 \times 10^9$

Change in potential energy $= U_f - U_i$

Now percentage change in potential energy $= \frac{U_f - U_i}{u_i} \times 100$

$= \frac{q_2 q_3\left(\frac{1}{0.2} - 1\right) \times 100}{q_3\left(\frac{q_1}{0.8} + \frac{q_2}{1}\right)}$ On putting the values $\simeq -76\%$



Tricky example: 10

Three charged particles are initially in position 1. They are free to move and they come in position 2 after some time. Let U_1 and U_2 be the electrostatics potential energies in position 1 and 2. Then

(a) $U_1 > U_2$ (b) $U_2 > U_1$

(c) $U_1 = U_2$ (d) $U_2 \geq U_1$

Solution: (a) Particles move in a direction where potential energy of the system is decreased.

Motion of Charged Particle in an Electric Field.

(1) **When charged particle initially at rest is placed in the uniform field :**

Let a charge particle of mass *m* and charge *Q* be initially at rest in an electric field of strength *E*

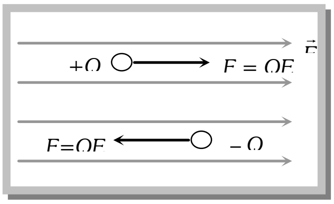


Fig. (A)

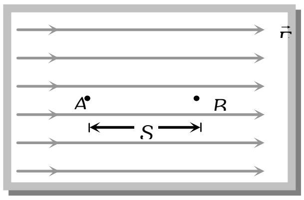


Fig. (B)

(i) **Force and acceleration :** The force experienced by the charged particle is $F = QE$. Positive charge experiences force in the direction of electric field while negative charge experiences force in the direction opposite to the field. [Fig. (A)]

Acceleration produced by this force is $a = \frac{F}{m} = \frac{QE}{m}$

Since the field *E* in constant the acceleration is constant, thus motion of the particle is uniformly accelerated.

(ii) **Velocity :** Suppose at point *A* particle is at rest and in time *t*, it reaches the point *B* [Fig. (B)]

V = Potential difference between *A* and *B*; *S* = Separation between *A* and *B*

(a) By using $v = u + at$, $\quad v = 0 + Q\frac{E}{m}t$, $\quad \Rightarrow \quad v = \frac{QEt}{m}$

(b) By using $v^2 = u^2 + 2as$, $v^2 = 0 + 2\times\frac{QE}{m}\times s \quad v^2 = \frac{2QV}{m} \quad \left\{\because E = \frac{V}{s}\right\} \Rightarrow v = \sqrt{\frac{2QV}{m}}$

(iii) **Momentum :** Momentum $p = mv$, $p = m\times\frac{QEt}{m} = QEt$ or $p = m\times\sqrt{\frac{2QV}{m}} = \sqrt{2mQV}$

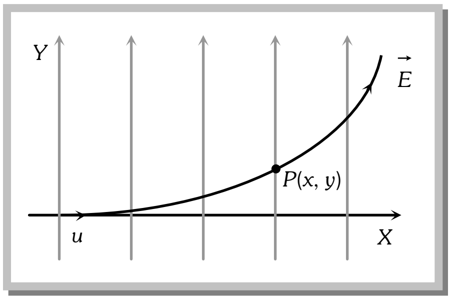
(iv) **Kinetic energy :** Kinetic energy gained by the particle in time *t* is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{(QEt)^2}{m} = \frac{Q^2E^2t^2}{2m}$ or $K = \frac{1}{2}m\times\frac{2QV}{m} = QV$

(2) **When a charged particle enters with an initial velocity at right angle to the uniform field :**

When charged particle enters perpendicularly in an electric field, it describe a parabolic path as shown

(i) Equation of trajectory : Throughout the motion particle has uniform velocity along *x*-axis and horizontal displacement (*x*) is given by the equation $x = ut$

Since the motion of the particle is accelerated along *y*–axis, we will use equation of motion for uniform acceleration to determine displacement *y*. From $S = ut + \frac{1}{2}at^2$

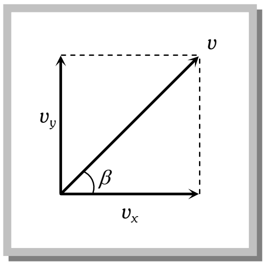


We have $u = 0$ (along y-axis) so $y = \frac{1}{2}at^2$

i.e., displacement along y-axis will increase rapidly with time (since $y \propto t^2$)

From displacement along x-axis $t = \frac{x}{u}$

So $y = \frac{1}{2}\left(\frac{QE}{m}\right)\left(\frac{x}{u}\right)^2$; this is the equation of parabola which shows $y \propto x^2$



(ii) Velocity at any instant : At any instant t, $v_x = u$ and $v_y = \frac{QEt}{m}$

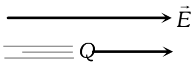
So $$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + \frac{Q^2E^2t^2}{m^2}}$$

If β is the angle made by v with x-axis than $\tan\beta = \frac{v_y}{v_x} = \frac{QEt}{mu}$.

