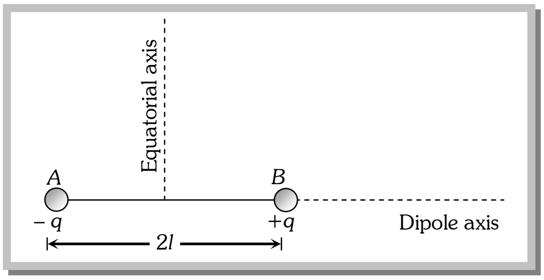
Electric Charges and Fields (Electrostatics Part 4)

Electric Dipole.

(1) **General information :** System of two equal and opposite charges separated by a small fixed distance is called a dipole.



(i) **Dipole axis :** Line joining negative charge to positive charge of a dipole is called its axis. It may also be termed as its longitudinal axis.

(ii) **Equatorial axis :** Perpendicular bisector of the dipole is called its equatorial or transverse axis as it is perpendicular to length.

(iii) **Dipole length :** The distance between two charges is known as dipole length ($L = 2l$)

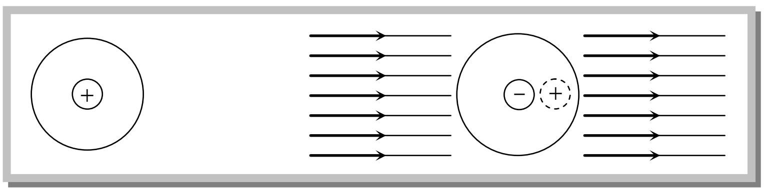
(iv) **Dipole moment :** It is a quantity which gives information about the strength of dipole. It is a vector quantity and is directed from negative charge to positive charge along the axis. It is denoted as $\vec{p}$ and is defined as the product of the magnitude of either of the charge and the dipole length.

i.e. $\vec{p} = q(2\vec{l})$

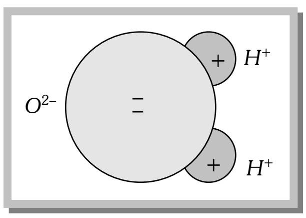
Its S.I. unit is ***coulomb-metre*** or ***Debye*** (1 *Debye* = 3.3×10^{-30} $C \times m$) and its dimensions are $M^0L^1T^1A^1$.

Note : ≅A region surrounding a stationary electric dipole has electric field only.

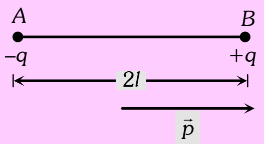
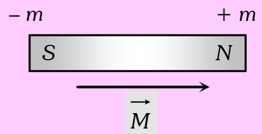
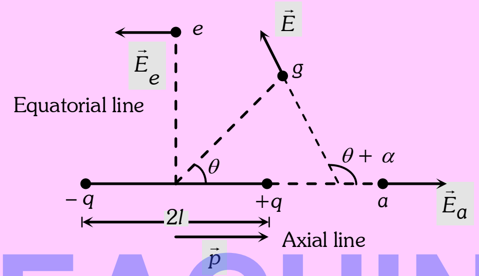
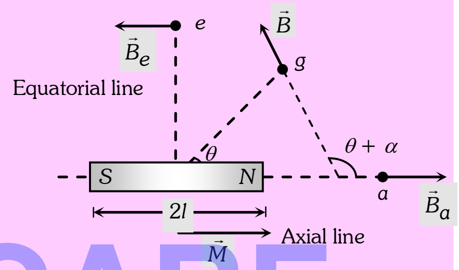
≅ When a dielectric is placed in an electric field, its atoms or molecules are considered as tiny dipoles.



≅ Water (H_2O), Chloroform ($CHCl_3$), Ammonia (NH_3), HCl, CO molecules are some example of permanent electric dipole.

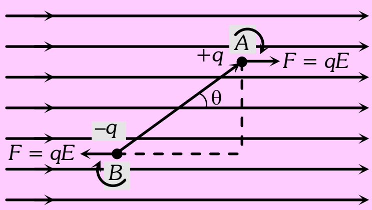
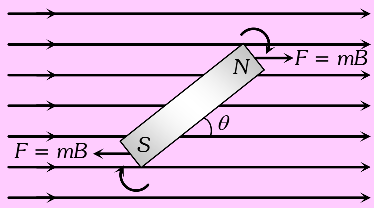


(2) **Electric field and potential due to an electric dipole :** It is better to understand electric dipole with magnetic dipole.

