Surface Tension

Intermolecular Force.

The force of attraction or repulsion acting between the molecules are known as intermolecular force. The nature of intermolecular force is electromagnetic.

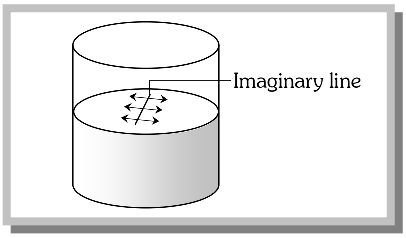
The intermolecular forces of attraction may be classified into two types.

Cohesive force	Adhesive force
The force of attraction between molecules of same substance is called the force of cohesion. This force is lesser in liquids and least in gases.	The force of attraction between the molecules of the different substances is called the force of adhesion.
Ex. (i) Two drops of a liquid coalesce into one when brought in mutual contact.	Ex. (i) Adhesive force enables us to write on the blackboard with a chalk.
(ii) It is difficult to separate two sticky plates of glass welded with water.	(ii) A piece of paper sticks to another due to large force of adhesion between the paper and gum molecules.
(iii) It is difficult to break a drop of mercury into small droplets because of large cohesive force between the mercury molecules.	(iii) Water wets the glass surface due to force of adhesion.

Note : ≅ Cohesive or adhesive forces are inversely proportional to the eighth power of distance between the molecules.

Surface Tension.

The property of a liquid due to which its free surface tries to have minimum surface area and behaves as if it were under tension some what like a stretched elastic membrane is called surface tension. A small liquid drop has spherical shape, as due to surface tension the liquid surface tries to have minimum surface area and for a given volume, the sphere has minimum surface area.



Surface tension of a liquid is measured by the force acting per unit length on either side of an imaginary line drawn on the free surface of liquid, the direction of this force being perpendicular to the line and tangential to the free surface of liquid. So if F is the force acting on one side of imaginary line of length L, then $T = (F/L)$

(1) It depends only on the nature of liquid and is independent of the area of surface or length of line considered.

(2) It is a scalar as it has a unique direction which is not to be specified.

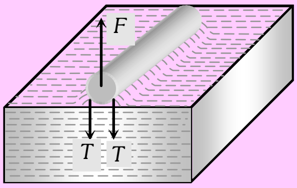
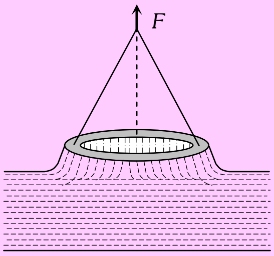
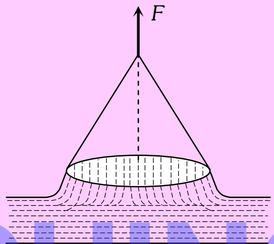
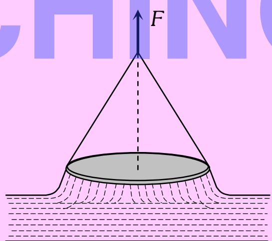
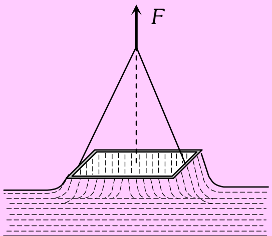
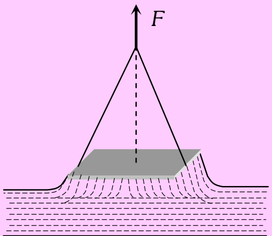
(3) Dimension : $[MT^{-2}]$. (Similar to force constant)

(4) Units : N/m (S.I.) and $Dyne/cm$ [C.G.S.]

(5) It is a molecular phenomenon and its root cause is the electromagnetic forces.

Force Due to Surface Tension.

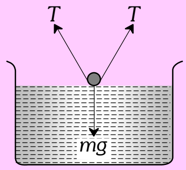
If a body of weight W is placed on the liquid surface, whose surface tension is T. If F is the minimum force required to pull it away from the water then value of F for different bodies can be calculated by the following table.

Body	Figure	Force
Needle (Length $= l$)		$F = 2l\,T + W$
Hollow disc (Inner radius $= r_1$ Outer radius $= r_2$)		$F = 2\pi(r_1 + r_2)T + W$
Thin ring (Radius $= r$)		$F = 2\pi(r + r)T + W$ $F = 4\pi rT + W$
Circular plate or disc (Radius $= r$)		$F = 2\pi rT + W$
Square frame (Side $= l$)		$F = 8l\,T + W$
Square plate		$F = 4l\,T + W$

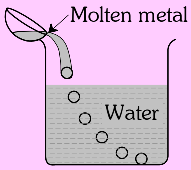
Examples of Surface Tension.

(1) When mercury is split on a clean glass plate, it forms globules. Tiny globules are spherical on the account of surface tension because force of gravity is negligible. The bigger globules get flattened from the middle but have round shape near the edges, figure

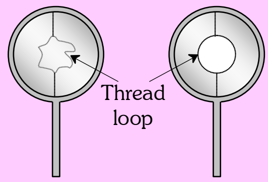
(2) When a greased iron needle is placed gently on the surface of water at rest, so that it does not prick the water surface, the needle floats on the surface of water despite it being heavier because the weight of needle is balanced by the vertical components of the forces of surface tension. If the water surface is pricked by one end of the needle, the needle sinks down.



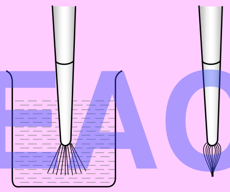
(3) When a molten metal is poured into water from a suitable height, the falling stream of metal breaks up and the detached portion of the liquid in small quantity acquire the spherical shape.



(4) Take a frame of wire and dip it in soap solution and take it out, a soap film will be formed in the frame. Place a loop of wet thread gently on the film. It will remain in the form, we place it on the film according to figure. Now, piercing the film with a pin at any point inside the loop, It immediately takes the circular form as shown in figure.



(5) Hair of shaving brush/painting brush when dipped in water spread out, but as soon as it is taken out, its hair stick together.



(6) If a small irregular piece of camphor is floated on the surface of pure water, it does not remain steady but dances about on the surface. This is because, irregular shaped camphor dissolves unequally and decreases the surface tension of the water locally. The unbalanced forces make it move haphazardly in different directions.

(7) Rain drops are spherical in shape because each drop tends to acquire minimum surface area due to surface tension, and for a given volume, the surface area of sphere is minimum.

(8) Oil drop spreads on cold water. Whereas it may remain as a drop on hot water. This is due to the fact that the surface tension of oil is less than that of cold water and is more than that of hot water.

Factors Affecting Surface Tension.

(1) **Temperature :** The surface tension of liquid decreases with rise of temperature. The surface tension of liquid is zero at its boiling point and it vanishes at critical temperature. At critical temperature, intermolecular forces for liquid and gases becomes equal and liquid can expand without any restriction. For small temperature differences, the variation in surface tension with temperature is linear and is given by the relation

$$T_t = T_0(1 - \alpha t)$$

where T_t, T_0 are the surface tensions at t^oC and 0^oC respectively and α is the temperature coefficient of surface tension.

Examples : (i) Hot soup tastes better than the cold soup.

(ii) Machinery parts get jammed in winter.

(2) **Impurities :** The presence of impurities either on the liquid surface or dissolved in it, considerably affect the force of surface tension, depending upon the degree of contamination. A highly soluble substance like sodium chloride when dissolved in water, increases the surface tension of water. But the sparingly soluble substances like phenol when dissolved in water, decreases the surface tension of water.

Applications of Surface Tension.

(1) The oil and grease spots on clothes cannot be removed by pure water. On the other hand, when detergents (like soap) are added in water, the surface tension of water decreases. As a result of this, wetting power of soap solution increases. Also the force of adhesion between soap solution and oil or grease on the clothes increases. Thus, oil, grease and dirt particles get mixed with soap solution easily. Hence clothes are washed easily.