Concepts

- *An electric field is completely characterized by two physical quantities Potential and Intensity. Force characteristic of the field is intensity and work characteristic of the field is potential.*

- *If a charge particle (say positive) is left free in an electric field, it experiences a force $(F = QE)$ in the direction of electric field and moves in the direction of electric field (which is desired by electric field), so its kinetic energy increases, potential energy decreases, then work is done by the electric field and it is negative.*



Examples based on motion of

Example: 79 An electron (mass $= 9.1 \times 10^{-31}\,kg$ and charge $= 1.6 \times 10^{-19}\,coul.$) is sent in an electric field of intensity $1 \times 10^6\,V/m$. How long would it take for the electron, starting from rest, to attain one–tenth the velocity of light

(a) $1.7 \times 10^{-12}\,sec$ (b) $1.7 \times 10^{-6}\,sec$ (c) $1.7 \times 10^{-8}\,sec$ (d) $1.7 \times 10^{-10}\,sec$

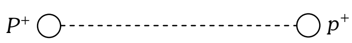
Solution: (b) By using $v = \frac{QEt}{m} \Rightarrow \frac{1}{10} \times 3 \times 10^8 = \frac{(1.6 \times 10^{-19}) \times 10^6 \times t}{9.1 \times 10^{-31}} \Rightarrow t = 1.7 \times 10^{-10}\,sec.$

Example: 80 Two protons are placed $10^{-10}\,m$ apart. If they are repelled, what will be the kinetic energy of each proton at very large distance

(a) $23 \times 10^{-19}\,J$ (b) $11.5 \times 10^{-19}\,J$ (c) $2.56 \times 10^{-19}\,J$ (d) $2.56 \times 10^{-28}\,J$

Solution: (d) Potential energy of the system when protons are separated by a distance of $10^{-10}\,m$ is

$$U = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{10^{-10}} = 23 \times 10^{-19}\,J$$



According to law of conservation of energy at very larger distance, this energy is equally distributed in both the protons as their kinetic energy hence *K.E.* of each proton will be 11.5×10^{-19} J.

Example: 81 A particle *A* has a charge $+q$ and particle *B* has charge $+4q$ with each of them having the same mass *m*. When allowed to fall from rest through the same electrical potential difference, the ratio of their speeds $\frac{v_A}{v_B}$ will becomes **[BHU 1995; MNR 1991]**

(a) 2 : 1 (b) 1 : 2 (c) 1 : 4 (d) 4 : 1

Solution: (b) We know that kinetic energy $K = \frac{1}{2}mv^2 = QV$. Since, *m* and *V* are same so, $v^2 \propto Q \Rightarrow$

$$\frac{v_A}{v_B} = \sqrt{\frac{Q_A}{Q_B}} = \sqrt{\frac{q}{4q}} = \frac{1}{2}.$$

Example: 82 How much kinetic energy will be gained by an α – particle in going from a point at 70 *V* to another point at 50 *V* **[RPET 1996]**

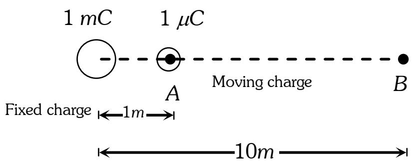
(a) 40 *eV* (b) 40 *keV* (c) 40 *MeV* (d) 0 *eV*

Solution: (a) Kinetic energy $K = Q\Delta V \Rightarrow K = (2e)(70-50)V = 40\,eV$

Example: 83 A particle of mass 2*g* and charge $1\mu C$ is held at a distance of 1 metre from a fixed charge of 1*mC*. If the particle is released it will be repelled. The speed of the particle when it is at a distance of 10 metres from the fixed charge is **[CPMT 1989]**

(a) 100 *m/s* (b) 90 *m/s* (c) 60 *m/s* (d) 45 *m/s*

Solution: (b) According to conservation of energy



Energy of moving charge at A = Energy of moving charge at B

$$9 \times 10^9 \times \frac{10^{-3} \times 10^{-6}}{1} = 9 \times 10^9 \times \frac{10^{-3} \times 10^{-6}}{10} + \frac{1}{2} \times (2 \times 10^{-3})v^2$$