S.No.	Electric dipole	Magnetic dipole
(i)	System of two equal and opposite charges separated by a small fixed distance.  	System of two equal and opposite magnetic poles (Bar magnet) separated by a small fixed distance. 
(ii)	**Electric dipole moment :** $\vec{p} = q(2\vec{l})$, directed from $-q$ to $+q$. It's S.I. unit is *coulomb* × *meter* or *Debye*.	**Magnetic dipole moment :** $\vec{M} = m(2\vec{l})$, directed from S to N. It's S.I. unit is *ampere* × *meter*^{2}.
(iii)	**Intensity of electric field**  If a, e and g are three points on axial, equatorial and general position at a distance r from the centre of dipole on axial point $E_a = \frac{1}{4\pi\varepsilon_0}.\frac{2p}{r^3}$ (directed from $-q$ to $+q$) on equatorial point $E_e = \frac{1}{4\pi\varepsilon_0}.\frac{p}{r^3}$ (directed from $+q$ to $-q$) on general point $E_a = \frac{1}{4\pi\varepsilon_0}.\frac{p}{r^3}\sqrt{(3\cos^2\theta + 1)}$ Angle between – $\vec{E}_a$ and $\vec{p}$ is 0°, $\vec{E}_e$ and $\vec{p}$ is 180°, $\vec{E}$ and $\vec{p}$ is $(\theta + \alpha)$ (where $\tan\alpha = \frac{1}{2}\tan\theta$) Electric Potential – At a $V_a = \frac{1}{4\pi\varepsilon_0}.\frac{p}{r^2}$, At e $V = 0$ At g $V = \frac{1}{4\pi\varepsilon_0}.\frac{p\cos\theta}{r^2}$	**Intensity of magnetic field**  If a, e and g are three points on axial, equatorial and general position at a distance r from the centre of dipole on axial point $B_a = \frac{\mu_0}{4\pi}.\frac{2M}{r^3}$ (directed from S to N) on equatorial point $B_e = \frac{\mu_0}{4\pi}.\frac{M}{r^3}$ (directed from N to S) on general point $B_a = \frac{\mu_0}{4\pi}.\frac{M}{r^3}\sqrt{(3\cos^2\theta + 1)}$ Angle between – $\vec{B}_a$ and $\vec{M}$ is 0°, $\vec{B}_e$ and $\vec{M}$ is 180°, $\vec{B}$ and $\vec{M}$ is $(\theta + \alpha)$ (where $\tan\alpha = \frac{1}{2}\tan\theta$)

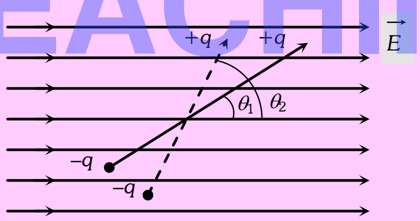
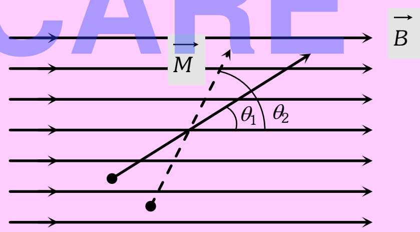
(3) **Dipole (electric/magnetic) in uniform field (electric/magnetic)**

(i) **Torque :** If a dipole is placed in an uniform field such that dipole (*i.e.* $\vec{p}$ or $\vec{M}$) makes an angle θ with direction of field then two equal and opposite force acting on dipole constitute a couple whose tendency is to rotate the dipole hence a torque is developed in it and dipole tries to align it self in the direction of field.

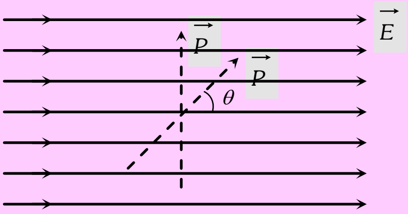
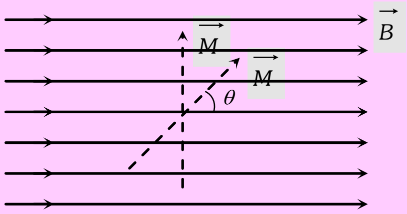
Electric dipole	Magnetic dipole
Consider an electric dipole in placed in an uniform electric field such that dipole (i.e. $\vec{p}$) makes an angle θ with the direction of electric field as shown	A magnetic dipole of magnetic moment M is placed in uniform magnetic field B by making an angle θ as shown
	
(a) Net force on electric dipole $F_{net} = 0$	(a) Net force on magnetic dipole $F_{net} = 0$
(b) Produced torque $\tau = pE\sin\theta$ $(\vec{\tau} = \vec{P} \times \vec{E})$	(b) torque $\tau = MB\sin\theta$ $(\vec{\tau} = \vec{M} \times \vec{B})$

(ii) **Work :** From the above discussion it is clear that in an uniform electric/magnetic field dipole tries to align itself in the direction of electric field (i.e. equilibrium position). To change it's angular position some work has to be done.

Suppose an electric/magnetic dipole is kept in an uniform electric/magnetic field by making an angle θ_1 with the field, if it is again turn so that it makes an angle θ_2 with the field, work done in this process is given by the formula

Electric dipole	Magnetic dipole
	
$W = pE(\cos\theta_1 - \cos\theta_2)$	$W = MB(\cos\theta_1 - \cos\theta_2)$
If $\theta_1 = 0°$ and $\theta_2 = \theta$ *i.e.* initially dipole is kept along the field then it turn through θ so work done $W = pE(1 - \cos\theta)$	If $\theta_1 = 0°$ and $\theta_2 = \theta$ then $W = MB(1 - \cos\theta)$

(iii) **Potential energy :** In case of a dipole (in a uniform field), potential energy of dipole is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction *i.e.* if $\theta_1 = 90°$ and $\theta_2 = \theta$ then –

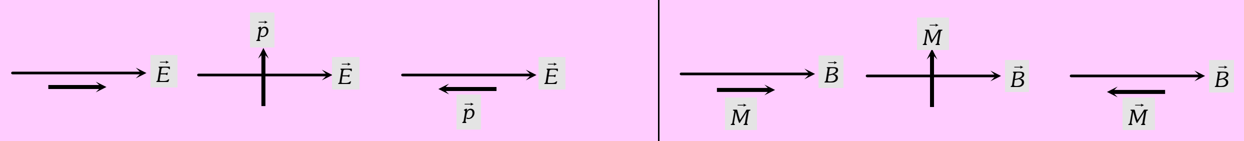
Electric dipole	Magnetic dipole
	
$W = U = pE(\cos 90 - \cos\theta) \Rightarrow U = -pE\cos\theta$	$W = U = MB(\cos 90 - \cos\theta) \Rightarrow U = -MB\cos\theta$

(iv) **Equilibrium of dipole :** We know that, for any equilibrium net torque and net force on a particle (or system) should be zero.

