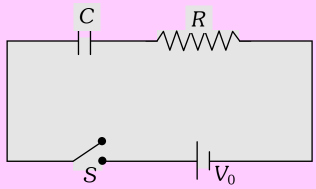
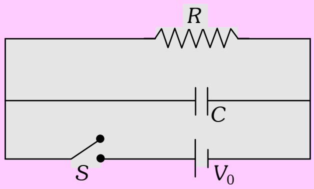
Circuit With Resistors and Capacitors.

(1) A resistor may be connected either in series or in parallel with the capacitor as shown below

Series RC Circuit	Parallel RC Circuit
	
In this combination capacitor takes longer time to charge.	Resistor has no effect on the charging of capacitor.
The charging current is maximum in the beginning; it decreases with time and becomes zero after a long time.	Resistor provides an alternative path for the electric current.

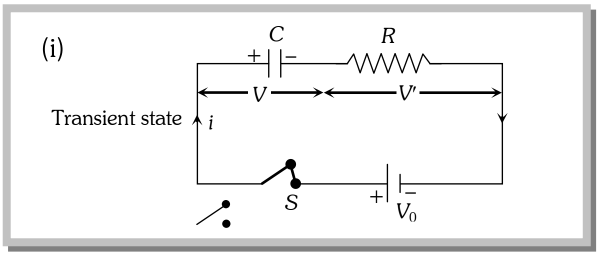
(2) **Three states of *RC* circuits**

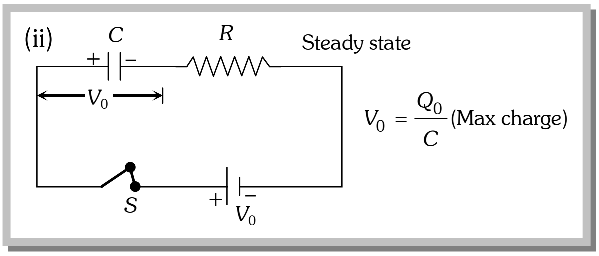
(i) Initial state : *i.e.*, just after closing the switch or just after opening the switch.

(ii) Transient state : or instantaneous state *i.e.*, any time after closing or opening the switch.

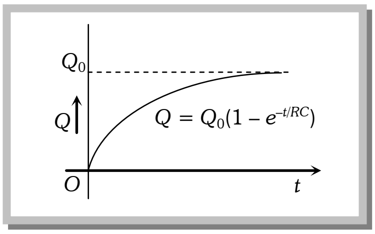
(iii) Steady state : *i.e.*, a long time after closing or opening the switch. In the steady state condition, the capacitor is charged or discharged.

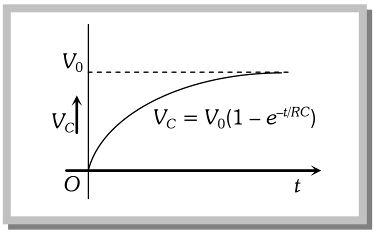
(3) **Charging and discharging of capacitor in series RC circuit :** As shown in the following figure (i) when switch *S* is closed, capacitor start charging. In this transient state potential difference appears across capacitor as well as resistor. When capacitor gets fully charged the entire potential difference appeared across the capacitor and nothing is left for the resistor. [shown in figure (ii)]



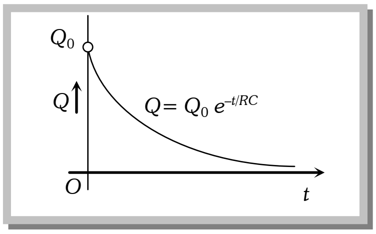


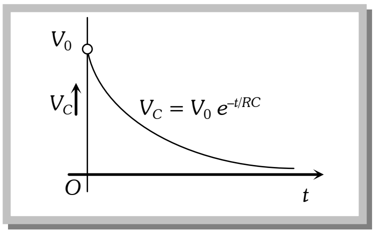
(i) **Charging :** In transient state of charging charge on the capacitor at any instant $Q = Q_0\left(1 - e^{\frac{-t}{RC}}\right)$ and potential difference across the capacitor at any instant $V = V_0\left(1 - e^{\frac{-t}{RC}}\right)$





(ii) **Discharging :** After the completion of charging, if battery is removed capacitor starts discharging. In transient state charge on the capacitor at any instant $Q = Q_0 e^{-t/RC}$ and potential difference cross the capacitor at any instant $V = V_0 e^{-t/CR}$.



