Example based on series and parallel grouping of capacitors

Example: 117 Three capacitors of $2\mu f$, $3\mu f$ and $6\mu f$ are joined in series and the combination is charged by means of a 24 *volt* battery. The potential difference between the plates of the $6\mu f$ capacitor is

[MP PMT 2002 Similar to MP PMT 1996]

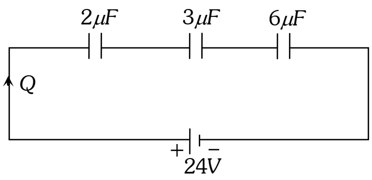
(a) 4 *volts* (b) 6 *volts* (c) 8 *volts* (d) 10 *volts*

Solution: (a) Equivalent capacitance of the network is $\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$

$$C_{eq} = 1\mu F$$

Charge supplied by battery $Q = C_{eq}.V \Rightarrow 1 \times 24 = 24\ \mu C$

Hence potential difference across $6\mu F$ capacitor $= \frac{24}{6} = 4\,volt$.



Example: 118 Two capacitors each of $1\ \mu f$ capacitance are connected in parallel and are then charged by 200 *V* D.C. supply. The total energy of their charges in joules is **[MP PMT 2002]**

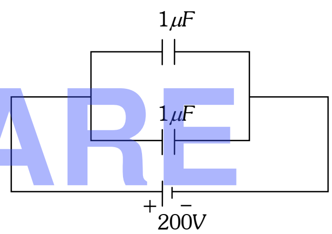
(a) 0.01 (b) 0.02 (c) 0.04 (d) 0.06

Solution: (c) By using formula $U = \frac{1}{2}C_{eq}V^2$

Here $C_{eq} = 2\mu F$

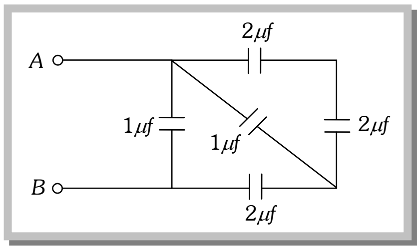
$\therefore \quad U = \frac{1}{2} \times 2 \times 10^{-6} \times (200)^2$

$= 0.04J$



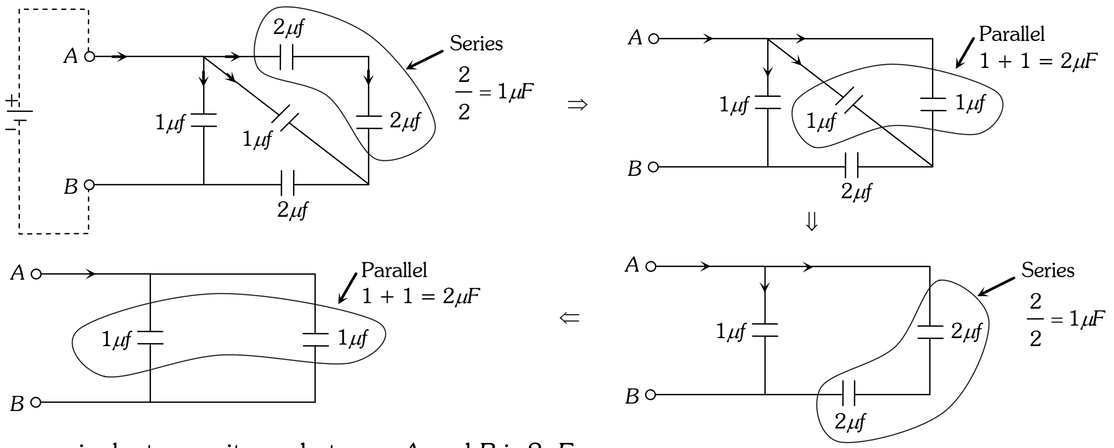
Example: 119 Five capacitors are connected as shown in the figure. The equivalent capacitance between the point *A* and *B* is

[MP PMT 2002; SCRA 1996; Pantnagar 1987]



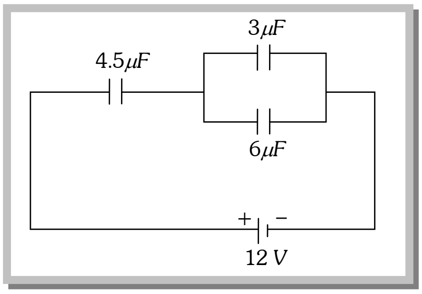
(a) $1\ \mu f$ (b) $2\ \mu f$ (c) $3\ \mu f$ (d) $4\ \mu f$

Solution: (b)



Hence equivalent capacitance between *A* and *B* is $2\mu F$.

