Capacitance.

(1) **Definition :** We know that charge given to a conductor increases it's potential *i.e.*, $Q \propto V \Rightarrow Q = CV$

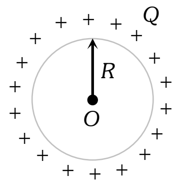
Where C is a proportionality constant, called capacity or capacitance of conductor. Hence capacitance is the ability of conductor to hold the charge.

(2) **Unit and dimensional formula** : S.I. unit is $\frac{Coulomb}{Volt} = Farad\ (F)$

Smaller S.I. units are mF, μF, nF and pF ($1mF = 10^{-3}F$, $1\mu F = 10^{-6}F$, $1nF = 10^{-9}F$, $1pF = 1\mu\mu F = 10^{-12}F$)

C.G.S. unit is *Stat Farad* $1F = 9 \times 10^{11}$ *Stat Farad*. Dimension : $[C] = [M^{-1}L^{-2}T^4A^2]$.

(3) **Capacity of an isolated spherical conductor :** When charge Q is given to a spherical conductor of radius R, then potential at the surface of sphere is



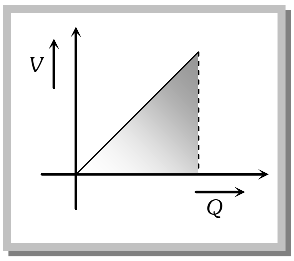
$$V = k.\frac{Q}{R} \qquad \left\{k = \frac{1}{4\pi\varepsilon_0}\right\}$$

Hence it's capacity $C = \frac{Q}{V} = 4\pi\varepsilon_0 R \qquad \Rightarrow C = 4\pi\varepsilon_0 R = \frac{1}{9 \times 10^9}.R$

in C.G.S. $C = R$

Note : ≅ If earth is assumed to be spherical having radius $R = 6400\ km$. It's theortical capacitance $C = \frac{1}{9 \times 10^9} \times 6400 \times 10^3 = 711\,\mu F$. But for all practical purpose capacitance of earth is taken infinity.

(4) **Energy of a charged conductor :** When a conductor is charged it's potential increases from 0 to V as shown in the graph; and work is done against repulsion, between charge stored in the conductor and charge coming from the source (battery). This work is stored as "electrostatic potential energy"



From graph : Work done = Area of graph $= \frac{1}{2}QV$

Hence potential energy $U = \frac{1}{2}QV$; By using $Q = CV$, we can write

$$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

(5) **Combination of drops :** Suppose we have n identical drops each having – Radius – r, Capacitance – c, Charge – q, Potential – v and Energy – u.

If these drops are combined to form a big drop of – Radius – R, Capacitance – C, Charge – Q, Potential – V and Energy – U then –

(i) **Charge on big drop :** $Q = nq$

(ii) **Radius of big drop :** Volume of big drop = $n \times$ volume of a single drop *i.e.*, $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$, $R = n^{1/3} r$

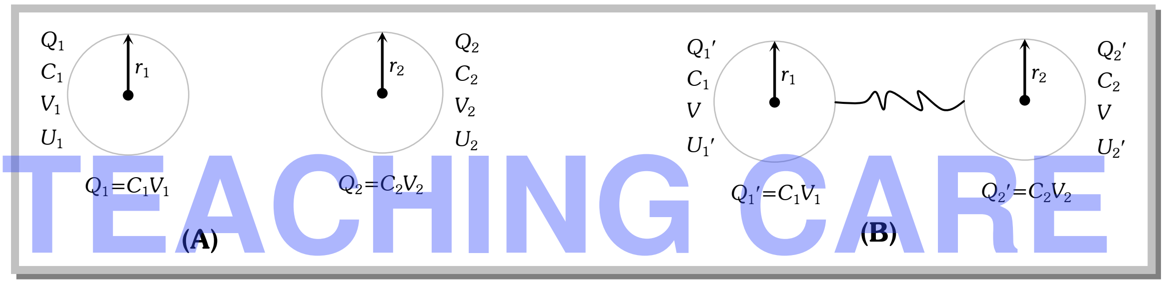
(iii) **Capacitance of big drop :** $C = n^{1/3} c$

(iv) **Potential of big drop :** $V = \frac{Q}{C} = \frac{nq}{n^{1/3} c}$ $\quad V = n^{2/3} v$

(v) **Energy of big drop :** $U = \frac{1}{2} CV^2 = \frac{1}{2}(n^{1/3} c)(n^{2/3} v)^2$ $\quad U = n^{5/3} u$

(6) **Sharing of charge :** When two conductors joined together through a conducting wire, charge begins to flow from one conductor to another till both have the same potential, due to flow of charge, loss of energy also takes place in the form of heat.

Suppose there are two spherical conductors of radii r_1 and r_2, having charge Q_1 and Q_2, potential V_1 and V_2, energies U_1 and U_2 and capacitance C_1 and C_2 respectively, as shown in figure. If these two spheres are connected through a conducting wire, then alteration of charge, potential and energy takes place.



(i) **New charge :** According to the conservation of charge $Q_1 + Q_2 = Q_1' + Q_2' = Q$ (say), also

$$\frac{Q_1'}{Q_2'} = \frac{C_1 V}{C_2 V} = \frac{4\pi\varepsilon_0 r_1}{4\pi\varepsilon_0 r_2}, \quad \frac{Q_1'}{Q_2'} = \frac{r_1}{r_2} \quad \Rightarrow \quad 1 + \frac{Q_1'}{Q_2'} = 1 + \frac{r_1}{r_2} \quad \Rightarrow \quad \frac{Q_1' + Q_2'}{Q_2'} = \frac{r_1 + r_2}{r_2}$$

$$\Rightarrow \quad Q_2' = Q\left[\frac{r_2}{r_1 + r_2}\right] \quad \text{and similarly} \quad Q_1' = Q\left[\frac{r_1}{r_1 + r_2}\right]$$

(ii) **Common potential :** Common potential $(V) = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{Q_1' + Q_2'}{C_1 + C_2} \Rightarrow V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

(iii) **Energy loss :** As electrical energy stored in the system before and after connecting the spheres is

$$U_i = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 \quad \text{and} \quad U_f = \frac{1}{2}(C_1 + C_2).V^2 = \frac{1}{2}(C_1 + C_2)\left(\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}\right)^2$$

so energy loss $\quad \Delta U = U_i - U_f = \frac{C_1 C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

Concept

- *Capacity of a conductor is a constant term, it does not depend upon the charge Q, and potential (V) and nature of the material of the conductor.*

Examples based on sharing of charge, drops and general concept of capacity

Example: 95 Eight drops of mercury of same radius and having same charge coalesce to form a big drop. Capacitance of big drop relative to that of small drop will be **[MP PMT 2002, 1990; MNR 1999, 87; DCE 1998]**

(a) 16 times (b) 8 times (c) 4 times (d) 2 times

Solution: (d) By using relation $C = n^{1/3}.c \Rightarrow C = (8)^{1/3}.c = 2c$

Example: 96 Two spheres A and B of radius 4 *cm* and 6 *cm* are given charges of $80\mu C$ and $40\mu C$ respectively. If they are connected by a fine wire, the amount of charge flowing from one to the other is **[MP PET 1991]**

(a) $20\,\mu C$ from A to B (b) $16\,\mu C$ from A to B (c) $32\mu C$ from B to A (d) $32\,\mu C$ from A to B

Solution: (d) Total charge $Q = 80 + 40 = 120\,\mu C$. By using the formula $Q_1' = Q\left[\frac{r_1}{r_1 + r_2}\right]$. New charge on sphere A is

$Q_A' = Q\left[\frac{r_A}{r_A + r_B}\right] = 120\left[\frac{4}{4+6}\right] = 48\,\mu C$. Initially it was $80\,\mu C$, *i.e.*, $32\,\mu C$ charge flows from A to B.