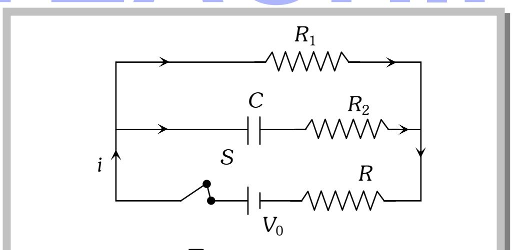


(iii) **Time constant (τ) :** The quantity RC is called the time constant *i.e.*, $\tau = RC$.

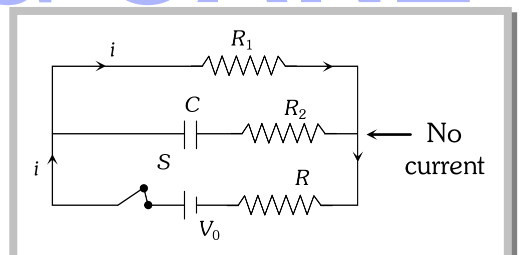
In charging : It is defined as the time during which charge on the capacitor rises to 0.63 times (63%) the maximum value. That is when $t = \tau = RC$, $Q = Q_0(1 - e^{-1}) = 0.639\,Q_0$ **or**

In discharging : It is defined as the time during which charge on a capacitor falls to 0.37 times (37%) of the initial charge on the capacitor that is when $t = \tau = RC$, $Q = Q_0\,(e^{-1}) = 0.37Q_0$

(iv) **Mixed *RC* circuit :** In a mixed RC circuit as shown below, when switch S is closed current flows through the branch containing resistor as well as through the branch contains capacitor and resistor (because capacitor is in the process of charging)



Transient state



Steady state

When capacitor gets fully charged (steady state), no current flows through the line in which capacitor is connected. Therefore the current through resistor R_1 is $\dfrac{V_0}{(R_1 + r)}$, hence potential difference across resistance will be equal to $\dfrac{V_0}{(R_1 + r)} R_1$. The same potential difference will appear across the capacitor, hence charge on capacitor in steady state $Q = \dfrac{CV_0 R_1}{(R_1 + r)}$

Concepts

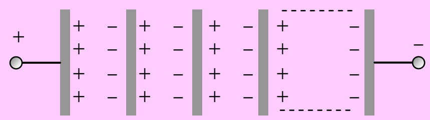
- *In series combination equivalent capacitance is always lesser than that of either of the individual capacitors e.g. If two capacitors having capacitances 3μF and 6μF respectively are connected in series then equivalent capacitance*

  $C_{eq} = \frac{3 \times 6}{3+6} = 2\mu F$ *Which is lesser than the smallest capacitance (3μF) of the network.*

- *In parallel combination, equivalent capacitance is always greater than the maximum capacitance of either capacitor in network.*

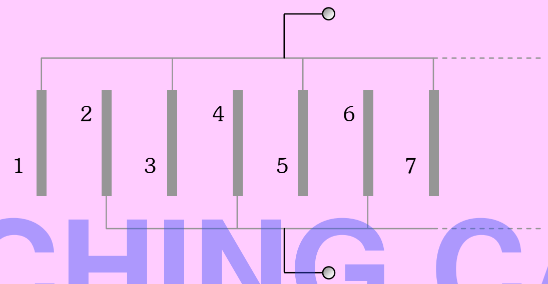
- *If n identical plates arranged as shown below, constitutes (n – 1) capacitors in series. If each capacitors having capacitance* $\frac{\varepsilon_0 A}{d}$

  then $C_{eq} = \frac{\varepsilon_0 A}{(n-1)d}$



In this situation except two extreme plates each plate is common to adjacent capacitors.

- *If n identical plates are arranged such that even no. of plates are connected together and odd number of plates are connected together, then (n – 1) capacitors will be formed and they will be in parallel grouping.*



Equivalent capacitance $C' = (n-1)C$ *Where C = capacitance of a capacitor* $= \frac{\varepsilon_0 A}{d}$

- *If n identical capacitors are connected in parallel which are charged to a potential V. If these are separated and connected in series then potential difference of combination will be nV.*

Network Solving.

To solve capacitive network for equivalent capacitance following guidelines should be followed.