(2) The antiseptics have very low value of surface tension. The low value of surface tension prevents the formation of drops that may otherwise block the entrance to skin or a wound. Due to low surface tension, the antiseptics spreads properly over wound.

(3) Surface tension of all lubricating oils and paints is kept low so that they spread over a large area.

(4) Oil spreads over the surface of water because the surface tension of oil is less than the surface tension of cold water.

(5) A rough sea can be calmed by pouring oil on its surface.

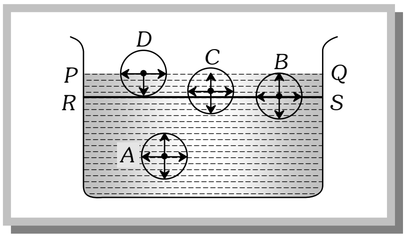
(6) In soldering, addition of 'flux' reduces the surface tension of molten tin, hence, it spreads.

Molecular Theory of Surface Tension.

The maximum distance upto which the force of attraction between two molecules is appreciable is called molecular range $(\approx 10^{-9} m)$. A sphere with a molecule as centre and radius equal to molecular range is called the sphere of influence. The liquid enclosed between free surface (*PQ*) of the liquid and an imaginary plane (*RS*) at a distance *r* (equal to molecular range) from the free surface of the liquid form a liquid film.

To understand the tension acting on the free surface of a liquid, let us consider four liquid molecules like *A*, *B*, *C* and *D*. Their sphere of influence are shown in the figure.

(1) Molecule *A* is well within the liquid, so it is attracted equally in all directions. Hence the net force on this molecule is zero and it moves freely inside the liquid.



(2) Molecule *B* is little below the free surface of the liquid and it is also attracted equally in all directions. Hence the resultant force on it is also zero.

(3) Molecule *C* is just below the upper surface of the liquid film and the part of its sphere of influence is outside the free liquid surface. So the number of molecules in the upper half (attracting the molecules upward) is less than the number of molecule in the lower half (attracting the molecule downward). Thus the molecule *C* experiences a net downward force.

(4) Molecule *D* is just on the free surface of the liquid. The upper half of the sphere of influence has no liquid molecule. Hence the molecule *D* experiences a maximum downward force.

Thus all molecules lying in surface film experiences a net downward force. Therefore, free surface of the liquid behaves like a stretched membrane.

***Problem* 1.** A wooden stick 2*m* long is floating on the surface of water. The surface tension of water 0.07 *N*/*m*. By putting soap solution on one side of the sticks the surface tension is reduced to 0.06 *N*/*m*. The net force on the stick will be **[Pb. PMT 2002]**

(a) 0.07 *N* (b) 0.06 *N* (c) 0.01 *N* (d) 0.02 *N*

Solution : (d) Force on one side of the stick $F_1 = T_1 \times L = 0.07 \times 2 = 0.14N$

and force on other side of the stick $F_2 = T_2 \times L = 0.06 \times 2 = 0.12N$

So net force on the stick $= F_1 - F_2 = 0.14 - 0.12 = 0.02N$

Problem 2. A thin metal disc of radius r floats on water surface and bends the surface downwards along the perimeter making an angle θ with vertical edge of disc. If the disc displaces a weight of water W and surface tension of water is T, then the weight of metal disc is **[AMU (Med.) 1999]**

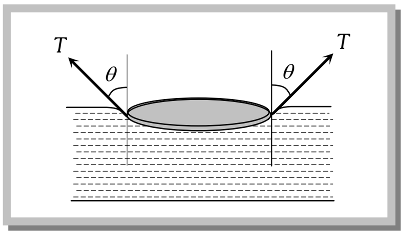
(a) $2\pi rT + W$ (b) $2\pi rT \cos\theta - W$ (c) $2\pi rT \cos\theta + W$ (d) $W - 2\pi rT \cos\theta$

Solution : (c) Weight of metal disc = total upward force

= upthrust force + force due to surface tension

= weight of displaced water + $T \cos\theta\,(2\pi r)$

$= W + 2\pi rT \cos\theta$



Problem 3. A 10 *cm* long wire is placed horizontally on the surface of water and is gently pulled up with a force of $2 \times 10^{-2} N$ to keep the wire in equilibrium. The surface tension in Nm^{-1} of water is **[AMU (Med.) 1999]**

(a) 0.1 *N/m* (b) 0.2 *N/m* (c) 0.001 *N/m* (d) 0.002 *N/m*

Solution : (a) Force on wire due to surface tension $F = T \times 2l$

$$\therefore T = \frac{F}{2l} = \frac{2 \times 10^{-2}}{2 \times 10 \times 10^{-2}} = 0.1\,N/m$$

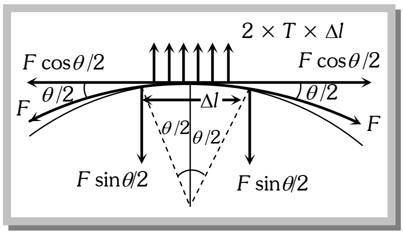
Problem 4. There is a horizontal film of soap solution. On it a thread is placed in the form of a loop. The film is pierced inside the loop and the thread becomes a circular loop of radius R. If the surface tension of the loop be T, then what will be the tension in the thread **[RPET 1996]**

(a) $\pi R^2 / T$ (b) $\pi R^2 T$ (c) $2\pi RT$ (d) $2RT$

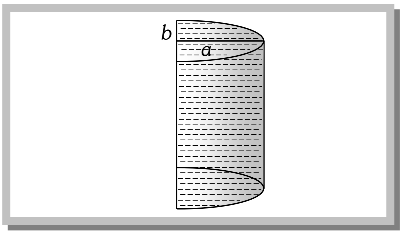
Solution : (d) Suppose tension in thread is F, then for small part Δl of thread

$\Delta l = R\theta$ and $2F \sin\theta/2 = 2T\Delta l = 2TR\theta$

$$\Rightarrow F = \frac{TR\theta}{\sin\theta/2} = \frac{TR\theta}{\theta/2} = 2TR \quad (\sin\theta/2 \approx \theta/2)$$



Problem 5. A liquid is filled into a tube with semi-elliptical cross-section as shown in the figure. The ratio of the surface tension forces on the curved part and the plane part of the tube in vertical position will be



(a) $\frac{\pi(a+b)}{4b}$

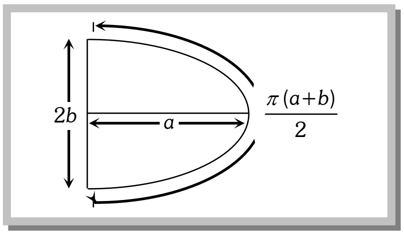
(b) $\frac{2\pi a}{b}$

(c) $\frac{\pi a}{4b}$

(d) $\frac{\pi(a-b)}{4b}$

Solution : (a) From the figure Curved part = semi perimeter $= \frac{\pi(a+b)}{2}$

and the plane part = minor axis = $2b$

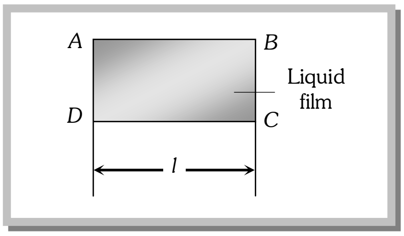


$\therefore$ Force on curved part $= T \times \frac{\pi(a+b)}{2}$

and force on plane part $= T \times 2b$

$\therefore$ Ratio $= \frac{\pi(a+b)}{4b}$

***Problem* 6.** A liquid film is formed over a frame *ABCD* as shown in figure. Wire *CD* can slide without friction. The mass to be hung from *CD* to keep it in equilibrium is



(a) $\frac{Tl}{g}$

(b) $\frac{2Tl}{g}$

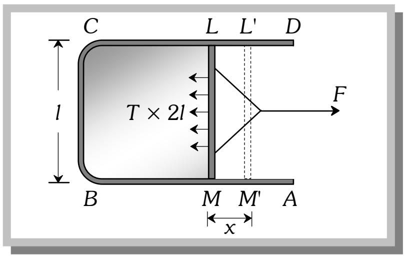
(c) $\frac{g}{2Tl}$

(d) $T \times l$

Solution : (b) Weight of the body hung from wire (mg) = upward force due to surface tension $(2Tl)$ $\Rightarrow m = \frac{2Tl}{g}$

Surface Energy.