Example: 97 Two insulated metallic spheres of $3\mu F$ and $5\mu F$ capacitances are charged to $300V$ and $500V$ respectively. The energy loss, when they are connected by a wire, is **[Pb PMT 1999; CPMT 1999; KCET (Engg.) 2000]**

(a) $0.012\,J$ (b) $0.0218\,J$ (c) $0.0375J$ (d) $3.75\,J$

Solution: (c) By using $\Delta U = \frac{C_1 C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$; $\Delta U = 0.375\,J$

Example: 98 64 small drops of mercury, each of radius r and charge q coalesce to form a big drop. The ratio of the surface density of charge of each small drop with that of the big drop is **[KCET 2002]**

(a) 1 : 64 (b) 64 : 1 (c) 4 : 1 (d) 1 : 4

Solution: (d) $\frac{\sigma_{Small}}{\sigma_{Big}} = \frac{q/4\pi r^2}{Q/4\pi R^2} = \left(\frac{q}{Q}\right)\left(\frac{R}{r}\right)^2$; since $R = n^{1/3}r$ and $Q = nq$

So $\frac{\sigma_{Small}}{\sigma_{Big}} = \frac{1}{n^{1/3}} \Rightarrow \frac{\sigma_{Small}}{\sigma_{Big}} = \frac{1}{4}$

Tricky example: 14

Two hollow spheres are charged positively. The smaller one is at 50 *V* and the bigger one is at 100 *V*. How should they be arranged so that the charge flows from the smaller to the bigger sphere when they are connected by a wire **[Kerala PET 2002]**

(a) By placing them close to each other

(b) By placing them at very large distance from each other

(c) By placing the smaller sphere inside the bigger one

(d) Information is insufficient

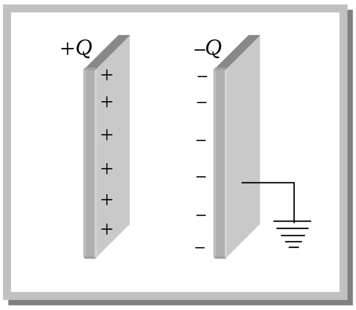
Solution: (c) By placing the smaller sphere inside the bigger one. The potential of the smaller one will now be 150 *V*. So on connecting it with the bigger one charge will flow from the smaller one to the bigger one.

Capacitor.

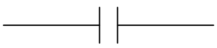
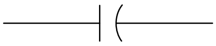
(1) **Definition :** A capacitor is a device that stores electric energy. It is also named condenser.

or

A capacitor is a pair of two conductors of any shape, which are close to each other and have equal and opposite charge.

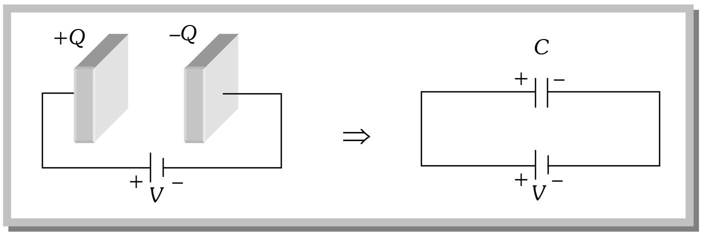


(2) **Symbol :** The symbol of capacitor are shown below

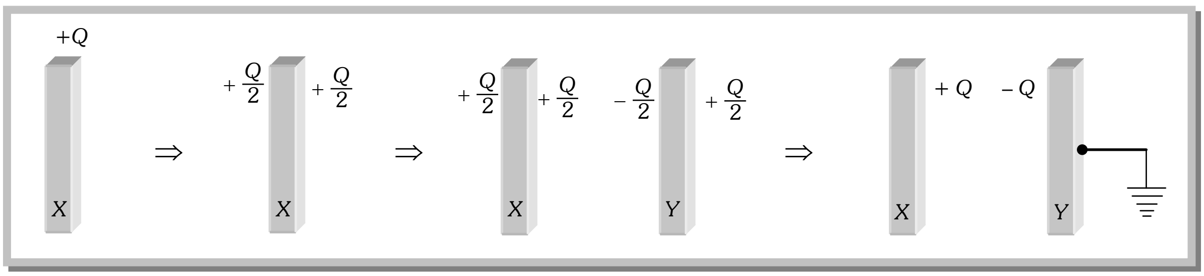
 or   variable capacitor

(3) **Capacitance :** The capacitance of a capacitor is defined as the magnitude of the charge Q on the positive plate divided by the magnitude of the potential difference V between the plates *i.e.*, $\boldsymbol{C = \frac{Q}{V}}$

(4) **Charging :** A capacitor get's charged when a battery is connected across the plates. The plate attached to the positive terminal of the battery get's positively charged and the one joined to the negative terminal get's negatively charged. Once capacitor get's fully charged, flow of charge carriers stops in the circuit and in this condition potential difference across the plates of capacitor is same as the potential difference across the terminals of battery (say V).



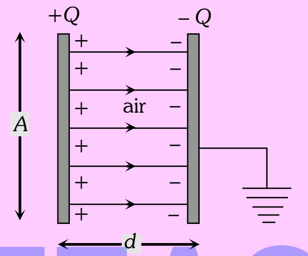
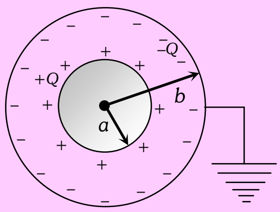
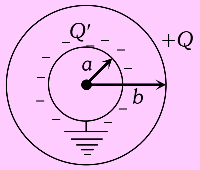
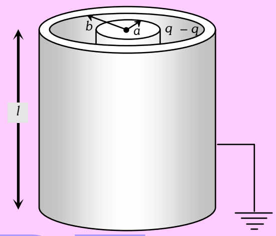
(5) **Charge on capacitor :** Net charge on a capacitor is always zero, but when we speaks of the charge Q on a capacitor, we are referring to the magnitude of the charge on each plate. Charge distribution in making of parallel plate capacitor can easily be understand by reading carefully the following sequence of figures –



(6) **Energy stored :** When a capacitor is charged by a voltage source (say battery) it stores the electric energy. If C = Capacitance of capacitor; Q = Charge on capacitor and V = Potential difference across capacitor then energy stored in capacitor $U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$

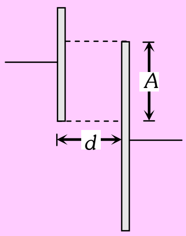
Note : ≅ In charging capacitor by battery half the energy supplied is stored in the capacitor and remaining half energy (1/2 QV) is lost in the form of heat.