We already discussed when a dipole is placed in an uniform electric/magnetic field net force on dipole is always zero. But net torque will be zero only when $\theta = 0^o$ or 180^o

When $\theta = 0^o$ *i.e.* dipole is placed along the electric field it is said to be in stable equilibrium, because after turning it through a small angle, dipole tries to align itself again in the direction of electric field.

When $\theta = 180^o$ *i.e.* dipole is placed opposite to electric field, it is said to be in unstable equilibrium.



$\theta = 0^o$	$\theta = 90^O$	$\theta = 180^o$	$\theta = 0^o$	$\theta = 90^O$	$\theta = 180^o$
Stable equilibrium		Unstable equilibrium	Stable equilibrium		Unstable equilibrium
$\tau = 0$	$\tau_{max} = pE$	$\tau = 0$	$\tau = 0$	$\tau_{max} = MB$	$\tau = 0$
$W = 0$	$W = pE$	$W_{max} = 2pE$	$W = 0$	$W = MB$	$W_{max} = 2MB$
$U_{min} = -pE$	$U = 0$	$U_{max} = pE$	$U_{min} = -MB$	$U = 0$	$U_{max} = MB$

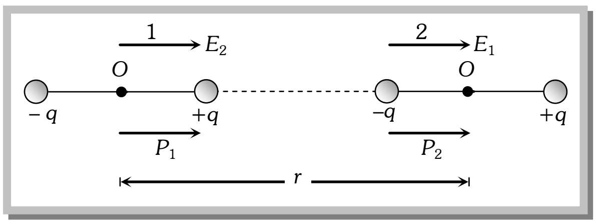
(v) **Angular SHM :** In a uniform electric/magnetic field (intensity E/B) if a dipole (electric/magnetic) is slightly displaced from it's stable equilibrium position it executes angular SHM having period of oscillation. If I = moment of inertia of dipole about the axis passing through it's centre and perpendicular to it's length.

For electric dipole : $T = 2\pi\sqrt{\frac{I}{pE}}$ and For Magnetic dipole : $T = 2\pi\sqrt{\frac{I}{MB}}$

(vi) **Dipole-point charge interaction :** If a point charge/isolated magnetic pole is placed in dipole field at a distance r from the mid point of dipole then force experienced by point charge/pole varies according to the relation $F \propto \frac{1}{r^3}$

(vii) **Dipole-dipole interaction :** When two dipoles placed closed to each other, they experiences a force due to each other. If suppose two dipoles (1) and (2) are placed as shown in figure then

Both the dipoles are placed in the field of one another hence potential energy dipole (2) is



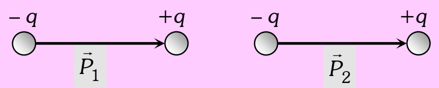
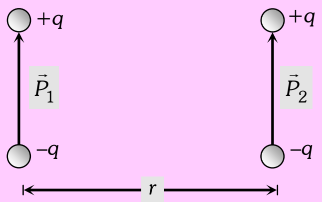
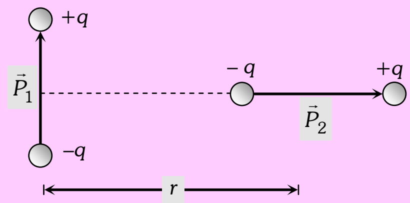
$$U_2 = -p_2E_1\cos 0 = -p_2E_1 = -p_2 \times \frac{1}{4\pi\varepsilon_0}.\frac{2p_1}{r^3}$$

then by using $F = -\frac{dU}{dr}$, Force on dipole (2) is $F_2 = -\frac{dU_2}{dr}$

$$\Rightarrow F_2 = -\frac{d}{dr}\left\{\frac{1}{4\pi\varepsilon_0}.\frac{2p_1p_2}{r^3}\right\} = -\frac{1}{4\pi\varepsilon_0}.\frac{6p_1p_2}{r^4}$$

Similarly force experienced by dipole (1) $F_1 = -\frac{1}{4\pi\varepsilon_0}.\frac{6p_1p_2}{r^4}$ so $F_1 = F_2 = -\frac{1}{4\pi\varepsilon_0}.\frac{6p_1p_2}{r^4}$

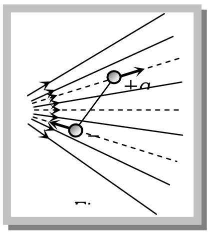
Negative sign indicates that force is attractive. $|F| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{6p_1p_2}{r^4}$ and $\boldsymbol{F} \propto \frac{\boldsymbol{1}}{\boldsymbol{r^4}}$

S. No.	Relative position of dipole	Force	Potential energy
(i)		$\frac{1}{4\pi\varepsilon_0} \cdot \frac{6p_1p_2}{r^4}$ (attractive)	$\frac{1}{4\pi\varepsilon_0} \cdot \frac{2p_1p_2}{r^3}$
(ii)		$\frac{1}{4\pi\varepsilon_0} \cdot \frac{3p_1p_2}{r^4}$ (repulsive)	$\frac{1}{4\pi\varepsilon_0} \cdot \frac{p_1p_2}{r^3}$
(iii)		$\frac{1}{4\pi\varepsilon_0} \cdot \frac{3p_1p_2}{r^4}$ (perpendicular to r)	0

Note : ≅ Same result can also be obtained for magnetic dipole.