Example: 120 In the following network potential difference across capacitance of 4.5 μF is **[RPET 2001; MP PET 1992]**

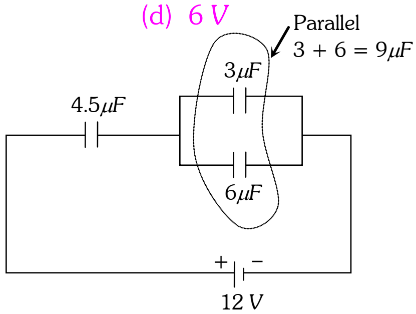


(a) 8 V (b) 4 V (c) 2 V (d) 6 V

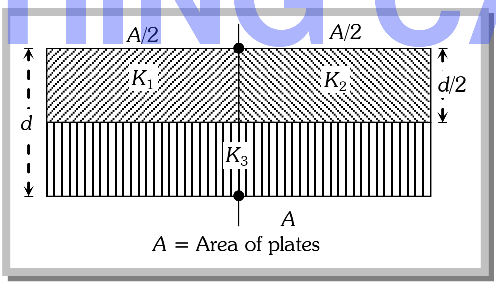
Solution: (a) Equivalent capacitance $C_{eq} = \dfrac{9 \times 4.5}{9 + 4.5} = 3\mu F$

Charge supplied by battery $Q = C_{eq} \times V = 3 \times 12 = 36\ \mu C$

Hence potential difference across 4.5 $\mu F = \dfrac{36}{4.5} = 8V$.



Example: 121 A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants K_1, K_2 and K_3 as shown in fig. If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant K is given by **[IIT Screening 2000]**



(a) $\dfrac{1}{K} = \dfrac{1}{K_1} + \dfrac{1}{K_2} + \dfrac{1}{2K_3}$ (b) $\dfrac{1}{K} = \dfrac{1}{K_1 + K_2} + \dfrac{1}{2K_3}$

(c) $K = \dfrac{K_1 K_2}{K_1 + K_2} + 2K_3$ (d) $K = K_1 + K_2 + 2K_3$

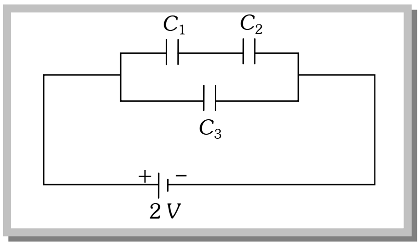
Solution: (b) The effective capacitance is given by $\dfrac{1}{C_{eq}} = \dfrac{d}{\varepsilon_0 A}\left[\dfrac{1}{2K_3} + \dfrac{1}{(K_1 + K_2)}\right]$

The capacitance of capacitor with single dielectric of dielectric constant K is $C = \dfrac{K\varepsilon_0 A}{d}$

According to question $C_{eq} = C$ *i.e.*, $\dfrac{\varepsilon_0 A}{d\left[\dfrac{1}{2K_3} + \dfrac{1}{K_1 + K_2}\right]} = \dfrac{K\varepsilon_0 A}{d}$

$\Rightarrow \dfrac{1}{K} = \dfrac{1}{2K_3} + \dfrac{1}{K_1 + K_2}$.

Example: 122 Two capacitors $C_1 = 2\mu F$ and $C_2 = 6\mu F$ in series, are connected in parallel to a third capacitor $C_3 = 4\mu F$. This arrangement is then connected to a battery of e.m.f. = 2 V, as shown in the fig. How much energy is lost by the battery in charging the capacitors ? **[MP PET 2001]**

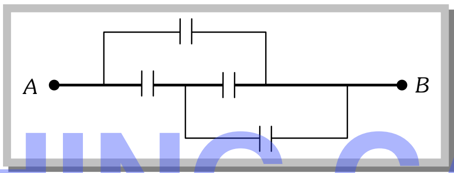


(a) $22 \times 10^{-6} J$ (b) $11 \times 10^{-6} J$ (c) $\left(\frac{32}{3}\right) \times 10^{-6} J$ (d) $\left(\frac{16}{3}\right) \times 10^{-6} J$