(7) **Types of capacitors :** Capacitors are of mainly three types as described in given table

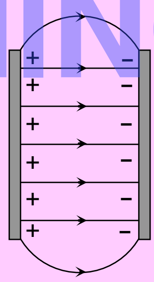
Parallel Plate Capacitor	Spherical Capacitor	Cylindrical Capacitor
It consists of two parallel metallic plates (may be circular, rectangular, square) separated by a small distance  A = Effective overlapping area of each plate **d = Separation between the plates** Q = Magnitude of charge on the inner side of each plate σ = Surface density of charge of each plate $\left(= \frac{Q}{A}\right)$ V = Potential difference across the plates E = Electric field between the plates $\left(= \frac{\sigma}{\varepsilon_0}\right)$ **Capacitance :** $C = \frac{\varepsilon_0 A}{d}$ in C.G.S. : $C = \frac{A}{4\pi d}$ If a dielectric medium of dielectric constant K is filled completely between the plates then capacitance increases by K times $C' = KC$	It consists of two concentric conducting spheres of radii a and b ($a < b$). Inner sphere is given charge $+Q$, while outer sphere is earthed  **Capacitance** $C = 4\pi\varepsilon_0 \cdot \frac{ab}{b-a}$ in C.G.S. $C = \frac{ab}{b-a}$. In the presence of dielectric medium (dielectric constant K) between the spheres $C' = 4\pi\varepsilon_0 K \frac{ab}{b-a}$ **Special Case :** If outer sphere is given a charge $+Q$ while inner sphere is earthed  Induced charge on the inner sphere $Q' = -\frac{a}{b}.Q$, $C' = 4\pi\varepsilon_0 \cdot \frac{b^2}{b-a}$ This arrangement is not a capacitor. But it's capacitance is equivalent to the sum of capacitance of spherical capacitor and spherical conductor i.e. $4\pi\varepsilon_0 \cdot \frac{b^2}{b-a} = 4\pi\varepsilon_0 \frac{ab}{b-a} + 4\pi\varepsilon_0 b$	It consists of two concentric cylinders of radii a and b ($a < b$), inner cylinder is given charge $+Q$ while outer cylinder is earthed. Common length of the cylinders is l then  **Capacitance** $C = \frac{2\pi\varepsilon_0 l}{\log_e\left(\frac{b}{a}\right)}$ In the presence of dielectric medium (dielectric constant K) capacitance increases by K times and $C' = \frac{2\pi\varepsilon_0 K l}{\log_e\left(\frac{b}{a}\right)}$

Concepts

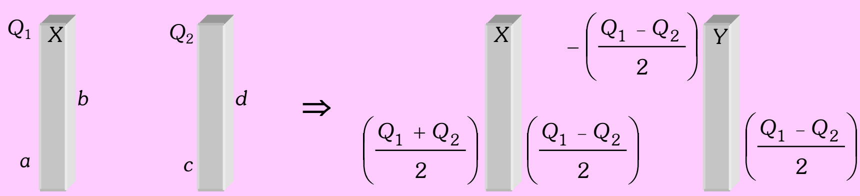
- *It is a very common misconception that a capacitor stores charge but actually a capacitor stores electric energy in the electrostatic field between the plates.*
- *Two plates of unequal area can also form a capacitor because effective overlapping area is considered.*



- *If two plates are placed side by side then three capacitors are formed. One between distant earthed bodies and the first face of the first plate, the second between the two plates and the third between the second face of the second plate and distant earthed objects. However the capacitances of the first and third capacitors are negligibly small in comparision to that between the plates which is the main capacitance.*
- *Capacitance of a parallel plate capacitor depends upon the effective overlapping area of plates* $(C \propto A)$, *separation between the plates* $\left(C \propto \frac{1}{d}\right)$ *and dielectric medium filled between the plates. While it is independent of charge given, potential raised or nature of metals and thickness of plates.*
- *The distance between the plates is kept small to avoid fringing or edge effect (non-uniformity of the field) at the bounderies of the plates.*



- *Spherical conductor is equivalent to a spherical capacitor with it's outer sphere of infinite radius.*
- *A spherical capacitor behaves as a parallel plate capacitor if it's spherical surfaces have large radii and are close to each other.*
- *The intensity of electric field between the plates of a parallel plate capacitor* $(E = \sigma/\varepsilon_0)$ *does not depends upon the distance between them.*
- *The plates of a parallel plate capacitor are being moved away with some velocity. If the plate separation at any instant of time is 'd' then the rate of change of capacitance with time is proportional to* $\frac{1}{d^2}$.
- *Radial and non-uniform electric field exists between the spherical surfaces of spherical capacitor.*
- *Two large conducting plates X and Y kept close to each other. The plate X is given a charge* Q_1 *while plate Y is given a charge* Q_2 $(Q_1 > Q_2)$, *the distribution of charge on the four faces a, b, c, d will be as shown in the following figure.*



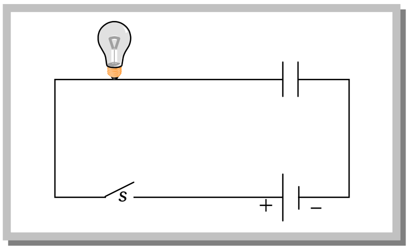
Example based on simple concepts of capacitor

Example: 99 The capacity of pure capacitor is 1 *farad*. In D.C. circuit, its effective resistance will be **[MP PMT 2000]**

(a) Zero (b) Infinite (c) 1 *ohm* (d) $\frac{1}{2}ohm$

Solution: (b) Capacitor does not work in D.C. for D.C. it's effective resistance is infinite i.e. it blocks the current to flow in the circuit.

Example: 100 A light bulb, a capacitor and a battery are connected together as shown here, with switch *S* initially open. When the switch *S* is closed, which one of the following is true **[MP PMT 1995]**



(a) The bulb will light up for an instant when the capacitor starts charging

(b) The bulb will light up when the capacitor is fully charged

(c) The bulb will not light up at all

(d) The bulb will light up and go off at regular intervals

Solution: (a) Current through the circuit can flow only for the small time of charging, once capacitor get's charged it blocks the current through the circuit and bulb will go off.