(4) **Electric dipole in non-uniform electric field :** When an electric dipole is placed in a non-uniform field, the two charges of dipole experiences unequal forces, therefore the net force on the dipole is not equal to zero. The magnitude of the force is given by the negative derivative of the potential energy w.r.t. distance along the axis of the dipole *i.e.* $\vec{F} = -\frac{dU}{dr} = -\vec{p} \cdot \frac{d\vec{E}}{dr}$.

Due to two unequal forces, a torque is produced which rotate the dipole so as to align it in the direction of field. When the dipole gets aligned with the field, the torque becomes zero and then the unbalanced force acts on the dipole and the dipole then moves linearly along the direction of field from weaker portion of the field to the stronger portion of the field. So in non-uniform electric field



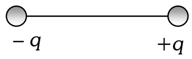
(i) Motion of the dipole is translatory and rotatory

(ii) Torque on it may be zero.

Concepts

☞ *For a short dipole, electric field intensity at a point on the axial line is double than at a point on the equatorial line on electric dipole i.e.* $E_{axial} = 2E_{equatorial}$

☞ *It is intresting to note that dipole field* $E \propto \frac{1}{r^3}$ *decreases much rapidly as compared to the field of a point charge* $\left(E \propto \frac{1}{r^2}\right)$.



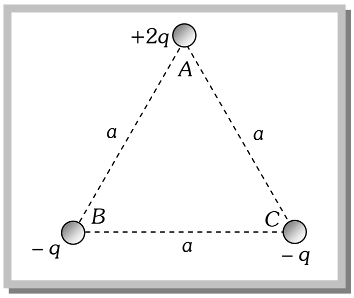
Examples based on electric dipole

Example: 84 If the magnitude of intensity of electric field at a distance x on axial line and at a distance y on equatorial line on a given dipole are equal, then $x : y$ is **[EAMCET 1994]**

(a) $1 : 1$ (b) $1 : \sqrt{2}$ (c) $1 : 2$ (d) $\sqrt[3]{2} : 1$

Solution: (d) According to the question $\frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{x^3} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{y^3} \Rightarrow \frac{x}{y} = (2)^{1/3} : 1$

Example: 85 Three charges of $(+2q)$, $(-q)$ and $(-q)$ are placed at the corners A, B and C of an equilateral triangle of side a as shown in the adjoining figure. Then the dipole moment of this combination is **[MP PMT 1994; CPMT 1994]**



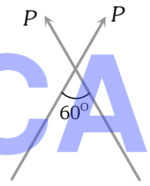
(a) qa

(b) Zero

(c) $q a\sqrt{3}$

(d) $\frac{2}{\sqrt{3}} qa$

Solution: (c) The charge $+2q$ can be broken in $+q$, $+q$. Now as shown in figure we have two equal dipoles inclined at an angle of 60°. Therefore resultant dipole moment will be

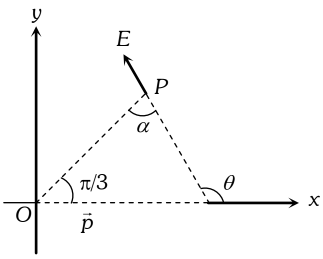


$$p_{net} = \sqrt{p^2 + p^2 + 2pp\cos 60}$$
$$= \sqrt{3}p$$
$$= \sqrt{3}\, qa$$

Example: 86 An electric dipole is placed along the x-axis at the origin O. A point P is at a distance of 20 cm from this origin such that OP makes an angle $\frac{\pi}{3}$ with the x-axis. If the electric field at P makes an angle θ with x-axis, the value of θ would be **[MP PMT 1997]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{3} + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (c) $\frac{2\pi}{3}$ (d) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Solution: (b) According to question we can draw following figure.



As we have discussed earlier in theory $\theta = \frac{\pi}{3} + \alpha$

$$\tan\alpha = \frac{1}{2}\tan\frac{\pi}{3} \Rightarrow \alpha = \tan^{-1}\frac{\sqrt{3}}{2}$$

So, $$\theta = \frac{\pi}{3} + \tan^{-1}\frac{\sqrt{3}}{2}$$

Example: 87 An electric dipole in a uniform electric field experiences **[RPET 2000]**

(a) Force and torque both (b) Force but no torque (c) Torque but no force (d) No force and no torque

Solution: (c) In uniform electric field $F_{net} = 0$, $\tau_{net} \neq 0$

Example: 89 Two opposite and equal charges 4×10^{-8} *coulomb* when placed 2×10^{-2} *cm* away, form a dipole. If this dipole is placed in an external electric field 4×10^{8} *newton/coulomb*, the value of maximum torque and the work done in rotating it through 180° will be **[MP PET 1996 Similar to MP PMT 1987]**