Solution: (b) Equivalent capacitance $C_{eq} = \frac{C_1 C_2}{C_1 + C_2} + C_3 = \frac{2 \times 6}{8} + 4 = 5.5\mu F$

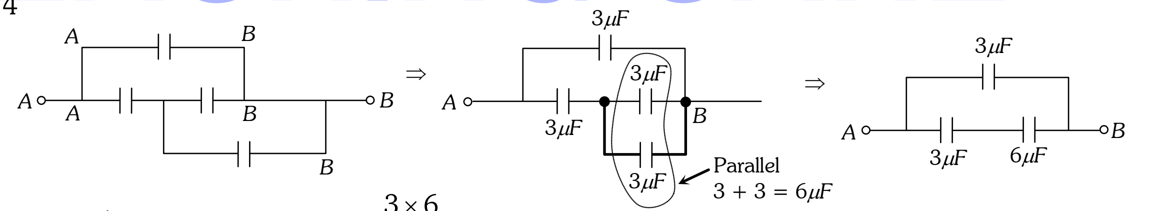
$\therefore$ $U = \frac{1}{2} C_{eq} . V^2 = \frac{1}{2} \times 5.5 \times (2)^2 = 11 \times 10^{-6} J$

Example: 123 In the circuit shown in the figure, each capacitor has a capacity of $3\mu F$. The equivalent capacity between A and B is **[MP PMT 2000]**



(a) $\frac{3}{4} \mu F$ (b) $3 \mu F$ (c) $6 \mu F$ (d) $5 \mu F$

Solution: (d)



Hence equivalent capacitance $C_{eq} = \frac{3 \times 6}{3 + 6} + 3 = 5\mu F$.

Example: 124 Given a number of capacitors labelled as $8\mu F$, $250\ V$. Find the minimum number of capacitors needed to get an arrangement equivalent to $16\ \mu F$, $1000\ V$ **[AIIMS 2000]**

(a) 4 (b) 16 (c) 32 (d) 64

Solution: (c) Let $C = 8\ \mu F$, $C' = 16\ \mu F$ and $V = 250\ volt$, $V' = 1000\ V$

Suppose m rows of given capacitors are connected in parallel which each row contains n capacitor then

Potential difference across each capacitors $V = \frac{V'}{n}$ and equivalent capacitance of network $C' = \frac{mC}{n}$.

On putting the values, we get $n = 4$ and $m = 8$. Hence total capacitors $= m \times n = 8 \times 4 = 32$.

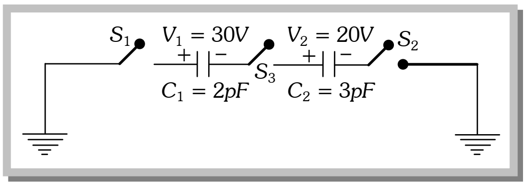
Short Trick : For such type of problem number of capacitors $n = \frac{C'}{C} \times \left(\frac{V'}{V}\right)^2$. Here $n = \frac{16}{8}\left(\frac{1000}{250}\right)^2 = 32$

Example: 125 Ten capacitors are joined in parallel and charged with a battery up to a potential V. They are then disconnected from battery and joined again in series then the potential of this combination will be **[RPET 2000]**

(a) V (b) $10\ V$ (c) $5\ V$ (d) $2\ V$

Solution: (b) By using the formula $V' = nV \Rightarrow V' = 10V$.

Example: 126 For the circuit shown, which of the following statements is true **[IIT-JEE 1999]**



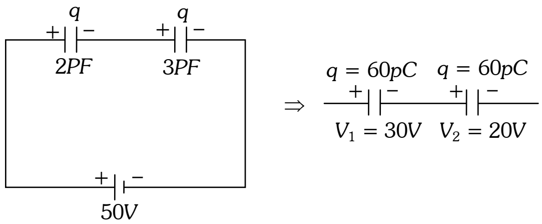
(a) With S_1 closed, $V_1 = 15\,V$, $V_2 = 20\,V$ (b) With S_3 closed, $V_1 = V_2 = 25\,V$

(c) With S_1 and S_2 closed $V_1 = V_2 = 0$ (d) With S_1 and S_3 closed $V_1 = 30\,V$, $V_2 = 20\,V$

Solution: (d) When S_3 is closed, due to attraction with opposite charge, no flow of charge takes place through S_3. Therefore, potential difference across capacitor plates remains unchanged or $V_1 = 30\,V$ and $V_2 = 20\,V$.