The molecules on the liquid surface experience net downward force. So to bring a molecule from the interior of the liquid to the free surface, some work is required to be done against the intermolecular force of attraction, which will be stored as potential energy of the molecule on the surface. The potential energy of surface molecules per unit area of the surface is called surface energy.



Unit : $Joule/m^2$ (S.I.) erg/cm^2 (C.G.S.)

Dimension : $[MT^{-2}]$

If a rectangular wire frame *ABCD*, equipped with a sliding wire *LM* dipped in soap solution, a film is formed over the frame. Due to the surface tension, the film will have a tendency to shrink and thereby, the sliding wire *LM* will be pulled in inward direction. However, the sliding wire can be held in this position under a force *F*, which is equal and opposite to the force acting on the sliding wire *LM* all along its length due to surface tension in the soap film.

If *T* is the force due to surface tension per unit length, then $F = T \times 2l$

Here, *l* is length of the sliding wire *LM*. The length of the sliding wire has been taken as $2l$ for the reason that the film has got two free surfaces.

Suppose that the sliding wire *LM* is moved through a small distance *x*, so as to take the position $L'M'$. In this process, area of the film increases by $2l \times x$ (on the two sides) and to do so, the work done is given by

$$W = F \times x = (T \times 2l) \times x = T \times (2lx) = T \times \Delta A$$

$\therefore \quad W = T \times \Delta A$ $\qquad$ [ΔA = Total increase in area of the film from both the sides]

If temperature of the film remains constant in this process, this work done is stored in the film as its surface energy.

From the above expression $T = \frac{W}{\Delta A}$ or $T = W$ $\qquad$ [If $\Delta A = 1$]

i.e. surface tension may be defined as the amount of work done in increasing the area of the liquid surface by unity against the force of surface tension at constant temperature.

Work Done in Blowing a Liquid Drop or Soap Bubble.

(1) If the initial radius of liquid drop is r_1 and final radius of liquid drop is r_2 then

$W = T \times$ Increment in surface area

$W = T \times 4\pi[r_2^2 - r_1^2]$ $\qquad$ [drop has only one free surface]

(2) In case of soap bubble

$W = T \times 8\pi[r_2^2 - r_1^2]$ $\qquad$ [Bubble has two free surfaces]

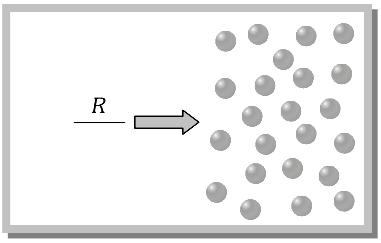
Splitting of Bigger Drop.

When a drop of radius R splits into n smaller drops, (each of radius r) then surface area of liquid increases. Hence the work is to be done against surface tension.

Since the volume of liquid remains constant therefore $\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3$ $\quad \therefore \; R^3 = nr^3$

Work done $= T \times \Delta A = T$ [Total final surface area of n drops – surface area of big drop] $= T[n4\pi r^2 - 4\pi R^2]$

Various formulae of work done			
$4\pi T[nr^2 - R^2]$	$4\pi R^2 T[n^{1/3} - 1]$	$4\pi T r^2 n^{2/3}[n^{1/3} - 1]$	$4\pi T R^3\left[\frac{1}{r} - \frac{1}{R}\right]$



If the work is not done by an external source then internal energy of liquid decreases, subsequently temperature decreases. This is the reason why spraying causes cooling.

By conservation of energy, Loss in thermal energy = work done against surface tension

$$JQ = W$$

$$\Rightarrow \quad JmS\Delta\theta = 4\pi T R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$\Rightarrow \quad J\frac{4}{3}\pi R^3 d\, S\Delta\theta = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$$ $\qquad$ [As $m = V \times d = \frac{4}{3}\pi R^3 \times d$]

$\therefore$ Decrease in temperature $\Delta\theta = \frac{3T}{JSd}\left[\frac{1}{r} - \frac{1}{R}\right]$

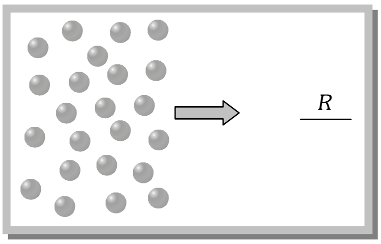
where J = mechanical equivalent of heat, S = specific heat of liquid, d = density of liquid.

Formation of Bigger Drop.

If n small drops of radius r coalesce to form a big drop of radius R then surface area of the liquid decreases.

Amount of surface energy released = Initial surface energy – final surface energy

$$E = n4\pi r^2 T - 4\pi R^2 T$$



Various formulae of released energy			
$4\pi T[nr^2 - R^2]$	$4\pi R^2 T(n^{1/3} - 1)$	$4\pi T r^2 n^{2/3}(n^{1/3} - 1)$	$4\pi T R^3\left[\frac{1}{r} - \frac{1}{R}\right]$

(i) If this released energy is absorbed by a big drop, its temperature increases and rise in temperature can be given by $\Delta\theta = \frac{3T}{JSd}\left[\frac{1}{r} - \frac{1}{R}\right]$

(ii) If this released energy is converted into kinetic energy of a big drop without dissipation then by the law of conservation of energy.

$$\frac{1}{2}mv^2 = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right] \Rightarrow \frac{1}{2}\left[\frac{4}{3}\pi R^3 d\right]v^2 = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right] \Rightarrow v^2 = \frac{6T}{d}\left[\frac{1}{r} - \frac{1}{R}\right]$$

$\therefore$ $v = \sqrt{\frac{6T}{d}\left(\frac{1}{r} - \frac{1}{R}\right)}$

Problem 7. Two small drops of mercury, each of radius R, coalesce to form a single large drop. The ratio of the total surface energies before and after the change is **[AIIMS 2003]**

(a) $1 : 2^{1/3}$ (b) $2^{1/3} : 1$ (c) 2 : 1 (d) 1 : 2

Solution : (b) As $R = n^{1/3}r = 2^{1/3}r \Rightarrow R^2 = 2^{2/3}r^2 \Rightarrow \frac{r^2}{R^2} = 2^{-2/3}$

$$\frac{\text{Initial surface energy}}{\text{Final surface energy}} = \frac{2(4\pi r^2 T)}{(4\pi R^2 T)} = 2\left(\frac{r^2}{R^2}\right) = 2 \times 2^{-2/3} = 2^{1/3}$$

Problem 8. Radius of a soap bubble is increased from R to $2R$ work done in this process in terms of surface tension is **[CPMT 1991; RPET 2001; BHU 2003]**

(a) $24\pi R^2 S$ (b) $48\pi R^2 S$ (c) $12\pi R^2 S$ (d) $36\pi R^2 S$

Solution : (a) $W = 8\pi T\left(R_2^2 - R_1^2\right) = 8\pi S[(2R)^2 - (R)^2] = 24\pi R^2 S$

Problem 9. The work done in blowing a soap bubble of 10*cm* radius is (surface tension of the soap solution is $\frac{3}{100} N/m$) **[MP PMT 1995; MH CET 2002]**

(a) $75.36 \times 10^{-4} J$ (b) $37.68 \times 10^{-4} J$ (c) $150.72 \times 10^{-4} J$ (d) $75.36\, J$

Solution : (a) $W = 8\pi R^2 T = 8\pi (10 \times 10^{-2})^2 \frac{3}{100} = 75.36 \times 10^{-4} J$