(a) 64×10^{-4} *Nm* and 64×10^{-4} *J* (b) 32×10^{-4} *Nm* and 32×10^{-4} *J*

(c) $64 \times 10^{-4}\ Nm$ and $32 \times 10^{-4}\ J$ (d) $32 \times 10^{-4}\ Nm$ and $64 \times 10^{-4}\ J$

Solution: (d) $\tau_{max} = pE$ and $W_{max} = 2pE$ $\because$ $p = Q \times 2l = 4 \times 10^{-8} \times 2 \times 10^{-2} \times 10^{-2} = 8 \times 10^{-12}\ C\text{-}m$

So, $\tau_{max} = 8 \times 10^{-12} \times 4 \times 10^{8} = 32 \times 10^{-4}\ N\text{-}m$ and $W_{max} = 2 \times 32 \times 10^{-4} = 64 \times 10^{-4}\ J$

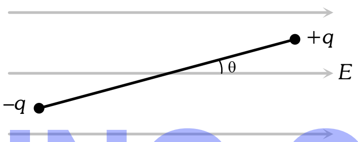
Example: 90 A point charge placed at any point on the axis of an electric dipole at some large distance experiences a force F. The force acting on the point charge when it's distance from the dipole is doubled is

[CPMT 1991; MNR 1986]

(a) F (b) $\frac{F}{2}$ (c) $\frac{F}{4}$ (d) $\frac{F}{8}$

Solution: (d) Force acting on a point charge in dipole field varies as $F \propto \frac{1}{r^3}$ where r is the distance of point charge from the centre of dipole. Hence if r makes double so new force $F' = \frac{F}{8}$.

Example: 91 A point particle of mass M is attached to one end of a massless rigid non-conducting rod of length L. Another point particle of the same mass is attached to other end of the rod. The two particles carry charges $+q$ and $-q$ respectively. This arrangement is held in a region of a uniform electric field E such that the rod makes a small angle θ (say of about 5 degrees) with the field direction (see figure). Will be minimum time, needed for the rod to become parallel to the field after it is set free



(a) $t = 2\pi\sqrt{\frac{mL}{2pE}}$ (b) $t = \frac{\pi}{2}\sqrt{\frac{mL}{2qE}}$ (c) $t = \frac{3\pi}{2}\sqrt{\frac{mL}{2pE}}$ (d) $t = \pi\sqrt{\frac{2mL}{qE}}$

Solution: (b) In the given situation system oscillate in electric field with maximum angular displacement θ. It's time period of oscillation (similar to dipole)

$$T = 2\pi\sqrt{\frac{I}{pE}}$$ where I = moment of inertia of the system and $p = qL$

Hence the minimum time needed for the rod becomes parallel to the field is $t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{I}{pE}}$

Here $I = M\left(\frac{L}{2}\right)^2 + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{2} \Rightarrow t = \frac{\pi}{2}\sqrt{\frac{ML^2}{2 \times qL \times E}} = \frac{\pi}{2}\sqrt{\frac{ML}{2qE}}$

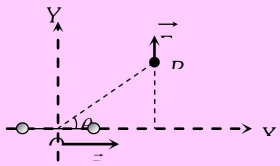
Tricky example: 12

An electric dipole is placed at the origin O and is directed along the x-axis. At a point P, far away from the dipole, the electric field is parallel to y-axis. OP makes an angle θ with the x-axis then

(a) $\tan\theta = \sqrt{3}$ (b) $\tan\theta = \sqrt{2}$ (c) $\theta = 45^\circ$ (d) $\tan\theta = \frac{1}{\sqrt{2}}$

Solution: (b) As we know that in this case electric field makes an angle $\theta + \alpha$ with the direction of dipole

Where $\tan\alpha = \frac{1}{2}\tan\theta$



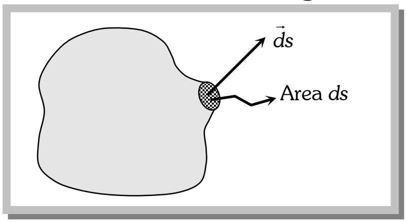
Here $\theta + \alpha = 90^\circ \Rightarrow \alpha = 90 - \theta$

Hence $\tan(90 - \theta) = \frac{1}{2}\tan\theta \Rightarrow \cot\theta = \frac{1}{2}\tan\theta$

$\Rightarrow \quad \tan^2\theta = 2 \quad \Rightarrow \tan\theta = \sqrt{2}$

Electric Flux.

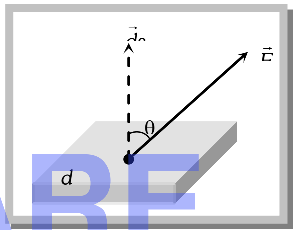
(1) **Area vector :** In many cases, it is convenient to treat area of a surface as a vector. The length of the vector represents the magnitude of the area and its direction is along the outward drawn normal to the area.



(2) **Electric flux :** The electric flux linked with any surface in an electric field is basically a measure of total number of lines of forces passing normally through the surface. **or**

Electric flux through an elementary area $\vec{ds}$ is defined as the scalar product of area of field i.e. $d\phi = \vec{E}\cdot\vec{ds} = E\,ds\cos\theta$

Hence flux from complete area (S) $\phi = \int E\,ds\cos\theta = ES\cos\theta$



If $\theta = 0^\circ$, *i.e.* surface area is perpendicular to the electric field, so flux linked with it will be max.