Alternate Solution

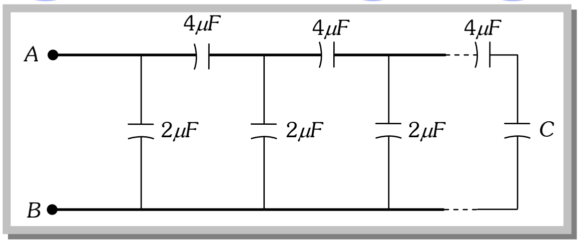
Charges on the capacitors are – $q_1 = (30)(2) = 60pC$, $q_2 = (20)(3) = 60\,pC$ or $q_1 = q_2 = q$ (say)



The situation is similar as the two capacitors in series are first charged with a battery of emf 50 V and then disconnected.

When S_3 is closed, $V_1 = 30\,V$ and $V_2 = 20\,V$.

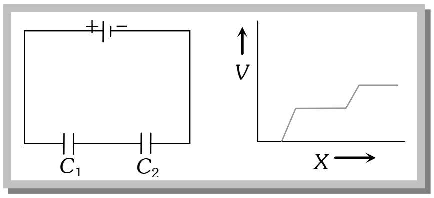
Example: 127 A finite ladder is constructed by connecting several sections of $2\mu F, 4\mu F$ capacitor combinations as shown in the figure. It is terminated by a capacitor of capacitance C. What value should be chosen for C, such that the equivalent capacitance of the ladder between the points A and B becomes independent of the number of sections in between



(a) $4\mu F$ (b) $2\mu F$ (c) $18\,\mu F$ (d) $6\,\mu F$

Solution: (a) By using formula $C = \frac{C_2}{2}\left[\sqrt{1 + 4\left(\frac{C_1}{C_2}\right)} + 1\right]$; $C_1 = 4\mu F$, $C_2 = 2\mu F$ We get $C = 4\,\mu F$.

Example: 128 Figure shows two capacitors connected in series and joined to a battery. The graph shows the variation in potential as one moves from left to right on the branch containing the capacitors. **[MP PMT 1999]**



(a) $C_1 > C_2$

(b) $C_1 = C_2$

(c) $C_1 < C_2$

(d) The information is insufficient to decide the relation between C_1 and C_2

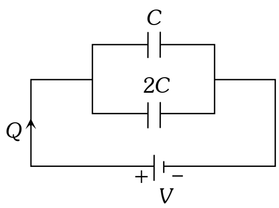
Solution: (c) According to graph we can say that potential difference across the capacitor C_1 is more than that across C_2.

Since charge Q is same *i.e.*, $Q = C_1V_1 = C_2V_2 \Rightarrow \frac{C_1}{C_2} = \frac{V_2}{V_1} \Rightarrow C_1 < C_2$ $(V_1 > V_2)$.

Example: 129 Two condensers of capacity C and $2C$ are connected in parallel and these are charged upto V volt. If the battery is removed and dielectric medium of constant K is put between the plates of first condenser, then the potential at each condenser is **[RPET 1998; IIT-JEE 1988]**

(a) $\frac{V}{K+2}$ (b) $2+\frac{K}{3V}$ (c) $\frac{2V}{K+2}$ (d) $\frac{3V}{K+2}$

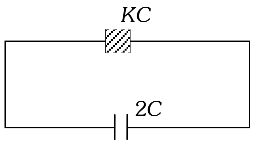
Solution: (d) Initially



Equivalent capacitance of the system $C_{eq} = 3C$

Total charge $Q = (3C)V$

Finally



Equivalent capacitance of the system $C_{eq} = KC + 2C$

Hence common potential $V = \frac{Q}{(KC+2C)} = \frac{3CV}{(K+2)C} = \frac{3V}{K+2}$.