Problem 10. A drop of mercury of radius 2*mm* is split into 8 identical droplets. Find the increase in surface energy. (Surface tension of mercury is 0.465 J/m^2) **[UPSEAT 2002]**

(a) $23.4\mu J$ (b) $18.5\mu J$ (c) $26.8\mu J$ (d) $16.8\mu J$

Solution : (a) Increase in surface energy $= 4\pi R^2 T(n^{1/3} - 1) = 4\pi(2\times 10^{-3})^2(0.465)(8^{1/3} - 1) = 23.4\times 10^{-6} J = 23.4\mu J$

Problem 11. The work done in increasing the size of a soap film from 10*cm* × 6*cm* to 10*cm* × 11*cm* is $3\times 10^{-4} J$. The surface tension of the film is **[MP PET 1999; MP PMT 2000; AIIMS 2000; JIPMER 2001, 02]**

(a) $1.5\times 10^{-2}\ Nm^{-1}$ (b) $3.0\times 10^{-2}\ Nm^{-1}$ (c) $6.0\times 10^{-2}\ Nm^{-1}$ (d) $11.0\times 10^{-2}\ Nm^{-1}$

Solution : (b) $A_1 = 10\times 6 = 60 cm^2 = 60\times 10^{-4} m^2$, $A_2 = 10\times 11 = 110 cm^2 = 110\times 10^{-4} m^2$

As the soap film has two free surfaces $\therefore W = T\times 2\Delta A$

$$\Rightarrow W = T\times 2\times(A_2 - A_1) \Rightarrow T = \frac{W}{2\times 50\times 10^{-4}} = \frac{3\times 10^{-4}}{2\times 50\times 10^{-4}} = 3\times 10^{-2} N/m$$

Problem 12. A film of water is formed between two straight parallel wires of length 10*cm* each separated by 0.5*cm*. If their separation is increased by 1 *mm* while still maintaining their parallelism, how much work will have to be done (Surface tension of water $= 7.2\times 10^{-2} N/m$) **[Roorkee 1986; MP PET 2001]**

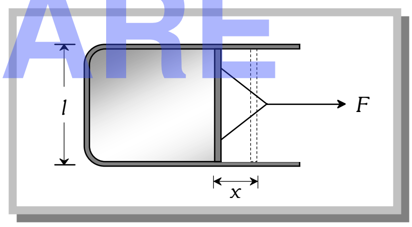
(a) $7.22\times 10^{-6} J$ (b) $1.44\times 10^{-5} J$ (c) $2.88\times 10^{-5} J$ (d) $5.76\times 10^{-5} J$

Solution : (b) As film have two free surfaces $W = T\times 2\Delta A$

$W = T\times 2l\times x$

$= 7.2\times 10^{-2}\times 2\times 0.1\times 1\times 10^{-3}$

$= 1.44\times 10^{-5} J$



Problem 13. If the work done in blowing a bubble of volume *V* is *W*, then the work done in blowing the bubble of volume 2*V* from the same soap solution will be **[MP PET 1989]**

(a) *W*/2 (b) $\sqrt{2}\,W$ (c) $\sqrt[3]{2}\,W$ (d) $\sqrt[3]{4}\,W$

Solution : (d) As volume of the bubble $V = \frac{4}{3}\pi R^3 \Rightarrow R = \left(\frac{3}{4\pi}\right)^{1/3} V^{1/3} \Rightarrow R^2 = \left(\frac{3}{4\pi}\right)^{2/3} V^{2/3} \Rightarrow R^2 \propto V^{2/3}$

Work done in blowing a soap bubble $W = 8\pi R^2 T \Rightarrow W \propto R^2 \propto V^{2/3}$

$$\therefore \frac{W_2}{W_1} = \left(\frac{V_2}{V_1}\right)^{2/3} = \left(\frac{2V}{V}\right)^{2/3} = (2)^{2/3} = (4)^{1/3} \Rightarrow W_2 = \sqrt[3]{4}\,W$$

Problem 14. Several spherical drops of a liquid of radius *r* coalesce to form a single drop of radius *R*. If *T* is surface tension and *V* is volume under consideration, then the release of energy is

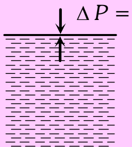
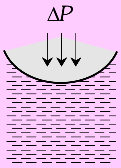
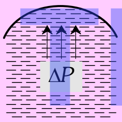
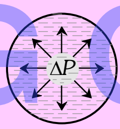
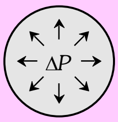
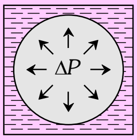
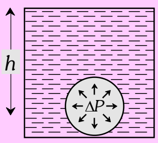
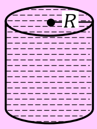
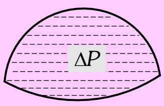
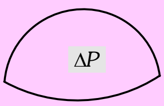
(a) $3VT\left(\frac{1}{r} + \frac{1}{R}\right)$ (b) $3VT\left(\frac{1}{r} - \frac{1}{R}\right)$ (c) $VT\left(\frac{1}{r} - \frac{1}{R}\right)$ (d) $VT\left(\frac{1}{r^2} + \frac{1}{R^2}\right)$

Solution : (b) Energy released $= 4\pi TR^3\left[\frac{1}{r} - \frac{1}{R}\right] = 3\left(\frac{4}{3}\pi R^3\right)T\left[\frac{1}{r} - \frac{1}{R}\right] = 3VT\left[\frac{1}{r} - \frac{1}{R}\right]$

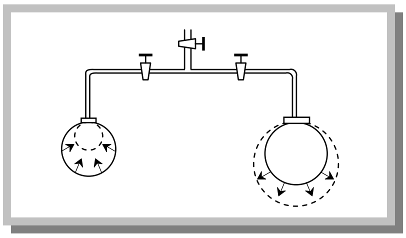
Excess Pressure.

Due to the property of surface tension a drop or bubble tries to contract and so compresses the matter enclosed. This in turn increases the internal pressure which prevents further contraction and equilibrium is achieved. So in equilibrium the pressure inside a bubble or drop is greater than outside and the difference of pressure between two sides of the liquid surface is called excess pressure. In case of a drop excess pressure is provided by hydrostatic pressure of the liquid within the drop while in case of bubble the gauge pressure of the gas confined in the bubble provides it.

Excess pressure in different cases is given in the following table :

Plane surface	Concave surface
 $\Delta P = 0$	 $\Delta P = \frac{2T}{R}$
Convex surface	Drop
 $\Delta P = \frac{2T}{R}$	 $\Delta P = \frac{2T}{R}$
Bubble in air	Bubble in liquid
 $\Delta P = \frac{4T}{R}$	 $\Delta P = \frac{2T}{R}$
Bubble at depth h below the free surface of liquid of density d	Cylindrical liquid surface
 $\Delta P = \frac{2T}{R} + hdg$	 $\Delta P = \frac{T}{R}$
Liquid surface of unequal radii	Liquid film of unequal radii
 $\Delta P = T\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$	 $\Delta P = 2T\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$

Note : ❑ Excess pressure is inversely proportional to the radius of bubble (or drop), *i.e.*, pressure inside a smaller bubble (or drop) is higher than inside a larger bubble (or drop). This is why when two bubbles of different sizes are put in communication with each other, the air will rush from smaller to larger bubble, so that the smaller will shrink while the larger will expand till the smaller bubble reduces to droplet.