Example: 101 Capacity of a parallel plate condenser is $10\mu F$ when the distance between its plates is $8\,cm$. If the distance between the plates is reduced to $4cm$, its capacity will be

[CBSE 2001; Similar to CPMT 1997; AFMC 2000]

(a) $10\mu F$ (b) $15\mu F$ (c) $20\mu F$ (d) $40\mu F$

Solution: (c) $C = \frac{\varepsilon_0 A}{d} \propto \frac{1}{d}$ $\therefore$ $\frac{C_1}{C_2} = \frac{d_2}{d_1}$ or $C_2 = \frac{d_1}{d_2} \times C_1 = \frac{8}{4} \times 10 = 20\mu F$

Example: 102 What is the area of the plates of a $3F$ parallel plate capacitor, if the separation between the plates is $5\,mm$

[BHU 2002; AIIMS 1998]

(a) $1.694 \times 10^9\, m^2$ (b) $4.529 \times 10^9\, m^2$ (c) $9.281 \times 10^9\, m^2$ (d) $12.981 \times 10^9\, m^2$

Solution: (a) By using the relation $C = \frac{\varepsilon_0 A}{d}$ $\Rightarrow$ $A = \frac{Cd}{\varepsilon_0} = \frac{3 \times 5 \times 10^{-3}}{8.85 \times 10^{-12}} = 1.694 \times 10^9 m^2$.

Example: 103 If potential difference of a condenser $(6\,\mu F)$ is changed from 10 *V* to 20 *V* then increase in energy is

[CPMT 1997, 87]

(a) $2 \times 10^{-4} J$ (b) $4 \times 10^{-4} J$ (c) $3 \times 10^{-4} J$ (d) $9 \times 10^{-4} J$

Solution: (d) Initial energy $U_i = \frac{1}{2}CV_1^2$; Final energy $U_f = \frac{1}{2}CV_2^2$

$\therefore$ Increase in energy $\Delta U = U_f - U_i = \frac{1}{2}C(V_2^2 - V_1^2) = \frac{1}{2} \times 6 \times 10^{-6}(20^2 - 10^2) = 9 \times 10^{-4} J$.

Example: 104 A spherical capacitor consists of two concentric spherical conductors. The inner one of radius R_1 maintained at potential V_1 and the outer conductor of radius R_2 at potential V_2. The potential at a point *P* at a distance *x* from the centre (where $R_2 > x > R_1$) is **[MP PMT 1997]**

(a) $\dfrac{V_1 - V_2}{R_2 - R_1}(x - R_1)$ (b) $\dfrac{V_1R_1(R_2 - x) + V_2R_2(x - R_1)}{(R_2 - R_1)x}$

(c) $V_1 + \dfrac{V_2 x}{(R_2 - R_1)}$ (d) $\dfrac{(V_1 + V_2)}{(R_1 + R_2)}x$

Solution: (b) Let Q_1 and Q_2 be the charges on the inner and the outer sphere respectively. Now V_1 is the total potential on the sphere of radius R_1,

So, $V_1 = \dfrac{Q_1}{R_1} + \dfrac{Q_2}{R_2}$ .... (i)

and V_2 is the total potential on the surface of sphere of radius R_2,

So, $V_2 = \dfrac{Q_2}{R_2} + \dfrac{Q_1}{R_2}$ .... (ii)

If *V* be the potential at point *P* which lies at a distance *x* from the common centre then

$$V = \frac{Q_1}{x} + \frac{Q_2}{R_2} = \frac{Q_1}{x} + V_1 - \frac{Q_1}{R_1} = Q_1\left(\frac{1}{x} - \frac{1}{R_1}\right) + V_1 = \frac{Q_1(R_1 - x)}{xR_1} + V_1 \quad \text{........(iii)}$$

Substracting (ii) from (i)

$$V_1 - V_2 = \frac{Q_1}{R_1} - \frac{Q_2}{R_2} \Rightarrow (V_1 - V_2)R_1R_2 = R_2Q_1 - R_1Q_1 \Rightarrow Q_1 = \frac{(V_1 - V_2)R_1R_2}{R_2 - R_1}$$

Now substituting it in equation (iii), we have

$$V = \frac{(R_1 - x)(V_1 - V_2)R_1R_2}{xR_1(R_2 - R_1)} + V_1 \quad \Rightarrow V = \frac{V_1R_1(R_2 - x) + V_2R_2(x - R_1)}{x(R_2 - R_1)}$$

Example: 105 The diameter of each plate of an air capacitor is 4 *cm*. To make the capacity of this plate capacitor equal to that of 20 *cm* diameter sphere, the distance between the plates will be **[MP PET 1996]**

(a) $4 \times 10^{-3}\, m$ (b) $1 \times 10^{-3}\, m$ (c) 1 *cm* (d) $1 \times 10^{-3}\, cm$

Solution: (b) According to the question $\dfrac{\varepsilon_0 A}{d} = 4\pi\varepsilon_0 R \Rightarrow d = \dfrac{A}{4\pi R} = \dfrac{\pi(2 \times 10^{-2})^2}{4\pi \times 10 \times 10^{-2}} = 1 \times 10^{-3}\, m.$

Example: 106 A spherical condenser has inner and outer spheres of radii *a* and *b* respectively. The space between the two is filled with air. The difference between the capacities of two condensers formed when outer sphere is earthed and when inner sphere is earthed will be **[MP PET 1996]**

(a) Zero (b) $4\pi\varepsilon_0 a$ (c) $4\pi\varepsilon_0 b$ (d) $4\pi\varepsilon_0 a\left(\dfrac{b}{b-a}\right)$

Solution: (c) Capacitance when outer sphere is earthed $C_1 = 4\pi\varepsilon_0 \,.\dfrac{ab}{b-a}$ and capacitance when inner sphere is earthed $C_2 = 4\pi\varepsilon_0 \,.\dfrac{b^2}{b-a}$. Hence $C_2 - C_1 = 4\pi\varepsilon_0 \,.b$

Example: 107 After charging a capacitor of capacitance $4\mu F$ upto a potential 400 *V*, its plates are connected with a resistance of $1 k\Omega$. The heat produced in the resistance will be **[CBSE PMT 1994]**

(a) 0.16 *J* (b) 1.28 *J* (c) 0.64 *J* (d) 0.32 *J*

Solution: (d) This is the discharging condition of capacitor and in this condition energy released

$$U = \frac{1}{2}CV^2 \quad = \frac{1}{2} \times 4 \times 10^{-6} \times (400)^2 = 0.32J \quad = 0.32J.$$

Example: 108 The amount of work done in increasing the voltage across the plates of a capacitor from $5V$ to $10V$ is W. The work done in increasing it from $10V$ to $15V$ will be

(a) $0.6\ W$ (b) W (c) $1.25\ W$ (d) $1.67\ W$

Solution: (d) As we know that work done $= U_{final} - U_{initial} = \frac{1}{2}C(V_2^2 - V_1^2)$

When potential difference increases from $5V$ to $10V$ then

$$W = \frac{1}{2}C(10^2 - 5^2) \quad \text{........(i)}$$

When potential difference increases from $10V$ to $15V$ then

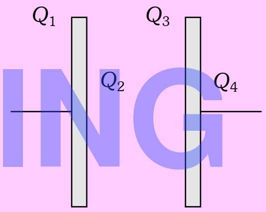
$$W' = \frac{1}{2}C(15^2 - 10^2) \quad \text{........(ii)}$$

On solving equation (i) and (ii) we get

$W' = 1.67W$.