Guideline 1. Identify the two points across which the equivalent capacitance is to be calculated.

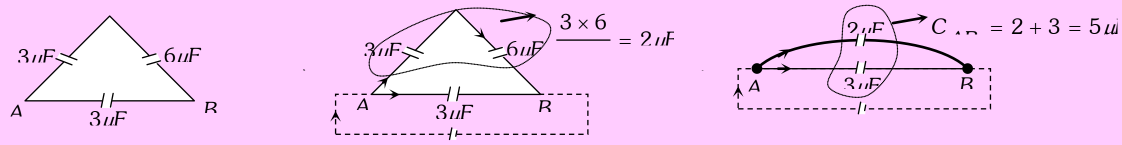
Guideline 2. Connect (Imagine) a battery between these points.

Guideline 3. Solve the network from the point (reference point) which is farthest from the points between which we have to calculate the equivalent capacitance. (The point is likely to be not a node)

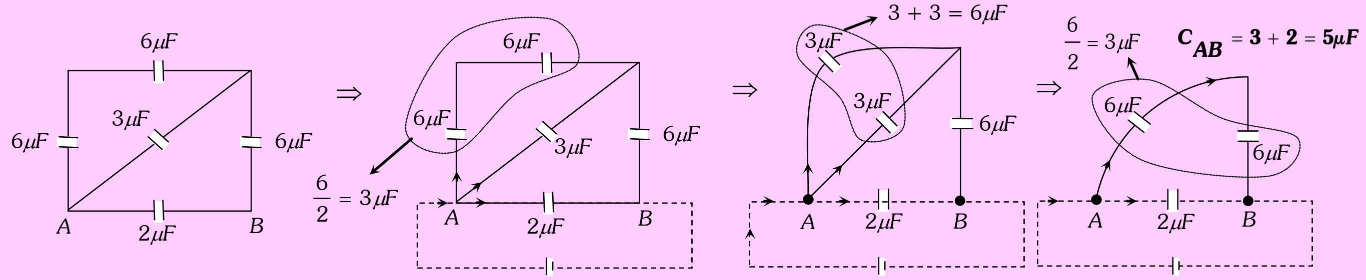
(1) **Simple circuits :** Suppose equivalent capacitance is to be determined in the following networks between points A and B

Suppose equivalent capacitance is to be determined in the following networks between points A and B

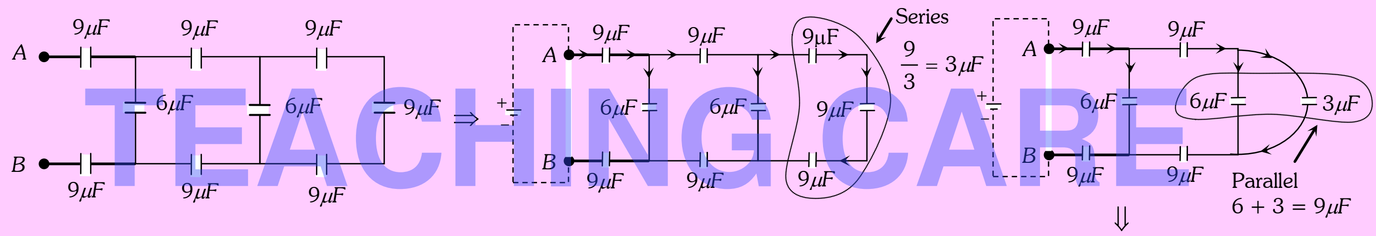
(i)



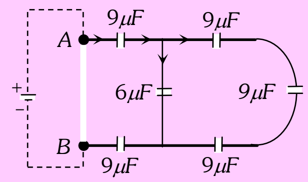
(ii)



(iii)