$\Rightarrow$ $v^2 = 8100$ $\Rightarrow$ $v = 90$ m/sec

Tricky example: 11

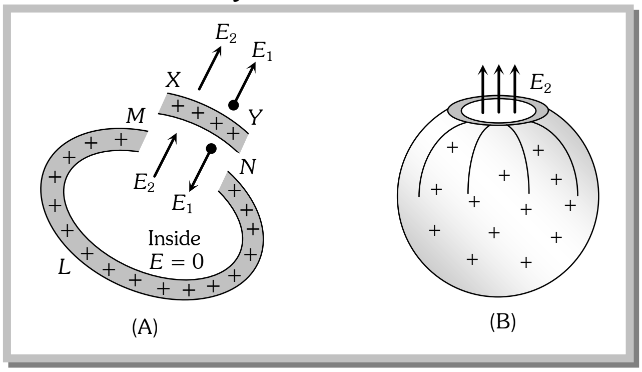
A mass of 1*g* carrying charge *q* falls through a potential difference *V*. The kinetic energy acquired by it is *E*. When a mass of 2*g* carrying the charge *q* falls through a potential difference *V*. What will be the kinetic energy acquired by it

(a) 0.25 *E* (b) 0.50 *E* (c) 0.75 *E* (d) *E*

Solution: (d) In electric field kinetic energy gain by the charged particle $K = qV$. Which depends charge and potential difference applied but not on the mass of the charged particle.

Force on a Charged Conductor.

To find force on a charged conductor (due to repulsion of like charges) imagine a small part XY to be cut and just separated from the rest of the conductor MLN. The field in the cavity due to the rest of the conductor is E_2, while field due to small part is E_1. Then



Inside the conductor $E = E_1 - E_2 = 0$ or $E_1 = E_2$

Outside the conductor $E = E_1 + E_2 = \frac{\sigma}{\varepsilon_0}$

Thus $E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$

To find force, imagine charged part XY (having charge $\sigma\, dA$ placed in the cavity MN having field E_2). Thus force $dF = (\sigma\, dA)E_2$ or $dF = \frac{\sigma^2}{2\varepsilon_0} dA$. The force per unit area or electric pressure is $\frac{dF}{dA} = \frac{\sigma^2}{2\varepsilon_0}$

The force is always outwards as $(\pm\sigma)^2$ is positive *i.e.*, whether charged positively or negatively, this force will try to expand the charged body.

A soap bubble or rubber balloon expands on given charge to it (charge of any kind + or –).

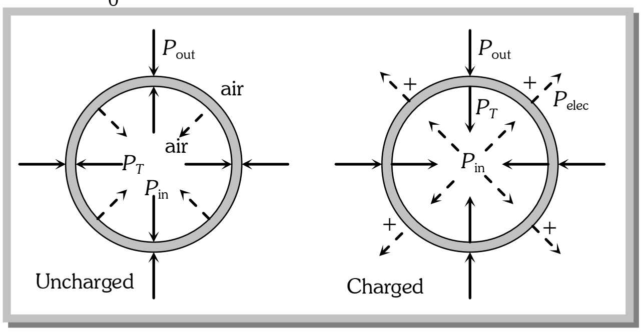
Equilibrium of Charged Soap Bubble.

For a charged soap bubble of radius R and surface tension T and charge density σ. The pressure due to surface tension $4\frac{T}{R}$ and atmospheric pressure P_{out} act radially inwards and the electrical pressure (P_{el}) acts radially outward.

The total pressure inside the soap bubble $P_{in} = P_{out} + \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}$

Excess pressure inside the charged soap bubble $P_{in} - P_{out} = P_{excess} = \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}$. If air pressure inside and outside are assumed equal then $P_{in} = P_{out}$ *i.e.*, $P_{excess} = 0$. So, $\frac{4T}{R} = \frac{\sigma^2}{2\varepsilon_0}$

This result give us the following formulae



(1) Radius of bubble $R = \frac{8\,\varepsilon_0\, T}{\sigma^2}$

(2) Surface tension $T = \frac{\sigma^2 R}{8\varepsilon_0}$

(3) Total charge on the bubble $Q = 8\pi R\sqrt{2\varepsilon_0 TR}$

(4) Electric field intensity at the surface of the bubble $E = \sqrt{\frac{8T}{\varepsilon_0 R}} = \sqrt{\frac{32\pi\, kT}{R}}$

(5) Electric potential at the surface $V = \sqrt{3\pi RTk} = \sqrt{\frac{8RT}{\varepsilon_0}}$