Problem 15. The pressure inside a small air bubble of radius 0.1mm situated just below the surface of water will be equal to (Take surface tension of water $70\times10^{-3}Nm^{-1}$ and atmospheric pressure $=1.013\times10^{5}Nm^{-2}$)

[AMU (Med.) 2002]

(a) $2.054\times10^{3}Pa$ (b) $1.027\times10^{3}Pa$ (c) $1.027\times10^{5}Pa$ (d) $2.054\times10^{5}Pa$

Solution : (c) Pressure inside a bubble when it is in a liquid $=P_o+\dfrac{2T}{R}=1.013\times10^{5}+2\times\dfrac{70\times10^{-3}}{0.1\times10^{-3}}=1.027\times10^{5}Pa.$

Problem 16. If the radius of a soap bubble is four times that of another, then the ratio of their excess pressures will be

[AIIMS 2000]

(a) 1 : 4 (b) 4 : 1 (c) 16 : 1 (d) 1 : 16

Solution : (a) Excess pressure inside a soap bubble $\Delta P=\dfrac{4T}{r}\Rightarrow\dfrac{\Delta P_1}{\Delta P_2}=\dfrac{r_2}{r_1}=1:4$

Problem 17. Pressure inside two soap bubbles are 1.01 and 1.02 atmospheres. Ratio between their volumes is

[MP PMT 1991]

(a) 102 : 101 (b) $(102)^3:(101)^3$ (c) 8 : 1 (d) 2 : 1

Solution : (c) Excess pressure $\Delta P=P_{in}-P_{out}=1.01atm-1atm=0.01atm$ and similarly $\Delta P_2=0.02atm$

and volume of air bubble $V=\dfrac{4}{3}\pi r^3$ $\therefore V\propto r^3\propto\dfrac{1}{(\Delta P)^3}$ [as $\Delta P\propto\dfrac{1}{r}$ or $r\propto\dfrac{1}{\Delta P}$]

$$\therefore\ \frac{V_1}{V_2}=\left(\frac{\Delta P_2}{\Delta P_1}\right)^3=\left(\frac{0.02}{0.01}\right)^3=\left(\frac{2}{1}\right)^3=\frac{8}{1}$$

Problem 18. The excess pressure inside an air bubble of radius r just below the surface of water is P_1. The excess pressure inside a drop of the same radius just outside the surface is P_2. If T is surface tension then

(a) $P_1=2P_2$ (b) $P_1=P_2$ (c) $P_2=2P_1$ (d) $P_2=0,\ P_1\neq0$

Solution : (b) Excess pressure inside a bubble just below the surface of water $P_1=\dfrac{2T}{r}$

and excess pressure inside a drop $P_2=\dfrac{2T}{r}$ $\therefore P_1=P_2$

Shape of Liquid Meniscus.

We know that a liquid assumes the shape of the vessel in which it is contained *i.e.* it can not oppose permanently any force that tries to change its shape. As the effect of force is zero in a direction perpendicular to it, the free surface of liquid at rest adjusts itself at right angles to the resultant force.

When a capillary tube is dipped in a liquid, the liquid surface becomes curved near the point of contact. This curved surface is due to the resultant of two forces *i.e.* the force of cohesion and the force of adhesion. The curved surface of the liquid is called meniscus of the liquid.

If liquid molecule A is in contact with solid (*i.e.* wall of capillary tube) then forces acting on molecule A are

(i) Force of adhesion F_a (acts outwards at right angle to the wall of the tube).

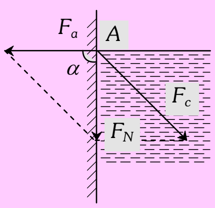
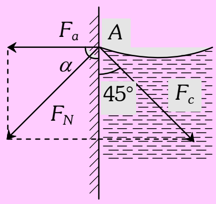
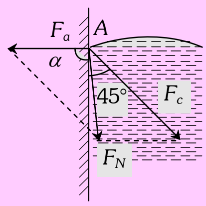
(ii) Force of cohesion F_c (acts at an angle 45° to the vertical).

Resultant force F_N depends upon the value of F_a and F_c.

If resultant force F_N make an angle α with F_a.

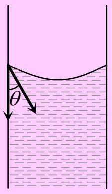
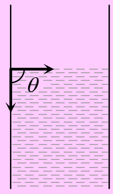
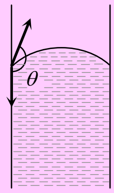
Then $\tan\alpha = \dfrac{F_c \sin 135^o}{F_a + F_c \cos 135^o} = \dfrac{F_c}{\sqrt{2}\,F_a - F_c}$

By knowing the direction of resultant force we can find out the shape of meniscus because the free surface of the liquid adjust itself at right angle to this resultant force.

<table>
<tr><td>If $F_c = \sqrt{2}Fa$</td><td>$F_c < \sqrt{2}Fa$</td><td>$F_c > \sqrt{2}Fa$</td></tr>
<tr><td>$\tan\alpha = \infty$ $\therefore\ \alpha = 90^o$</td><td>$\tan\,\alpha$ = positive $\therefore\ \alpha$ is acute angle</td><td>$\tan\,\alpha$ = negative $\therefore\ \alpha$ is obtuse angle</td></tr>
<tr><td>i.e. the resultant force acts vertically downwards. Hence the liquid meniscus must be horizontal.</td><td>i.e. the resultant force directed outside the liquid. Hence the liquid meniscus must be concave upward.</td><td>i.e. the resultant force directed inside the liquid. Hence the liquid meniscus must be convex upward.</td></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td>Example: Pure water in silver coated capillary tube.</td><td>Example: Water in glass capillary tube.</td><td>Example: Mercury in glass capillary tube.</td></tr>
</table>

Angle of Contact.

Angle of contact between a liquid and a solid is defined as the angle enclosed between the tangents to the liquid surface and the solid surface inside the liquid, both the tangents being drawn at the point of contact of the liquid with the solid.

$\theta < 90^o$	$\theta = 90^o$	$\theta > 90^o$
$F_a > \frac{F_c}{\sqrt{2}}$ 	$F_a = \frac{F_c}{\sqrt{2}}$ 	$F_a < \frac{F_c}{\sqrt{2}}$ 
concave meniscus.	plane meniscus.	convex meniscus.
Liquid wets the solid surface	Liquid does not wet the solid surface.	Liquid does not wet the solid surface.

(i) Its value lies between 0° and 180°

$\theta = 0^o$ for pure water and glass, $\theta = 8^o$ for tap water and glass, $\theta = 90^o$ for water and silver

$\theta = 138^o$ for mercury and glass, $\theta = 160^o$ for water and chromium

(ii) It is particular for a given pair of liquid and solid. Thus the angle of contact changes with the pair of solid and liquid.

(iii) It does not depends upon the inclination of the solid in the liquid.

(iv) On increasing the temperature, angle of contact decreases.

(v) Soluble impurities increases the angle of contact.

(vi) Partially soluble impurities decreases the angle of contact.

Capillarity.

If a tube of very narrow bore (called capillary) is dipped in a liquid, it is found that the liquid in the capillary either ascends or descends relative to the surrounding liquid. This phenomenon is called capillarity.

The root cause of capillarity is the difference in pressures on two sides of (concave and convex) curved surface of liquid.

Examples of capillarity :

(i) Ink rises in the fine pores of blotting paper leaving the paper dry.

(ii) A towel soaks water.

(iii) Oil rises in the long narrow spaces between the threads of a wick.

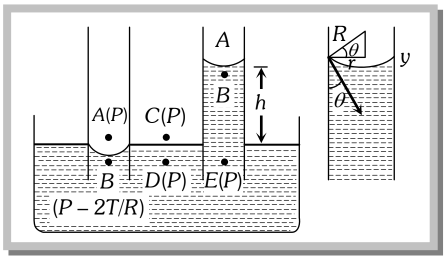
(iv) Wood swells in rainy season due to rise of moisture from air in the pores.

(v) Ploughing of fields is essential for preserving moisture in the soil.

(vi) Sand is drier soil than clay. This is because holes between the sand particles are not so fine as compared to that of clay, to draw up water by capillary action.

Ascent Formula.

When one end of capillary tube of radius r is immersed into a liquid of density d which wets the sides of the capillary tube (water and capillary tube of glass), the shape of the liquid meniscus in the tube becomes concave upwards.



R = radius of curvature of liquid meniscus.

T = surface tension of liquid

P = atmospheric pressure

Pressure at point $A = P$, Pressure at point $B = P - \dfrac{2T}{R}$

Pressure at points C and D just above and below the plane surface of liquid in the vessel is also P (atmospheric pressure). The points B and D are in the same horizontal plane in the liquid but the pressure at these points is different.