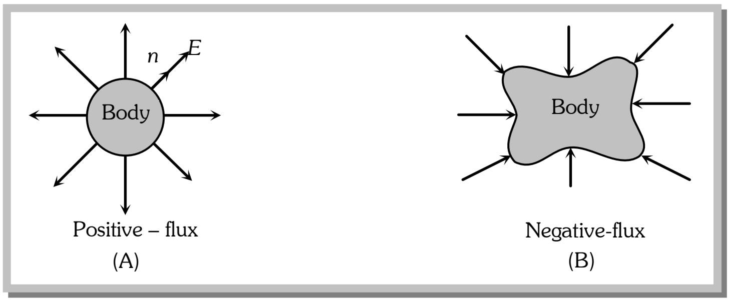
i.e. $\phi_{max} = E\,ds$ and if $\theta = 90^\circ$, $\phi_{min} = 0$

(3) **Unit and Dimensional Formula**

S.I. unit – (*volt* × *m*) or $\frac{N-C}{m^2}$

It's Dimensional formula – $(ML^3T^{-3}A^{-1})$

(4) **Types :** For a closed body outward flux is taken to be positive, while inward flux is to be negative

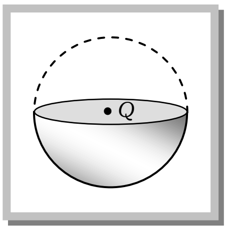


Gauss's Law.

(1) **Definition :** According to this law, total electric flux through a closed surface enclosing a charge is $\frac{1}{\varepsilon_0}$ times the magnitude of the charge enclosed i.e. $\phi = \frac{1}{\varepsilon_0}(Q_{enc.})$

(2) **Gaussian Surface :** Gauss's law is valid for symmetrical charge distribution. Gauss's law is very helpful in calculating electric field in those cases where electric field is symmetrical around the source producing it. Electric field can be calculated very easily by the clever choice of a closed surface that encloses the source charges. Such a surface is called "Gaussian surface". This surface should pass through the point where electric field is to be calculated and must have a shape according to the symmetry of source.

e.g. If suppose a charge Q is placed at the centre of a hemisphere, then to calculate the flux through this body, to encloses the first charge we will have to imagine a Gaussian surface. This imaginary Gaussian surface will be a hemisphere as shown.



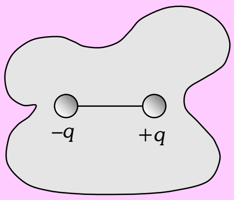
Net flux through this closed body $\phi = \dfrac{Q}{\varepsilon_0}$

Hence flux coming out from given hemisphere is $\phi = \dfrac{Q}{2\varepsilon_0}$.

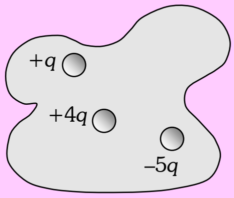
(3) **Zero flux :** The value of flux is zero in the following circumstances

(i) If a dipole is enclosed by a surface

$\phi = 0; Q_{enc} = 0$



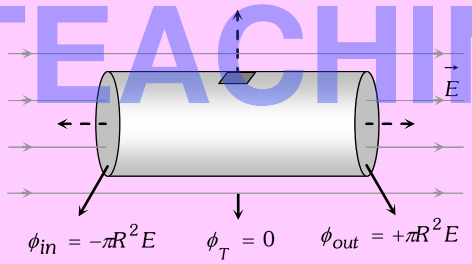
(ii) If the magnitude of positive and negative charges are equal inside a closed surface

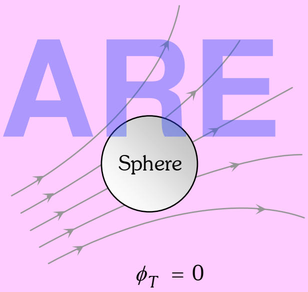


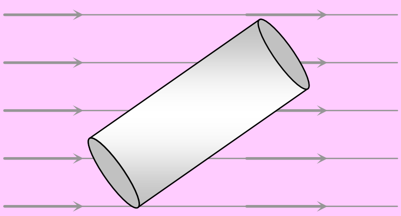
$Q_{enc} = 0$,

so, $\phi = 0$

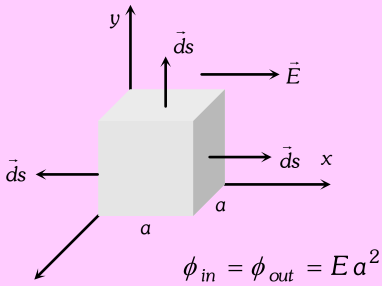
(iii) If a closed body (not enclosing any charge) is placed in an electric field (either uniform or non-uniform) total flux linked with it will be zero