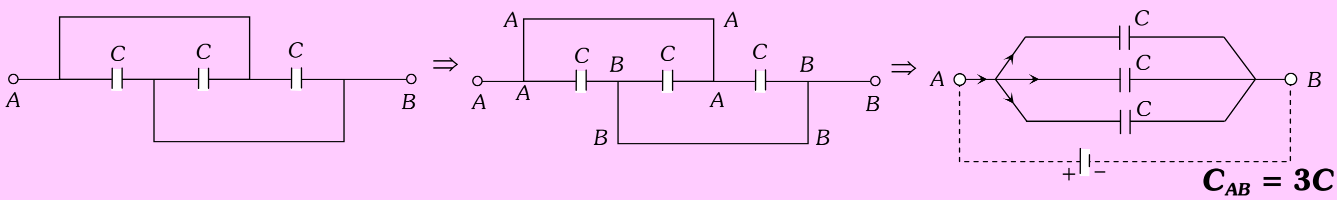


By similar process $C_{AB} = 3\mu F$ ⇐



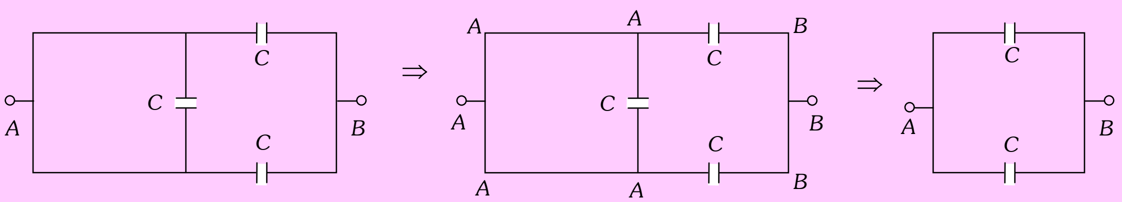
(2) **Circuits with extra wire :** If there is no capacitor in any branch of a network then every point of this branch will be at same potential. Suppose equivalent capacitance is to be determine in following cases

(i)



$C_{AB} = 3C$

(ii)

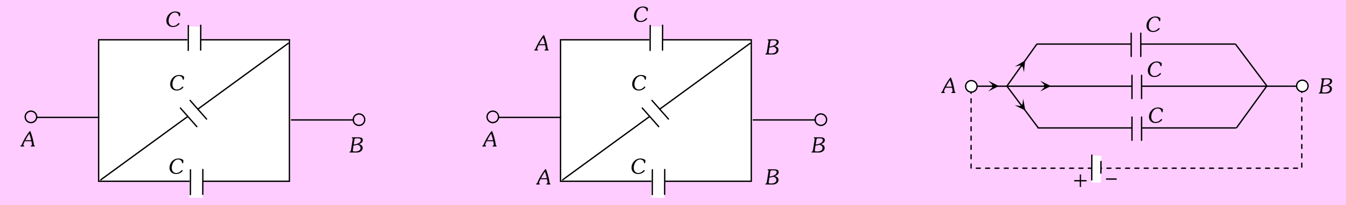


No p.d. across vertical branch so it is removed

$C_{AB} = 2C$

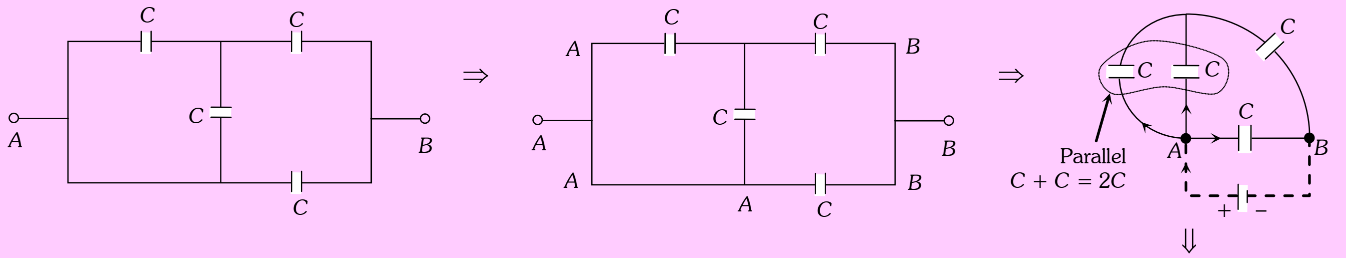
⇒

(iii)

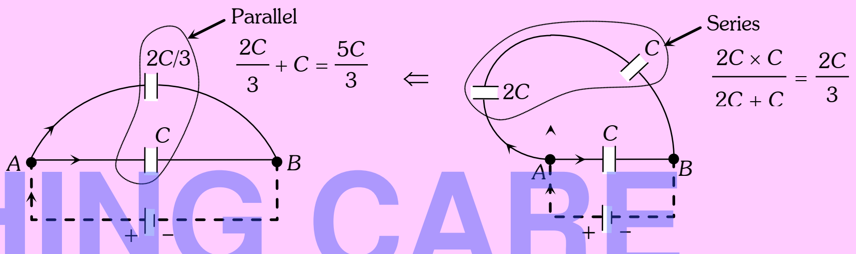


$\mathbf{C_{AB} = 3C}$

(iv)