Tricky example: 15

In an isolated parallel plate capacitor of capacitance C, the four surface have charges Q_1, Q_2, Q_3 and Q_4 as shown. The potential difference between the plates is **[UPSEAT 2003; IIT-JEE 1999]**



(a) $\frac{Q_1 + Q_2 + Q_3 + Q_4}{2C}$ (b) $\frac{Q_2 + Q_3}{2C}$ (c) $\frac{Q_2 - Q_3}{2C}$ (d) $\frac{Q_1 + Q_4}{2C}$

Solution: (c) Plane conducting surfaces facing each other must have equal and opposite charge densities. Here as the plate areas are equal, $Q_2 = -Q_3$.

The charge on a capacitor means the charge on the inner surface of the positive plate (here it is Q_2)

Potential difference between the plates $= \frac{\text{charge}}{\text{capacitance}} = \frac{Q_2}{C} = \frac{2Q_2}{2C}$

$$= \frac{Q_2 - (-Q_2)}{2C} = \frac{Q_2 - Q_3}{2C}.$$

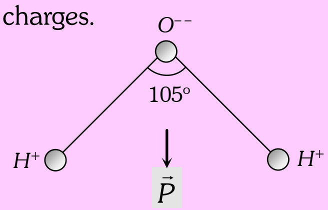
Dielectric.

Dielectrics are insulating (non-conducting) materials which transmits electric effect without conducting we know that in every atom, there is a positively charged nucleus and a negatively charged electron cloud surrounding it. The two oppositely charged regions have their own centres of charge. The centre of positive charge is the centre of mass of positively charged protons in the nucleus. The centre of negative charge is the centre of mass of negatively charged electrons in the atoms/molecules.

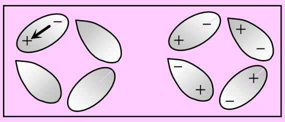
(1) **Type of Dielectrics :** Dielectrics are of two types –

(i) **Polar dielectrics :** Like water, Alcohol, CO_2, NH_3, *HCl* etc. are made of polar atoms/molecules.

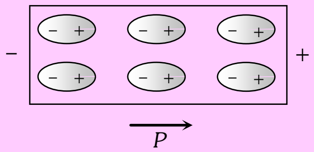
In polar molecules when no electric field is applied centre of positive charges does not coincide with the centre of negative charges.



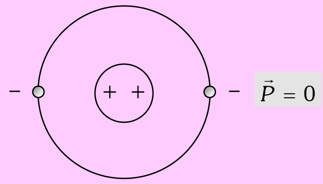
A polar molecule has permanent electric dipole moment $(\vec{p})$ in the absence of electric field also. But a polar dielectric has net dipole moment is zero in the absence of electric field because polar molecules are randomly oriented as shown in figure.



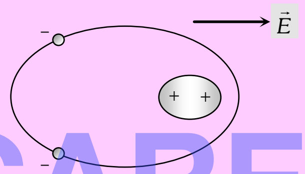
In the presence of electric field polar molecules tends to line up in the direction of electric field, and the substance has finite dipole moment.



(ii) **Non polar dielectric :** Like N_2, O_2, Benzene, Methane etc. are made of non-polar atoms/molecules. In non-polar molecules, when no electric field is applied the centre of positive charge coincides with the centre of negative charge in the molecule. Each molecule has zero dipole moment in its normal state.

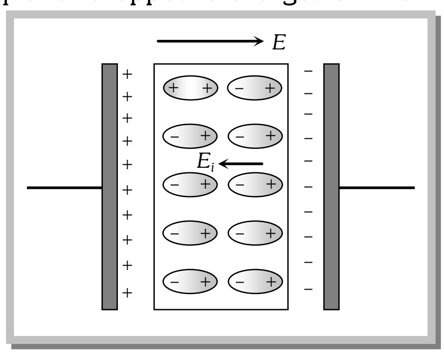


When electric field is applied, positive charge experiences a force in the direction of electric field and negative charge experiences a force in the direction opposite to the field *i.e.*, molecules becomes induced electric dipole.



❑ In general, any non-conducting, material can be called as a dielectric but broadly non conducting material having non polar molecules referred to as dielectric because induced dipole moment is created in the non polar molecule.

(2) **Polarization of a dielectric slab :** It is the process of inducing equal and opposite charges on the two faces of the dielectric on the application of electric field.



Suppose a dielectric slab is inserted between the plates of a capacitor. As shown in the figure.

Induced electric field inside the dielectric is E_i, hence this induced electric field decreases the main field E to $E - E_i$ *i.e.*, New electric field between the plates will be $E' = E - E_i$.

(3) **Dielectric constant :** After placing a dielectric slab in an electric field. The net field is decreased in that region hence

If E = Original electric field and E' = Reduced electric field. Then $\frac{E}{E'} = K$ where K is called dielectric constant

K is also known as relative permittivity (ε_r) of the material or **SIC** (Specific Inductive Capacitance)

The value of K is always greater than one. For vacuum there is no polarization and hence $E = E'$ and $K = 1$

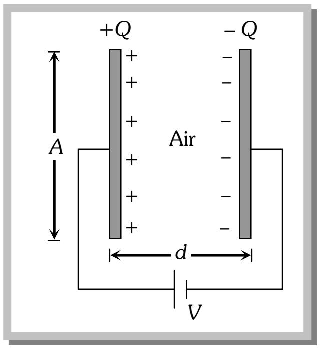
(4) **Dielectric breakdown and dielectric strength :** If a very high electric field is created in a dielectric, the outer electrons may get detached from their parent atoms. The dielectric then behaves like a conductor. This phenomenon is known as **dielectric breakdown.**

The maximum value of electric field (or potential gradient) that a dielectric material can tolerate without it's electric breakdown is called it's **dielectric strength.**

S.I. unit of dielectric strength of a material is $\frac{V}{m}$ but practical unit is $\frac{kV}{mm}$.

Variation of Different Variables (Q, C, V, E and U) of Parallel Plate Capacitor.