In order to maintain the equilibrium the liquid level rises in the capillary tube upto height h.

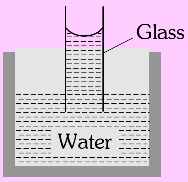
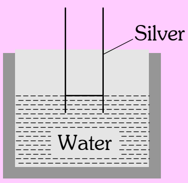
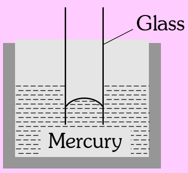
Pressure due to liquid column = pressure difference due to surface tension

$$\Rightarrow \qquad hdg = \frac{2T}{R}$$

$$\therefore \qquad h = \frac{2T}{Rdg} = \frac{2T\cos\theta}{rdg} \qquad \left[\text{As } R = \frac{r}{\cos\theta}\right]$$

(i) The capillary rise depends on the nature of liquid and solid both *i.e.* on T, d, θ and R.

(ii) Capillary action for various liquid-solid pair.

	Meniscus	**Angle of contact**	**Level**
	Concave	$\theta < 90^\circ$	Rises
	Plane	$\theta = 90^\circ$	No rise no fall
	Convex	$\theta > 90^\circ$	Fall

(iii) For a given liquid and solid at a given place

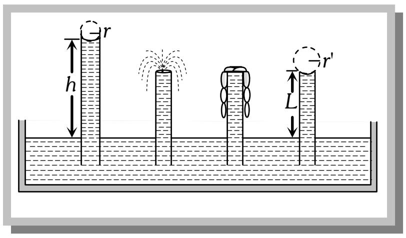
$$h \propto \frac{1}{r} \qquad \text{[As } T,\ \theta,\ d \text{ and } g \text{ are constant]}$$

i.e. lesser the radius of capillary greater will be the rise and vice-versa. This is called Jurin's law.

(iv) If the weight of the liquid contained in the meniscus is taken into consideration then more accurate ascent formula is given by

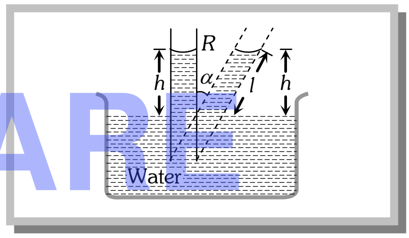
$$h = \frac{2T\cos\theta}{rdg} - \frac{r}{3}$$

(v) In case of capillary of insufficient length, *i.e.*, $L < h$, the liquid will neither overflow from the upper end like a fountain nor will it tickle along the vertical sides of the tube. The liquid after reaching the upper end will increase the radius of its meniscus without changing nature such that :



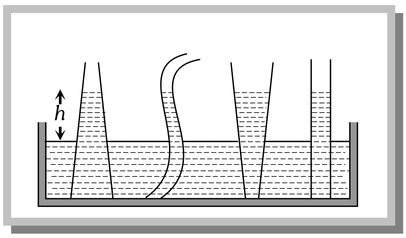
$$hr = Lr' \quad \because\ L < h \quad \therefore\ r' > r$$

(vi) If a capillary tube is dipped into a liquid and tilted at an angle α from vertical, then the vertical height of liquid column remains same whereas the length of liquid column (l) in the capillary tube increases.



$$h = l\cos\alpha \quad \text{or} \quad l = \frac{h}{\cos\alpha}$$

(vii) It is important to note that in equilibrium the height h is independent of the shape of capillary if the radius of meniscus remains the same. That is why the vertical height h of a liquid column in capillaries of different shapes and sizes will be same if the radius of meniscus remains the same.



***Problem* 19.** Water rises to a height of 10*cm* in a capillary tube and mercury falls to a depth of 3.5*cm* in the same capillary tube. If the density of mercury is 13.6 *gm/cc* and its angle of contact is 135° and density of water is 1 *gm/cc* and its angle of contact is 0^o, then the ratio of surface tensions of the two liquids is $(\cos 135^o = 0.7)$

[MP PMT 1988; EAMCET (Med.) 2003]

(a) 1 : 14 (b) 5 : 34 (c) 1 : 5 (d) 5 : 27

Solution : (b) $h = \frac{2T\cos\theta}{rdg}$ $\quad \therefore \frac{h_W}{h_{Hg}} = \frac{T_W}{T_{Hg}} \frac{\cos\theta_W}{\cos\theta_{Hg}} \frac{d_{Hg}}{d_W}$ [as r and g are constants]

$$\Rightarrow \frac{10}{3.5} = \frac{T_W}{T_{Hg}} \cdot \frac{\cos 0^o}{\cos 135} \frac{13.6}{1} \Rightarrow \frac{T_W}{T_{Hg}} = \frac{10 \times 0.7}{3.5 \times 13.6} = \frac{20}{136} = \frac{5}{34}$$

***Problem* 20.** Water rises in a vertical capillary tube upto a height of 2.0 *cm*. If the tube is inclined at an angle of 60^o with the vertical, then upto what length the water will rise in the tube **[UPSEAT 2002]**

(a) 2.0 *cm* (b) 4.0 *cm* (c) $\frac{4}{\sqrt{3}}$ *cm* (d) $2\sqrt{2}$ *cm*

Solution : (b) The height upto which water will rise $l = \frac{h}{\cos\alpha} = \frac{2cm}{\cos 60} = 4cm$. [$h$ = vertical height, α = angle with vertical]

Problem 21. Two capillary tubes of same diameter are kept vertically one each in two liquids whose relative densities are 0.8 and 0.6 and surface tensions are 60 and 50 *dyne/cm* respectively. Ratio of heights of liquids in the two tubes $\frac{h_1}{h_2}$ is **[MP PMT 2002]**

(a) $\frac{10}{9}$ (b) $\frac{3}{10}$ (c) $\frac{10}{3}$ (d) $\frac{9}{10}$

Solution : (d) $h = \frac{2T\cos\theta}{rdg}$ [If diameter of capillaries are same and taking value of θ same for both liquids]

$\therefore \frac{h_1}{h_2} = \left(\frac{T_1}{T_2}\right)\left(\frac{d_2}{d_1}\right) = \left(\frac{60}{50}\right) \times \left(\frac{0.6}{0.8}\right) = \left(\frac{36}{40}\right) = \frac{9}{10}$.

Problem 22. A capillary tube of radius *R* is immersed in water and water rises in it to a height *H*. Mass of water in the capillary tube is *M*. If the radius of the tube is doubled, mass of water that will rise in the capillary tube will now be **[RPMT 1997; RPET 1999; CPMT 2002]**

(a) *M* (b) 2*M* (c) *M*/2 (d) 4*M*

Solution : (b) Mass of the liquid in capillary tube $M = V\rho = (\pi r^2 h)\rho$ $\therefore M \propto r^2 h \propto r$ [As $h \propto \frac{1}{r}$]

So if radius of the tube is doubled, mass of water will becomes 2*M*, which will rise in capillary tube.

Problem 23. Water rises to a height *h* in a capillary at the surface of earth. On the surface of the moon the height of water column in the same capillary will be **[MP PMT 2001]**

(a) 6*h* (b) $\frac{1}{6}h$ (c) *h* (d) Zero

Solution : (a) $h = \frac{2T\cos\theta}{rdg}$ $\therefore h \propto \frac{1}{g}$ [If other quantities remains constant]

$\frac{h_{moon}}{h_{earth}} = \frac{g_{earth}}{g_{moon}} = 6 \Rightarrow h_{moon} = 6h$ [As $g_{earth} = 6g_{moon}$]

Problem 24. Water rises upto a height *h* in a capillary on the surface of earth in stationary condition. Value of *h* increases if this tube is taken **[RPET 2000]**

(a) On sun (b) On poles

(c) In a lift going upward with acceleration (d) In a lift going downward with acceleration

Solution : (d) $h \propto \frac{1}{g}$. In a lift going downward with acceleration (a), the effective acceleration decreases. So *h* increases.