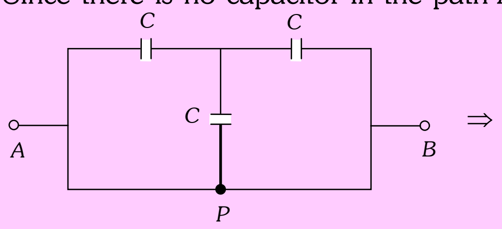


Hence equivalent capacitance between A and B is $\dfrac{\mathbf{5C}}{\mathbf{3}}$



$\frac{2C}{3} + C = \frac{5C}{3}$ $\Leftarrow$ $\frac{2C \times C}{2C + C} = \frac{2C}{3}$

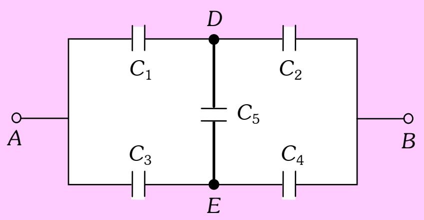
(v) Since there is no capacitor in the path APB, the points A, P and B are electrically same i.e., the input and output points are directly connected (short circuited).



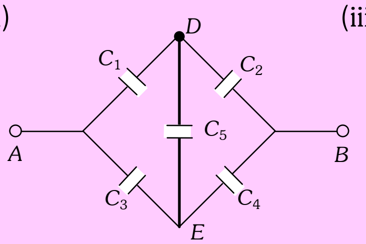
Thus, entire charge will prefer to flow along path APB. It means that the capacitors connected in the circuit will not receive any charge for storing. Thus equivalent capacitance of this circuit is zero.

(3) **Wheatstone bride based circuit :** If in a network five capacitors are arranged as shown in following figure, the network is called wheatstone bridge type circuit. If it is balanced then $\frac{C_1}{C_2} = \frac{C_3}{C_4}$ hence C_5 is removed and equivalent capacitance between A and B

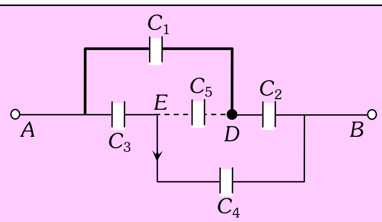
(i)



(ii)



(iii)

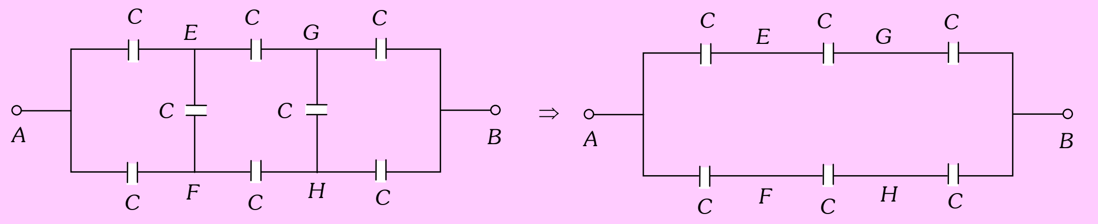


$$\mathbf{C_{AB}} = \frac{\mathbf{C_1 C_2}}{\mathbf{C_1 + C_2}} + \frac{\mathbf{C_3 C_4}}{\mathbf{C_3 + C_4}}$$

(4) **Extended wheatstone bridge :** The given figure consists of two wheatstone bridge connected together. One bridge is connected between points *AEGHFA* and the other is connected between points *EGBHFE*.

This problem is known as extended wheatstone bridge problem, it has two branches *EF* and *GH* to the left and right of which symmetry in the ratio of capacities can be seen.