Suppose we have an air filled charged parallel plate capacitor having variables are as follows :



Charge – Q, Surface charge density – $\sigma = \frac{Q}{A}$, Capacitance – $C = \frac{\varepsilon_0 A}{d}$

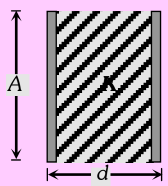
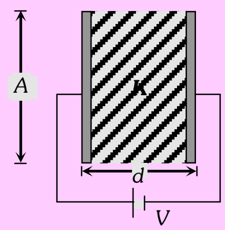
Potential difference across the plates – $V = E.d$

Electric field between the plates – $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

Energy stored – $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$

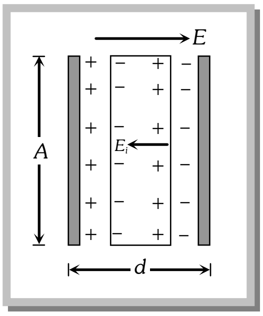
(1) **When dielectric is completely filled between plates :** If a dielectric slab is fills completely the gap between the plates, capacitance increases by K times *i.e.*, $C' = \frac{K\varepsilon_0 A}{d} \Rightarrow C' = KC$

The effect of dielectric on other variables such as charge. Potential difference field and energy associated with a capacitor depends on the fact that whether the charged capacitor is disconnected from the battery or battery is still connected.

Quantity	Battery is Removed	Battery Remains connected
		
Capacity	$C' = KC$	$C' = KC$
Charge	$Q' = Q$ (Charge is conserved)	$Q' = KQ$
Potential	$V' = V/K$	$V' = V$ (Since Battery maintains the potential difference)
Intensity	$E' = E/K$	$E' = E$
Energy	$U' = U/K$	$U' = U/K$

Note : ≅ If nothing is said it is to be assumed that battery is disconnected.

(2) **When dielectric is partially filled between the plates :** If a dielectric slab of thickness $t\,(t<d)$ is inserted between the plates as shown below, then $E=$ Main electric field between the plates, $E_i=$ Induced electric field in dielectric. $E'=(E-E_i)=$ The reduced value of electric field in the dielectric. Potential difference between the two plates of capacitor is given by

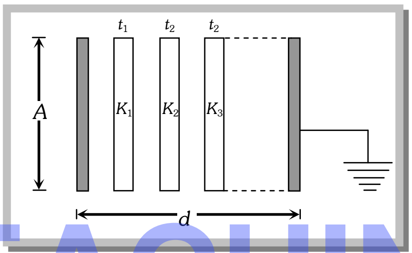


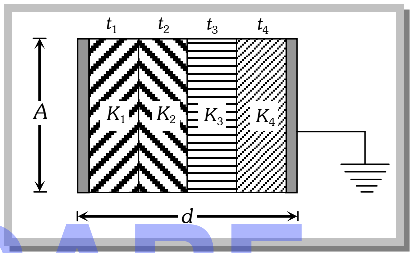
$$V'=E(d-t)+E't=E(d-t)+\frac{E}{K}.t$$

$$\Rightarrow V'=E\left(d-t+\frac{t}{K}\right)=\frac{\sigma}{\varepsilon_0}\left(d-t+\frac{t}{K}\right)=\frac{Q}{A\varepsilon_0}\left(d-t+\frac{t}{K}\right)$$

Now capacitance of the capacitor

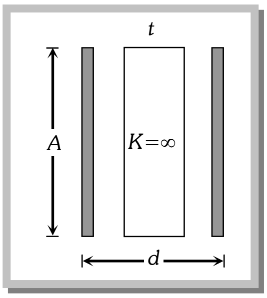
$$C'=\frac{Q}{V'} \qquad \Rightarrow \qquad C'=\frac{\varepsilon_0 A}{d-t+\dfrac{t}{K}}$$

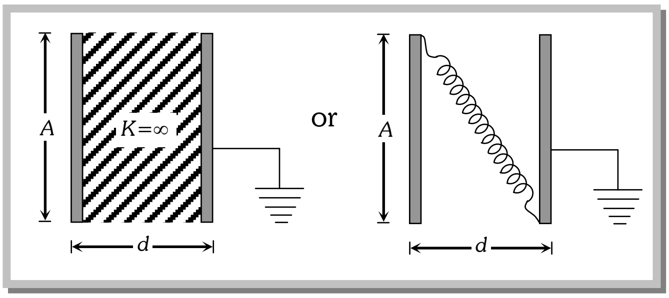




$$C'=\frac{\varepsilon_0 A}{d-(t_1+t_2+t_3+........)+\left(\frac{t_1}{K_1}+\frac{t_2}{K_2}+\frac{t_3}{K_3}+........\right)} \qquad C'=\frac{\varepsilon_0 A}{\left(\frac{t_1}{K_1}+\frac{t_2}{K_2}+\frac{t_3}{K_3}+\frac{t_4}{K_4}\right)}$$

(3) **When a metallic slab is inserted between the plates :**

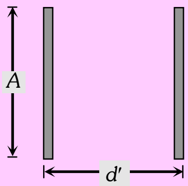
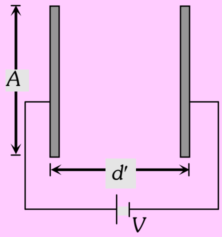




Capacitance $\quad C'=\dfrac{\varepsilon_0 A}{(d-t)}$ $\qquad$ $C'=\infty$ (In this case capacitor is said to be short circuited)

(4) **When separation between the plates is changing :** If separation between the plates changes then it's capacitance also changes according to $C\propto\dfrac{1}{d}$. The effect on other variables depends on the fact that whether the charged capacitor is disconnected from the battery or battery is still connected.

(i) **Separation is increasing**

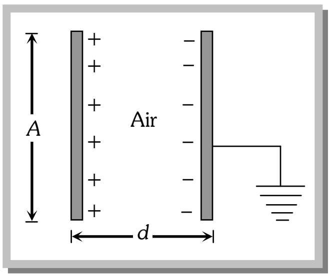
Quantity	**Battery is removed** 	**Battery remains connected** 
Capacity	Decreases because $C \propto \frac{1}{d}$ *i.e.*, $C' < C$	Decreases *i.e.*, $C' < C$
Charge	Remains constant because a battery is not present *i.e.*, $Q' = Q$	Decreases because battery is present *i.e.*, $Q' < Q$ Remaining charge $(Q - Q')$ goes back to the battery.
Potential difference	Increases because $V = \frac{Q}{C} \Rightarrow V \propto \frac{1}{C}$ *i.e.*, $V' > V$	$V' = V$ (Since Battery maintains the potential difference)
Electric field	Remains constant because $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$ *i.e.*, $E' = E$	Decrease because $E = \frac{Q}{A\varepsilon_0} \Rightarrow E \propto Q$ *i.e.*, $E' < E$
Energy	Increases because $U = \frac{Q^2}{2C} \Rightarrow U \propto \frac{1}{C}$ *i.e.*, $U' > U$	Decreases because $U = \frac{1}{2}CV^2 \Rightarrow U \propto C$ *i.e.*, $U' < U$

(ii) **Separation is decreasing**

Quantity	**Battery is removed**	**Battery remains connected**
Capacity	Increases because $C \propto \frac{1}{d}$ *i.e.*, $C' > C$	Increases *i.e.*, $C' > C$
Charge	Remains constant because battery is not present *i.e.*, $Q' = Q$	Increases because battery is present *i.e.*, $Q' > Q$ Remaining charge $(Q' - Q)$ supplied from the battery.
Potential difference	Decreases because $V = \frac{Q}{C} \Rightarrow V \propto \frac{1}{C}$ *i.e.*, $V' < V$	$V' = V$ (Since Battery maintains the potential difference)
Electric field	Remains constant because $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$ *i.e.*, $E' = E$	Increases because $E = \frac{Q}{A\varepsilon_0} \Rightarrow E \propto Q$ *i.e.*, $E' > E$
Energy	Decreases because $U = \frac{Q^2}{2C} \Rightarrow U \propto \frac{1}{C}$ *i.e.*, $U' < U$	Increases because $U = \frac{1}{2}CV^2 \Rightarrow U \propto C$ *i.e.*, $U' > U$

Force Between the Plates of a Parallel Plate Capacitor.