Problem 25. If the surface tension of water is 0.06 *N/m*, then the capillary rise in a tube of diameter 1*mm* is $(\theta = 0^o)$ **[AFMC 1998]**

(a) 1.22 *cm* (b) 2.44 *cm* (c) 3.12 *cm* (d) 3.86 *cm*

Solution : (b) $h = \frac{2T\cos\theta}{rdg}$, [$\theta = 0$, $r = \frac{1}{2}mm = 0.5\times 10^{-3}m$, $T = 0.06N/m$, $d = 10^3 kg/m^3$, $g = 9.8\ m/s^2$]

$$h = \frac{2 \times 0.06 \times \cos\theta}{0.5 \times 10^{-3} \times 10^3 \times 9.8} = 0.0244m = 2.44cm$$

Problem 26. Two capillaries made of same material but of different radii are dipped in a liquid. The rise of liquid in one capillary is 2.2*cm* and that in the other is 6.6*cm*. The ratio of their radii is **[MP PET 1990]**

(a) 9 : 1 (b) 1 : 9 (c) 3 : 1 (d) 1 : 3

Solution : (c) As $h \propto \frac{1}{r}$ $\therefore \frac{h_1}{h_2} = \frac{r_2}{r_1}$ or $\frac{r_1}{r_2} = \frac{h_2}{h_1} = \frac{6.6}{2.2} = \frac{3}{1}$

Problem 27. The lower end of a capillary tube is at a depth of 12*cm* and the water rises 3*cm* in it. The mouth pressure required to blow an air bubble at the lower end will be *X cm* of water column where *X* is **[CPMT 1989]**

(a) 3 (b) 9 (c) 12 (d) 15

Solution : (d) The lower end of capillary tube is at a depth of 12 + 3 = 15 *cm* from the free surface of water in capillary tube.

So, the pressure required = 15 *cm* of water column.

Problem 28. The lower end of a capillary tube of radius *r* is placed vertically in water. Then with the rise of water in the capillary, heat evolved is

(a) $+\frac{\pi^2 r^2 h^2}{J} dg$ (b) $+\frac{\pi r^2 h^2 dg}{2J}$ (c) $-\frac{\pi r^2 h^2 dg}{2J}$ (d) $-\frac{\pi r^2 h^2 dg}{J}$

Solution : (b) When the tube is placed vertically in water, water rises through height *h* given by $h = \frac{2T\cos\theta}{rdg}$

Upward force $= 2\pi r \times T\cos\theta$

Work done by this force in raising water column through height *h* is given by

$$\Delta W = (2\pi r T\cos\theta)h = (2\pi r h\cos\theta)T = (2\pi r h\cos\theta)\left(\frac{rhdg}{2\cos\theta}\right) = \pi r^2 h^2 dg$$

However, the increase in potential energy ΔE_p of the raised water column $= mg\frac{h}{2}$

where *m* is the mass of the raised column of water $\therefore m = \pi r^2 hd$

So, $\Delta E_P = (\pi r^2 hd)\left(\frac{hg}{2}\right) = \frac{\pi r^2 h^2 dg}{2}$

Further, $\Delta W - \Delta E_p = \frac{\pi r^2 h^2 dg}{2}$

The part $(\Delta W - \Delta E_P)$ is used in doing work against viscous forces and frictional forces between water and glass surface and appears as heat. So heat released $= \frac{\Delta W - \Delta E_p}{J} = \frac{\pi r^2 h^2 dg}{2J}$

Problem 29. Water rises in a capillary tube to a certain height such that the upward force due to surface tension is balanced by $75 \times 10^{-4} N$ force due to the weight of the liquid. If the surface tension of water is $6 \times 10^{-2}\, N/m$, the inner circumference of the capillary must be **[CPMT 1986, 88]**

(a) $1.25 \times 10^{-2}\, m$ (b) $0.50 \times 10^{-2}\, m$ (c) $6.5 \times 10^{-2}\, m$ (d) $12.5 \times 10^{-2}\, m$

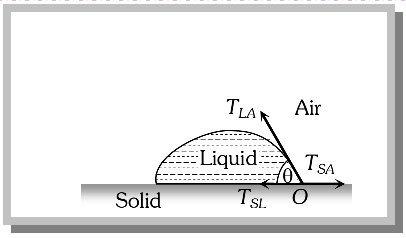
Solution : (d) Weight of liquid = upward force due to surface tension

$$75 \times 10^{-4} = 2\pi r T$$

$$\text{Circumference } 2\pi r = \frac{75\times10^{-4}}{T} = \frac{75\times10^{-4}}{6\times10^{-2}} = 0.125 = 12.5\times10^{-2} m$$

Shape of Drops.

Whether the liquid will be in equilibrium in the form of a drop or it will spread out; depends on the relative strength of the force due to surface tension at the three interfaces.

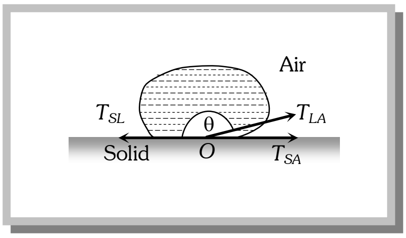


T_{LA} = surface tension at liquid-air interface, T_{SA} = surface tension at solid-air interface.

T_{SL} = surface tension at solid-liquid interface, θ = angle of contact between the liquid and solid.

For the equilibrium of molecule

$$T_{SL} + T_{LA}\cos\theta = T_{SA} \text{ or } \cos\theta = \frac{T_{SA} - T_{SL}}{T_{LA}} \qquad .....(i)$$



Special Cases

$T_{SA} > T_{SL}$, $\cos\theta$ is positive *i.e.* $0^o < \theta < 90^o$.

This condition is fulfilled when the molecules of liquid are strongly attracted to that of solid.

Example : (i) Water on glass.

(ii) Kerosene oil on any surface.

$T_{SA} < T_{SL}$, $\cos\theta$ is negative *i.e.* $90^o < \theta < 180^o$.

This condition is fulfilled when the molecules of the liquid are strongly attracted to themselves and relatively weakly to that of solid.

Example : (i) Mercury on glass surface.

(ii) Water on lotus leaf (or a waxy or oily surface)

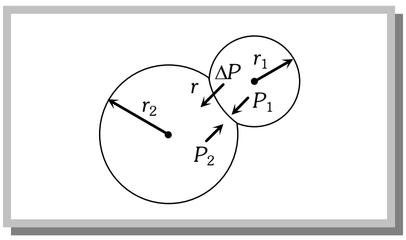
$(T_{SL} + T_{LA}\cos\theta) > T_{SA}$

In this condition, the molecule of liquid will not be in equilibrium and experience a net force at the interface. As a result, the liquid spreads.

Example : (i) Water on a clean glass plate.

Useful Facts and Formulae.

(1) Formation of double bubble : If r_1 and r_2 are the radii of smaller and larger bubble and P_0 is the atmospheric pressure, then the pressure inside them will be $P_1 = P_0 + \frac{4T}{r_1}$ and $P_2 = P_0 + \frac{4T}{r_2}$.



Now as $r_1 < r_2 \; \therefore \; P_1 > P_2$

So for interface $\Delta P = P_1 - P_2 = 4T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$ .....(i)

As excess pressure acts from concave to convex side, the interface will be concave towards the smaller bubble and convex towards larger bubble and if r is the radius of interface.

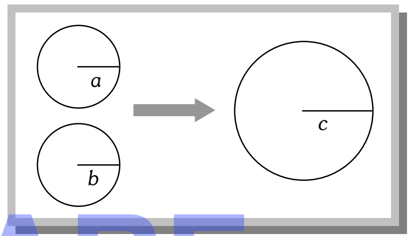
$$\Delta P = \frac{4T}{r} \quad .....(ii)$$

From (i) and (ii) $\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$

$\therefore$ Radius of the interface $r = \frac{r_1 r_2}{r_2 - r_1}$

(2) Formation of a single bubble

(i) Under isothermal condition two soap bubble of radii 'a' and 'b' coalesce to form a single bubble of radius 'c'.