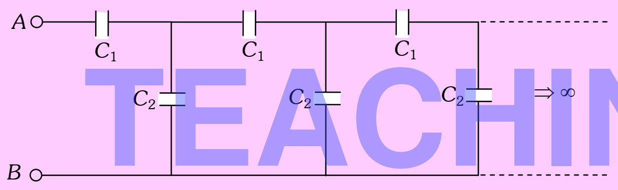
It can be seen that ratio of capacitances in branches *AE* and *EG* is same as that between the capacitances of the branches *AF* and *FH*. Thus, in the bridge *AEGHFA*; the branch *EF* can be removed. Similarly in the bridge *EGBHFE* branch *GH* can be removed



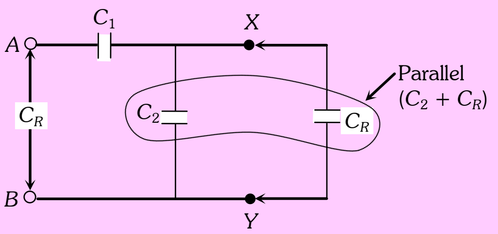
$$C_{AB} = \frac{2C}{3}$$

(5) **Infinite chain of capacitors :** In the following figure equivalent capacitance between *A* and *B*

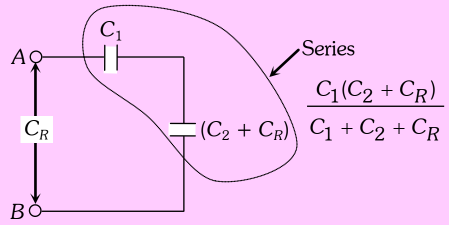
(i)



Suppose the effective capacitance between *A* and *B* is C_R. Since the network is infinite, even if we remove one pair of capacitors from the chain, remaining network would still have infinite pair of capacitors, i.e., effective capacitance between *X* and *Y* would also be C_R



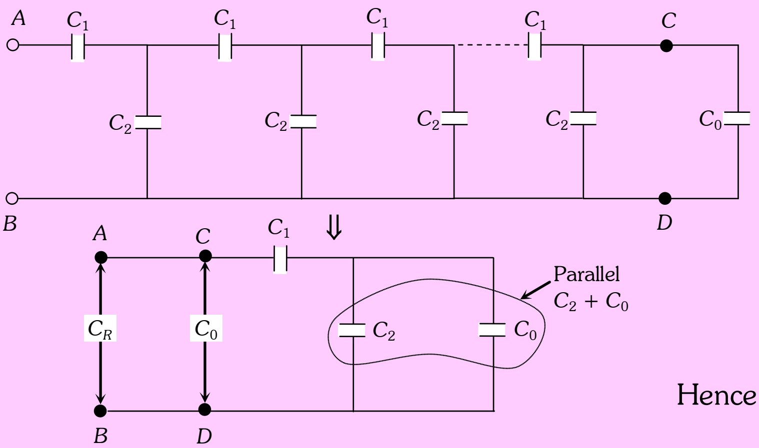
⇒



Hence equivalent capacitance between *A* and *B*

$$C_{AB} = \frac{C_1(C_2 + C_R)}{C_1 + C_2 + C_R} = C_R \quad \Rightarrow \quad \mathbf{C_{AB} = \frac{C_2}{2}\left[\sqrt{\left(1 + 4\frac{C_1}{C_2}\right)} - 1\right]}$$

(ii) For what value of C_0 in the circuit shown below will the net effective capacitance between A and B be independent of the number of sections in the chain



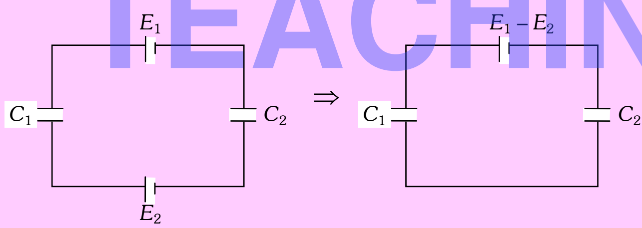
Suppose there are n sections between A and B and the network is terminated by C_0 with equivalent capacitance C_R. Now if we add one more sections to the network between D and C (as shown in the following figure), the equivalent capacitance of the network C_R will be independent of number of sections if the capacitance between D and C still remains C_0 *i.e.*,

Hence $C_0 = \dfrac{C_1 \times (C_2 + C_0)}{C_1 + C_2 + C_0} \Rightarrow C_0^{\,2} + C_2 C_0 - C_1 C_2 = 0$

On simplification $\mathbf{C_0} = \dfrac{\mathbf{C_2}}{2}\left[\sqrt{\left(\mathbf{1} + \mathbf{4}\dfrac{\mathbf{C_1}}{\mathbf{C_2}}\right)} - \mathbf{1}\right]$