Field due to charge on one plate on the other is $E = \frac{\sigma}{2\varepsilon_0}$, hence the force $F = QE$



$$F = -\sigma A \times \left(\frac{\sigma}{2\varepsilon_0}\right) = -\frac{\sigma^2}{2\varepsilon_0} A$$

$$\Rightarrow \qquad |F| = \frac{\sigma^2 A}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A}$$

Energy Density Between the Plates of a Parallel Plate Capacitor.

The energy stored in a capacitor is not localised on the charges or the plates but is distributed in the field. And as in case of a parallel plate capacitor field is only between the plates *i.e.* in a volume ($A \times d$), the so called **energy density**.

Hence Energy density $= \frac{\text{Energy}}{\text{Volume}} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{1}{2}\left[\frac{\varepsilon_0 A}{d}\right]\frac{V^2}{Ad} = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$.

Concepts

- *In the expression of capacitance of parallel plate capacitor filled partially with dielectric term* $\left(d - t + \frac{t}{K}\right)$ *is known as effective air separation between the plates.*
- *When dielectric is partially filled between the plates of a parallel plate capacitor then it's capacitance increases but potential difference decreases. To maintain the capacitance and potential difference of capacitor as before (i.e.,* $c = \frac{\varepsilon_0 A}{d}$, $V = \frac{\sigma}{\varepsilon_0} d$*) separation between the plates has to be increased. Suppose separation is increased by d' so in this case* $\frac{\varepsilon_0 A}{\left(d + d' - t + \frac{t}{K}\right)} = \frac{\varepsilon_0 A}{d}$ *which gives us* $\boldsymbol{K = \frac{t}{t - d'}}$

Example based on capacitor with dielectric

Example: 109 The mean electric energy density between the plates of a charged capacitor is (here Q = Charge on the capacitor and A = Area of the capacitor plate) **[MP PET 2002]**

(a) $\frac{Q^2}{2\varepsilon_0 A^2}$ (b) $\frac{Q}{2\varepsilon_0 A^2}$ (c) $\frac{Q^2}{2\varepsilon_0 A}$ (d) None of these

Solution: (a) Energy density $= \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{Q}{A\varepsilon_0}\right)^2 = \frac{Q^2}{2\varepsilon_0 A^2}$.

Example: 110 Plate separation of a $15\mu F$ capacitor is 2 *mm*. A dielectric slab (K = 2) of thickness 1 *mm* is inserted between the plates. Then new capacitance is given by **[BHU 1994, Similar to BHU 2000]**

(a) $15\mu F$ (b) $20\mu F$ (c) $30\mu F$ (d) $25\mu F$

Solution: (b) Given $C = \frac{\varepsilon_0 A}{d} = 15\mu F$ ....(i)

Then by using $C' = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} = \frac{\varepsilon_0 A}{2\times 10^{-3} - 10^{-3} + \frac{10^{-3}}{2}} = \frac{2}{3}\times \varepsilon_0 A \times 10^3$; From equation (i) $C' = 20\mu F$.

Example: 111 There is an air filled $1\, pF$ parallel plate capacitor. When the plate separation is doubled and the space is filled with wax, the capacitance increases to $2\, pF$. The dielectric constant of wax is **[MNR 1998]**

(a) 2 (b) 4 (c) 6 (d) 8

Solution: (b) Given that capacitance $C = 1\, pF$

After doubling the separation between the plates $C' = \frac{C}{2}$

and when dielectric medium of dielectric constant *k* filled between the plates then $C' = \frac{KC}{2}$

According to the question, $C' = \frac{KC}{2} = 2 \quad \Rightarrow \quad K = 4.$

Example: 112 If a slab of insulating material $4\times 10^{-5}\, m$ thick is introduced between the plate of a parallel plate capacitor, the distance between the plates has to be increased by $3.5\times 10^{-5}\, m$ to restore the capacity to original value. Then the dielectric constant of the material of slab is **[AMU 1999]**

(a) 10 (b) 12 (c) 6 (d) 8

Solution: (d) By using $K = \frac{t}{t - d'}$; here $t = 4\times 10^{-5}\, m$; $d' = 3.5\times 10^{-5}\, m \;\Rightarrow\; K = \frac{4\times 10^{-5}}{4\times 10^{-5} - 3.5\times 10^{-5}} = 8$

Example: 113 The force between the plates of a parallel plate capacitor of capacitance *C* and distance of separation of the plates *d* with a potential difference *V* between the plates, is **[MP PMT 1999]**

(a) $\frac{CV^2}{2d}$ (b) $\frac{C^2V^2}{2d^2}$ (c) $\frac{C^2V^2}{d^2}$ (d) $\frac{V^2 d}{C}$

Solution: (a) Since $F = \frac{Q^2}{2\varepsilon_0 A} \;\Rightarrow\; F = \frac{C^2V^2}{2\varepsilon_0 A} = \frac{CV^2}{2d}$.

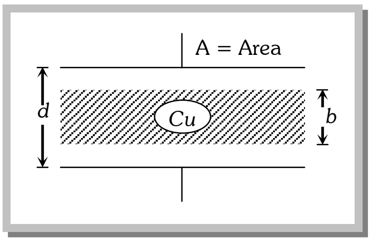
Example: 114 A capacitor when filled with a dielectric $K = 3$ has charge Q_0, voltage V_0 and field E_0. If the dielectric is replaced with another one having $K = 9$, the new values of charge, voltage and field will be respectively

(a) $3Q_0, 3V_0, 3E_0$ (b) $Q_0, 3V_0, 3E_0$ (c) $Q_0, \frac{V_0}{3}, 3E_0$ (d) $Q_0, \frac{V_0}{3}, \frac{E_0}{3}$

Solution: (d) Suppose, charge, potential difference and electric field for capacitor without dielectric medium are *Q*, *V* and *E* respectively

With dielectric medium of $K = 3$	With dielectric medium of $K = 9$
Charge $Q_0 = Q$	Charge $Q' = Q = Q_0$
Potential difference $V_0 = \frac{V}{3}$	Potential difference $V' = \frac{V}{9} = \frac{V_0}{3}$
Electric field $E_0 = \frac{E}{3}$	Electric field $E' = \frac{E}{9} = \frac{E_0}{3}$.