If the external pressure is P_0 then pressure inside bubbles

$$P_a = \left(P_0 + \frac{4T}{a}\right),\ P_b = \left(P_0 + \frac{4T}{b}\right) \text{ and } P_c = \left(P_0 + \frac{4T}{c}\right)$$

and volume of the bubbles

$$V_a = \frac{4}{3}\pi a^3,\ V_b = \frac{4}{3}\pi b^3,\ V_c = \frac{4}{3}\pi c^3$$

Now as mass is conserved $\mu_a + \mu_b = \mu_c \Rightarrow \frac{P_a V_a}{RT_a} + \frac{P_b V_b}{RT_b} = \frac{P_c V_c}{RT_c}$ $\left[\text{As } PV = \mu RT,\ i.e., \mu = \frac{PV}{RT}\right]$

$\Rightarrow \quad P_a V_a + P_b V_b = P_c V_c$ .....(i) [As temperature is constant, *i.e.*, $T_a = T_b = T_c$]

Substituting the value of pressure and volume

$$\Rightarrow \quad \left[P_0 + \frac{4T}{a}\right]\left[\frac{4}{3}\pi a^3\right] + \left[P_0 + \frac{4T}{b}\right]\left[\frac{4}{3}\pi b^3\right] = \left[P_0 + \frac{4T}{c}\right]\left[\frac{4}{3}\pi c^3\right]$$

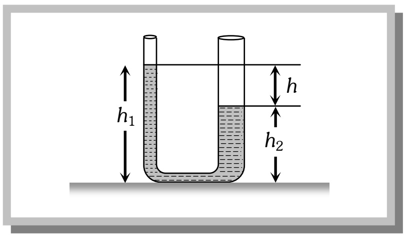
$$\Rightarrow \quad 4T(a^2 + b^2 - c^2) = P_0(c^3 - a^3 - b^3)$$

$\therefore$ Surface tension of the liquid $T = \frac{P_0(c^3 - a^3 - b^3)}{4(a^2 + b^2 - c^2)}$

(ii) If two bubble coalesce in vacuum then by substituting $P_0 = 0$ in the above expression we get

$$a^2 + b^2 - c^2 = 0 \quad \therefore\ c^2 = a^2 + b^2$$

Radius of new bubble $= c = \sqrt{a^2 + b^2}$ or can be expressed as $r = \sqrt{r_1^2 + r_2^2}$.



(3) The difference of levels of liquid column in two limbs of u-tube of unequal radii r_1 and r_2 is

$$h = h_1 - h_2 = \frac{2T\cos\theta}{dg}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

(4) A large force (F) is required to draw apart normally two glass plate enclosing a thin water film because the thin water film formed between the two glass plates will have concave surface all around. Since on the concave side of a liquid surface, pressure is more, work will have to be done in drawing the plates apart.

$F = \frac{2AT}{t}$ where T = surface tension of water film, t = thickness of film, A = area of film.

(5) When a soap bubble is charged, then its size increases due to outward force on the bubble.

(6) The materials, which when coated on a surface and water does not enter through that surface are known as water proofing agents. For example wax *etc*. Water proofing agent increases the angle of contact.

(7) Values of surface tension of some liquids.

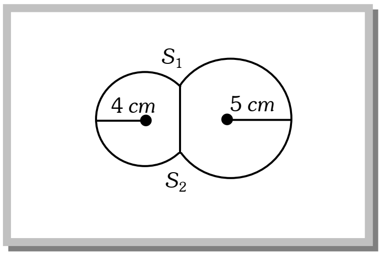
Liquid	**Surface tension *Newton/metre***
Mercury	0.465
Water	0.075
Soap solution	0.030
Glycerine	0.063
Carbon tetrachloride	0.027
Ethyl alcohol	0.022

Problem 30. The radii of two soap bubbles are r_1 and r_2. In isothermal conditions, two meet together in vacuum. Then the radius of the resultant bubble is given by **[RPET 1999; MP PMT 2001; EAMCET 2003]**

(a) $R = (r_1 + r_2)/2$ (b) $R = r_1(r_1 r_2 + r_2)$ (c) $R^2 = r_1^2 + r_2^2$ (d) $R = r_1 + r_2$

Solution : (c) Under isothermal condition surface energy remain constant $\therefore\ 8\pi r_1^2 T + 8\pi r_2^2 T = 8\pi R^2 T \Rightarrow R^2 = r_1^2 + r_2^2$

Problem 31. Two soap bubbles of radii r_1 and r_2 equal to 4*cm* and 5*cm* are touching each other over a common surface S_1S_2 (shown in figure). Its radius will be **[MP PMT 2002]**



(a) 4 *cm*

(b) 20 *cm*

(c) 5 *cm*

(d) 4.5 *cm*

Solution : (b) Radius of curvature of common surface of double bubble $r = \frac{r_2 r_1}{r_2 - r_1} = \frac{5 \times 4}{5 - 4} = 20cm$

Problem 32. An air bubble in a water tank rises from the bottom to the top. Which of the following statements are true **[Roorkee 2000]**

(a) Bubble rises upwards because pressure at the bottom is less than that at the top

(b) Bubble rises upwards because pressure at the bottom is greater than that at the top

(c) As the bubble rises, its size increases

(d) As the bubble rises, its size decreases

Solution : (b, c)

Problem 33. The radii of two soap bubbles are R_1 and R_2 respectively. The ratio of masses of air in them will be

(a) $\frac{R_1^3}{R_2^3}$ (b) $\frac{R_2^3}{R_1^3}$ (c) $\left(\frac{P+\frac{4T}{R_1}}{P+\frac{4T}{R_2}}\right)\frac{R_1^3}{R_2^3}$ (d) $\left(\frac{P+\frac{4T}{R_2}}{P+\frac{4T}{R_1}}\right)\frac{R_2^3}{R_1^3}$

Solution : (c) From $PV = \mu RT$.

At a given temperature, the ratio masses of air $\frac{\mu_1}{\mu_2} = \frac{P_1V_1}{P_2V_2} = \frac{\left(P+\frac{4T}{R_1}\right)\frac{4}{3}\pi R_1^3}{\left(P+\frac{4T}{R_2}\right)\frac{4}{3}\pi R_2^3} = \frac{\left(P+\frac{4T}{R_1}\right)}{\left(P+\frac{4T}{R_2}\right)}\frac{R_1^3}{R_2^3}$.

Problem 34. On dipping one end of a capillary in liquid and inclining the capillary at an angles 30^o and 60^o with the vertical, the lengths of liquid columns in it are found to be l_1 and l_2 respectively. The ratio of l_1 and l_2 is

(a) $1:\sqrt{3}$ (b) $1:\sqrt{2}$ (c) $\sqrt{2}:1$ (d) $\sqrt{3}:1$

Solution : (a) $l_1 = \frac{h}{\cos\alpha_1}$ and $l_2 = \frac{h}{\cos\alpha_2}$ $\therefore \frac{l_1}{l_2} = \frac{\cos\alpha_2}{\cos\alpha_1} = \frac{\cos 60^o}{\cos 30^o} = \frac{1/2}{\sqrt{3}/2} = 1:\sqrt{3}$

Problem 35. A drop of water of volume V is pressed between the two glass plates so as to spread to an area A. If T is the surface tension, the normal force required to separate the glass plates is

(a) $\frac{TA^2}{V}$ (b) $\frac{2TA^2}{V}$ (c) $\frac{4TA^2}{V}$ (d) $\frac{TA^2}{2V}$

Solution : (b) Force required to separate the glass plates $F = \frac{2AT}{t} \times \frac{A}{A} = \frac{2TA^2}{(A\times t)} = \frac{2TA^2}{V}$.