(6) **Network with more than one cell :**

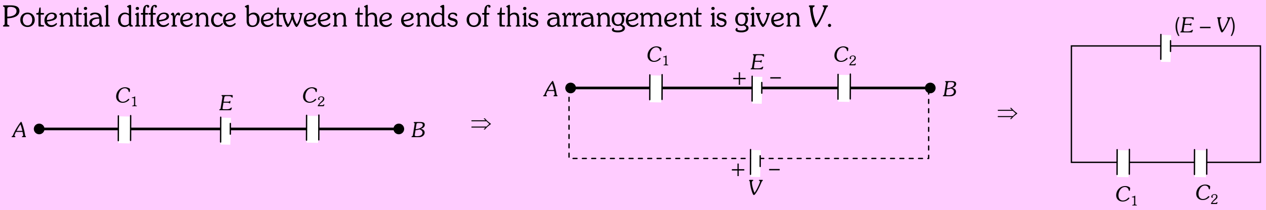
(i)



Potential difference across C_1 is $\left(\dfrac{C_2}{C_1 + C_2}\right)(E_1 - E_2)$

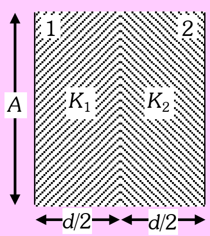
and potential difference across C_2 is $\left(\dfrac{C_1}{C_1 + C_2}\right)(E_1 - E_2)$

(ii) Potential difference between the ends of this arrangement is given V.



(7) **Advance case of compound dielectrics :** If several dielectric medium filled between the plates of a parallel plate capacitor in different ways as shown.

(i)

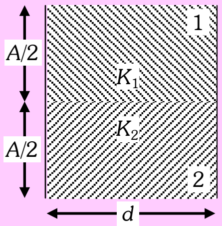


The system can be assumed to be made up of two capacitors C_1 and C_2 which may be said to connected in series

$C_1 = \dfrac{K_1 \varepsilon_0 A}{\frac{d}{2}}$, $C_2 = \dfrac{K_2 \varepsilon_0 A}{\frac{d}{2}}$ and $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} \Rightarrow C_{eq} = \left(\dfrac{2K_1K_2}{K_1 + K_2}\right) \cdot \dfrac{\varepsilon_0 A}{d}$

Also $K_{eq} = \dfrac{2K_1K_2}{K_1 + K_2}$

(ii)



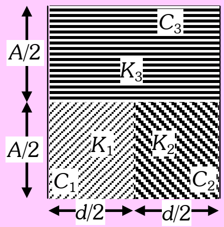
In this case these two capacitors are in parallel and

$$C_1 = \frac{K_1\varepsilon_0 A}{2d}, \quad C_2 = \frac{K_2\varepsilon_0 A}{2d}$$

Hence, $C_{eq} = C_1 + C_2 \Rightarrow C_{eq} = \left(\frac{K_1 + K_2}{2}\right).\frac{\varepsilon_0 A}{d}$

Also $K_{eq} = \frac{K_1 + K_2}{2}$

(iii)



In this case C_1 and C_2 are in series while this combination is in parallel with C_3

$$C_1 = \frac{K_1\varepsilon_0 \frac{A}{2}}{\frac{d}{2}} = \frac{K_1\varepsilon_0 A}{d}, \quad C_2 = \frac{K_1\varepsilon_0 \frac{A}{2}}{\frac{d}{2}} = \frac{K_2\varepsilon_0 A}{d} \text{ and } C_3 = \frac{K_3\varepsilon_0 \frac{A}{2}}{\frac{d}{2}} = \frac{K_3\varepsilon_0 A}{2d}$$

Hence, $C_{eq} = \frac{C_1 C_2}{C_1 + C_2} + C_3 = \frac{\frac{k_1\varepsilon_0 A}{d} \times \frac{k_2\varepsilon_0 A}{d}}{\frac{k_1\varepsilon_0 A}{d} + \frac{k_2\varepsilon_0 A}{d}} + \frac{k_3\varepsilon_0 A}{2d} \Rightarrow C_{eq} = \left(\frac{k_1 k_2}{k_1 + k_2} + \frac{k_3}{2}\right).\frac{\varepsilon_0 A}{d}$

Also $k_{eq} = \left(\frac{k_3}{2} + \frac{k_1 k_2}{k_1 + k_2}\right)$