Example: 115 A slab of copper of thickness *b* is inserted in between the plates of parallel plate capacitor as shown in the figure. The separation between the plates is *d*. If $b = \frac{d}{2}$ then the ratio of capacities of the capacitor after and before inserting the slab will be **[IIT-JEE 1976; Similar to Orissa JEE 2002, KCET 2001, MP PMT 1994]**



(a) $\sqrt{2}:1$ (b) $2:1$ (c) $1:1$ (d) $1:\sqrt{2}$

Solution: (b) Capacitance before inserting the slab $C = \frac{\varepsilon_0 A}{d}$ and capacitance after inserting the slab $C' = \frac{\varepsilon_0 A}{d-t}$.

Where $t = b = \frac{d}{2}$ so $C' = \frac{2\varepsilon_0 A}{d}$ hence, $\frac{C'}{C} = \frac{2}{1}$.

Example: 116 The capacity of a parallel plate condenser is C_0. If a dielectric of relative permitivity ε_r and thickness equal to one fourth the plate separation is placed between the plates, then its capacity becomes *C*. The value of $\frac{C}{C_0}$ will be

(a) $\frac{5\varepsilon_r}{4\varepsilon_r + 1}$ (b) $\frac{4\varepsilon_r}{3\varepsilon_r + 1}$ (c) $\frac{3\varepsilon_r}{2\varepsilon_r + 1}$ (d) $\frac{2\varepsilon_r}{\varepsilon_r + 1}$

Solution: (b) Initially capacitance $C_0 = \frac{\varepsilon_0 A}{d}$ ....(i) Finally capacitance $C = \frac{\varepsilon_0 A}{d - \frac{d}{4} + \frac{d/4}{\varepsilon_r}}$ ....(ii)

By dividing equation (ii) by equation (i) $\frac{C}{C_0} = \frac{4\varepsilon_r}{3\varepsilon_r + 1}$

Tricky example: 16

An air capacitor of capacity $C = 10\mu F$ is connected to a constant voltage battery of 12 *V*. Now the space between the plates is filled with a liquid of dielectric constant 5. The charge that flows now from battery to the capacitor is **[MP PMT 1997]**

(a) $120\mu C$ (b) $600\mu C$ (c) $480\mu C$ (d) $24\mu C$

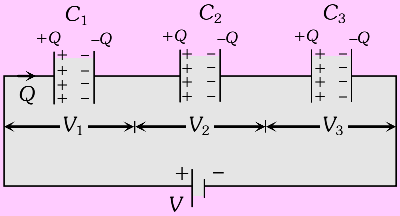
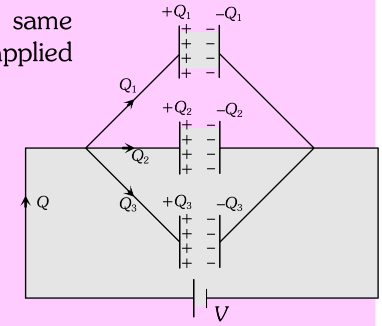
Solution: (c) Initially charge on the capacitor $Q_i = 10 \times 12 = 120\mu C$

When dielectric medium is filled, so capacitance becomes *K* times, *i.e.* new capacitance $C' = 5 \times 10 = 50\mu C$

Final charge on the capacitor $Q_f = 50 \times 12 = 600\mu C$

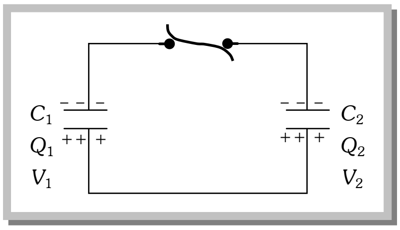
Hence additional charge supplied by the battery $= Q_f - Q_i = 480\mu C$.

Grouping of Capacitors.

Series grouping	Parallel grouping
(1) Charge on each capacitor remains same and equals to the main charge supplied by the battery $V = V_1 + V_2 + V_3$ 	(1) Potential difference across each capacitor remains same and equal to the applied potential difference $Q = Q_1 + Q_2 + Q_3$ 
(2) Equivalent capacitance $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ or $C_{eq} = (C_1^{-1} + C_2^{-1} + C_3^{-1})^{-1}$	(2) $C_{eq} = C_1 + C_2 + C_3$
(3) In series combination potential difference and energy distribution in the reverse ratio of capacitance i.e., $V \propto \frac{1}{C}$ and $U \propto \frac{1}{C}$.	(3) In parallel combination charge and energy distributes in the ratio of capacitance *i.e.* $Q \propto C$ and $U \propto C$
(4) If two capacitors having capacitances C_1 and C_2 are connected in series then $\boldsymbol{C_{eq} = \frac{C_1C_2}{C_1 + C_2}} = \frac{\text{Multiplication}}{\text{Addition}}$ $V_1 = \left(\frac{C_2}{C_1 + C_2}\right).V$ and $V_2 = \left(\frac{C_2}{C_1 + C_2}\right).V$	(4) If two capacitors having capacitance C_1 and C_2 respectively are connected in parallel then $\boldsymbol{C_{eq} = C_1 + C_2}$ $Q_1 = \left(\frac{C_1}{C_1 + C_2}\right).Q$ and $Q_2 = \left(\frac{C_2}{C_1 + C_2}\right).Q$
(5) If n identical capacitors each having capacitances C are connected in series with supply voltage V then Equivalent capacitance $C_{eq} = \frac{C}{n}$ and Potential difference across each capacitor $V' = \frac{V}{n}$.	(5) If n identical capacitors are connected in parallel Equivalent capacitance $C_{eq} = nC$ and Charge on each capacitor $Q' = \frac{Q}{n}$

Redistribution of Charge Between Two Capacitors.

When a charged capacitor is connected across an uncharged capacitor, then redistribution of charge occur to equalize the potential difference across each capacitor. Some energy is also wasted in the form of heat.



Suppose we have two charged capacitors C_1 and C_2 after disconnecting these two from their respective batteries. These two capacitors are connected to each other as shown below (positive plate of one capacitor is connected to positive plate of other while negative plate of one is connected to negative plate of other)

Charge on capacitors redistributed and new charge on them will be $Q_1' = Q\left(\frac{C_1}{C_1 + C_2}\right)$, $Q_2' = Q\left(\frac{C_2}{C_1 + C_2}\right)$

The common potential $V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$ and loss of energy $\Delta U = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

Note : ≅Two capacitors of capacitances C_1 and C_2 are charged to potential of V_1 and V_2 respectively. After disconnecting from batteries they are again connected to each other with reverse polarity *i.e.*, positive plate of a capacitor connected to negative plate of other. So common potential $V = \frac{Q_1 - Q_2}{C_1 + C_2} = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$.