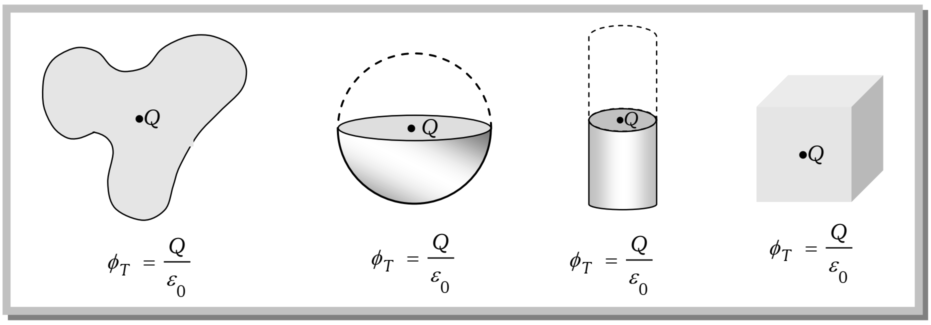




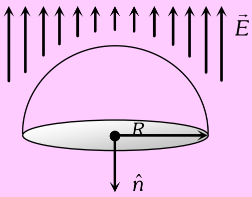
$\phi_T = 0$



(4) **Flux emergence :** Flux linked with a closed body is independent of the shape and size of the body and position of charge inside it

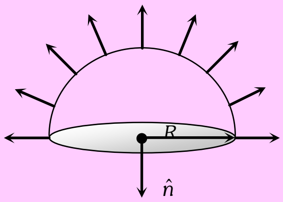


(i) If a hemispherical body is placed in uniform electric field then flux linked with the curved surface



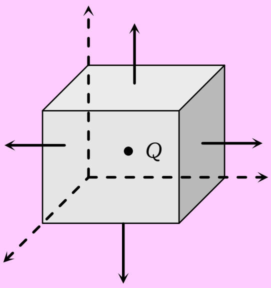
$\phi_{curved} = +\pi R^2 E$

(ii) If a hemispherical body is placed in non-uniform electric field as shown below. then flux linked with the curved surface.



$\phi_{curved} = 2\pi R^2 E$

(v) If charge is kept at the centre of cube

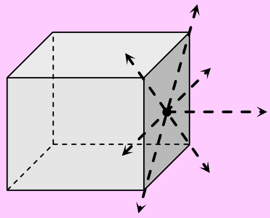


$\phi_{total} = \frac{1}{\varepsilon_0}.(Q)$

$\phi_{face} = \frac{Q}{6\varepsilon_0}$

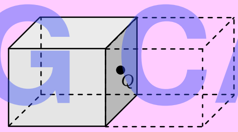
$\phi_{corner} = \frac{Q}{8\varepsilon_0}$ $\phi_{edge} = \frac{Q}{12\varepsilon_0}$

(iv) If charge is kept at the centre of a face



First we should enclosed the charge by assuming a Gaussian surface (an identical imaginary cube)

$\phi_{total} = \frac{Q}{\varepsilon_0}$



$\phi_{cube} = \frac{Q}{2\varepsilon_0}$ (i.e. from 5 face only)

$\phi_{face} = \frac{1}{5}\left(\frac{Q}{2\varepsilon_0}\right) = \frac{Q}{10\varepsilon_0}$.

Concept

- *In C.G.S. $\varepsilon_0 = \frac{1}{4\pi}$. Hence if 1C charge is enclosed by a closed surface so flux through the surface will be $\phi = 4\pi$.*

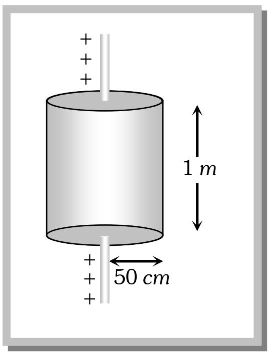
Example based on electric flux and Gauss's

Example: 91 Electric charge is uniformly distributed along a long straight wire of radius 1 *mm*. The charge per *cm* length of the wire is *Q coulomb*. Another cylindrical surface of radius 50 *cm* and length 1 *m* symmetrically encloses the wire as shown in the figure. The total electric flux passing through the cylindrical surface is **[MP PET 2001]**

(a) $\frac{Q}{\varepsilon_0}$

(b) $\frac{100\,Q}{\varepsilon_0}$

(c) $\frac{10\,Q}{(\pi\varepsilon_0)}$

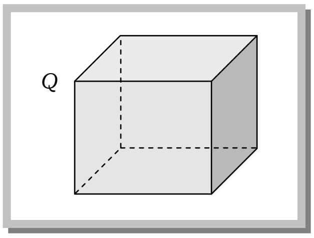


(d) $\frac{100Q}{(\pi\varepsilon_0)}$

Solution: (b) Given that charge per *cm* length of the wire is *Q*. Since 100 *cm* length of the wire is enclosed so $Q_{enc} = 100Q$

$\Rightarrow$ Electric flux emerging through cylindrical surface $\phi = \frac{100Q}{\varepsilon_0}$.

Example: 92 A charge *Q* is situated at the corner *A* of a cube, the electric flux through the one face of the cube is **[CPMT 2000]**



(a) $\frac{Q}{6\varepsilon_0}$ (b) $\frac{Q}{8\varepsilon_0}$ (c) $\frac{Q}{24\varepsilon_0}$ (d) $\frac{Q}{2\varepsilon_0}$

Solution: (c) For the charge at the corner, we require eight cube to symmetrically enclose it in a Gaussian surface. The total flux $\phi_T = \frac{Q}{\varepsilon_0}$. Therefore the flux through one cube will be $\phi_{cube} = \frac{Q}{8\varepsilon_0}$. The cube has six faces and flux linked with three faces (through *A*) is zero, so flux linked with remaining three faces will $\frac{\phi}{8\varepsilon_0}$. Now as the remaining three are identical so flux linked with each of the three faces will be $= \frac{1}{3} \times \left[\frac{1}{8}\left(\frac{Q}{\varepsilon_0}\right)\right] = \frac{1}{24}\frac{Q}{\varepsilon_0}$.

Example: 93 A square of side 20 *cm* is enclosed by a surface of sphere of 80 *cm* radius. Square and sphere have the same centre. Four charges $+2 \times 10^{-6}$ *C*, -5×10^{-6} *C*, -3×10^{-6} *C*, $+6 \times 10^{-6}$ C are located at the four corners of a square, then out going total flux from spherical surface in $N\text{–}m^2/C$ will be

(a) Zero (b) $(16\pi) \times 10^{-6}$ (c) $(8\pi) \times 10^{-6}$ (d) $36\pi \times 10^{-6}$

Solution: (a) Since charge enclosed by Gaussian surface is

$$\phi_{enc.} = (2 \times 10^{-6} - 5 \times 10^{-6} - 3 \times 10^{-6} + 6 \times 10^{-6}) = 0 \quad \text{so} \quad \phi = 0$$

Example: 94 In a region of space, the electric field is in the *x*-direction and proportional to *x*, *i.e.*, $\vec{E} = E_0 x\hat{i}$. Consider an imaginary cubical volume of edge *a*, with its edges parallel to the axes of coordinates. The charge inside this cube is

(a) Zero (b) $\varepsilon_0 E_0 a^3$ (c) $\frac{1}{\varepsilon_0} E_0 a^3$ (d) $\frac{1}{6}\varepsilon_0 E_0 a^2$

Solution: (b) The field at the face *ABCD* = $E_0 x_0 \hat{i}$.

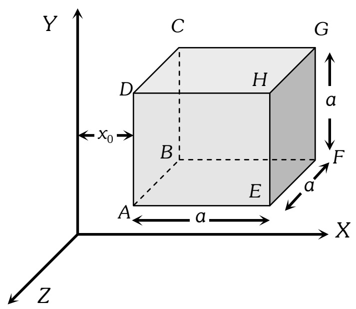
$\therefore$ Flux over the face *ABCD* = $-(E_0 x_0) a^2$

The negative sign arises as the field is directed into the cube.

The field at the face EFGH = $E_0(x_0 + a)\hat{i}$.

$\therefore$ Flux over the face *EFGH* = $E_0(x_0 + a)a^2$

The flux over the other four faces is zero as the field is parallel to the surfaces.



$\therefore$ Total flux over the cube $= E_0 a^2 = \frac{1}{2} q$

where q is the total charge inside the cube. $\therefore\ q = \varepsilon_0 E_0 a^3$.

Tricky example: 13

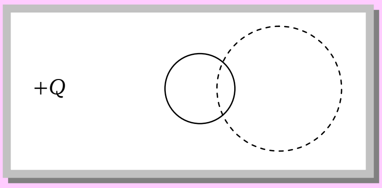
In the electric field due to a point charge $+Q$ a spherical closed surface is drawn as shown by the dotted circle. The electric flux through the surface drawn is zero by Gauss's law. A conducting sphere is inserted intersecting the previously drawn Gaussian surface. The electric flux through the surface

(a) Still remains zero

(b) Non zero but positive

(c) Non-zero but negative

(d) Becomes infinite

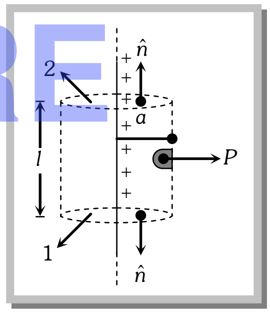


Solution: (b) Due to induction some positive charge will lie within the Gaussian surface drawn and hence flux becomes something positive.

Application of Gauss's Law.

Gauss's law is a powerful tool for calculating electric field in case of symmetrical charge distribution by choosing a Gaussian surface in such away that $\vec{E}$ is either parallel or perpendicular to it's various faces.

***e.g.* Electric field due to infinitely long line of charge :** Let us consider a uniformly charged wire of infinite length having a constant linear charge density is $\lambda \left(\lambda = \frac{\text{charge}}{\text{length}} \right)$. Let P be a point distant r from the wire at which the electric field is to be calculated.



Draw a cylinder (Gaussian surface) of radius r and length l around the line charge which encloses the charge Q $(Q = \lambda . l)$. Cylindrical Gaussian surface has three surfaces; two circular and one curved for surfaces (1) and (2) angle between electric field and normal to the surface is 90° i.e., $\theta = 90^\circ$.

So flux linked with these surfaces will be zero. Hence total flux will pass through curved surface and it is

$$\phi = \int E\, ds \cos\theta \qquad \text{.... (i)}$$

According to Gauss's law

$$\phi = \frac{Q}{\varepsilon_0} \qquad \text{.... (ii)}$$

Equating equation (i) and (ii) $\int E\, ds = \frac{Q}{\varepsilon_0}$

$$\Rightarrow \quad E\int ds = \frac{Q}{\varepsilon_0} \Rightarrow Ex2\pi rl = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{Q}{2\pi\varepsilon_0 rl} = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r} \left\{ K = \frac{1}{4\pi\varepsilon_0} \right\}$$