Example: 130 Condenser A has a capacity of 15 μF when it is filled with a medium of dielectric constant 15. Another condenser B has a capacity 1 μF with air between the plates. Both are charged separately by a battery of 100V. after charging, both are connected in parallel without the battery and the dielectric material being removed. The common potential now is **[MNR 1994]**

(a) 400V (b) 800V (c) 1200V (d) 1600V

Solution: (b) Charge on capacitor A is given by $Q_1 = 15\times10^{-6}\times100 = 15\times10^{-4}C$

Charge on capacitor B is given by $Q_2 = 1\times10^{-6}\times100 = 10^{-4}C$

Capacity of capacitor A after removing dielectric $= \frac{15\times10^{-6}}{15} = 1\mu F$

Now when both capacitors are connected in parallel their equivalent capacitance will be $C_{eq} = 1+1 = 2\mu F$

So common potential $= \frac{(15\times10^{-4})+(1\times10^{-4})}{2\times10^{-6}} = 800V$.

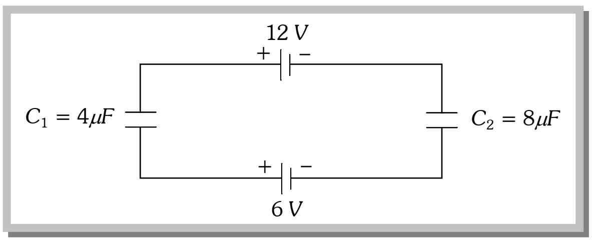
Example: 131 A capacitor of $20\mu F$ is charged upto 500V is connected in parallel with another capacitor of $10\mu F$ which is charged upto 200V. The common potential is **[CBSE 2000; CPMT 1999; BHU 1997]**

(a) 500V (b) 400V (c) 300V (d) 200V

Solution: (b) By using $V = \frac{C_1V_1 + C_2V_2}{C_1V_2}$; $C_1 = 20\ \mu F$, $V_1 = 500\ V$, $C_2 = 10\ \mu F$ and $V_2 = 200\ V$

$V = \frac{20\times500+10\times200}{20+10} = 400V$.

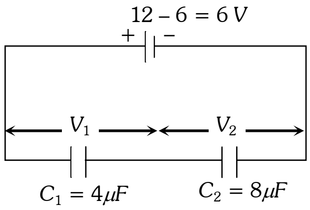
Example: 132 In the circuit shown **[DCE 1995]**



(a) The charge on C_2 is greater than that of C_1 (b) The charge on C_2 is smaller than that of C_1

(c) The potential drop across C_1 is smaller than C_2 (d) The potential drop across C_1 is greater than C_2

Solution: (d) Given circuit can be redrawn as follows

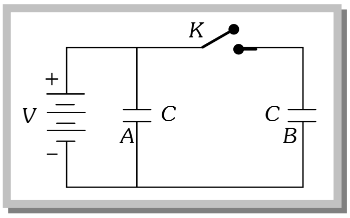


$$C_{eq} = \frac{4\times 8}{12} = \frac{8}{3}\mu F$$

So $Q = \frac{8}{3}\times 6 = 16\mu C$

Hence potential difference $V_1 = \frac{16}{4} = 4volt$ and $V_2 = \frac{16}{8} = 2volt$ i.e. $V_1 > V_2$

Example: 133 As shown in the figure two identical capacitors are connected to a battery of V volts in parallel. When capacitors are fully charged, their stored energy is U_1. If the key K is opened and a material of dielectric constant $K = 3$ is inserted in each capacitor, their stored energy is now U_2. $\frac{U_1}{U_2}$ will be **[IIT 1983]**



(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) 3 (d) $\frac{1}{3}$

Solution: (a) Initially potential difference across both the capacitor is same hence energy of the system is

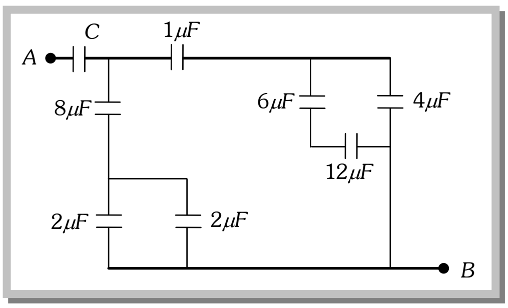
$$U_1 = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2 \qquad \text{........(i)}$$

In the second case when key K is opened and dielectric medium is filled between the plates, capacitance of both the capacitors becomes 3C, while potential difference across A is V and potential difference across B is $\frac{V}{3}$ hence energy of the system now is

$$U_2 = \frac{1}{2}(3C)V^2 + \frac{1}{2}(3C)\left(\frac{V}{3}\right)^2 = \frac{10}{6}CV^2 \qquad \text{.......(ii)}$$

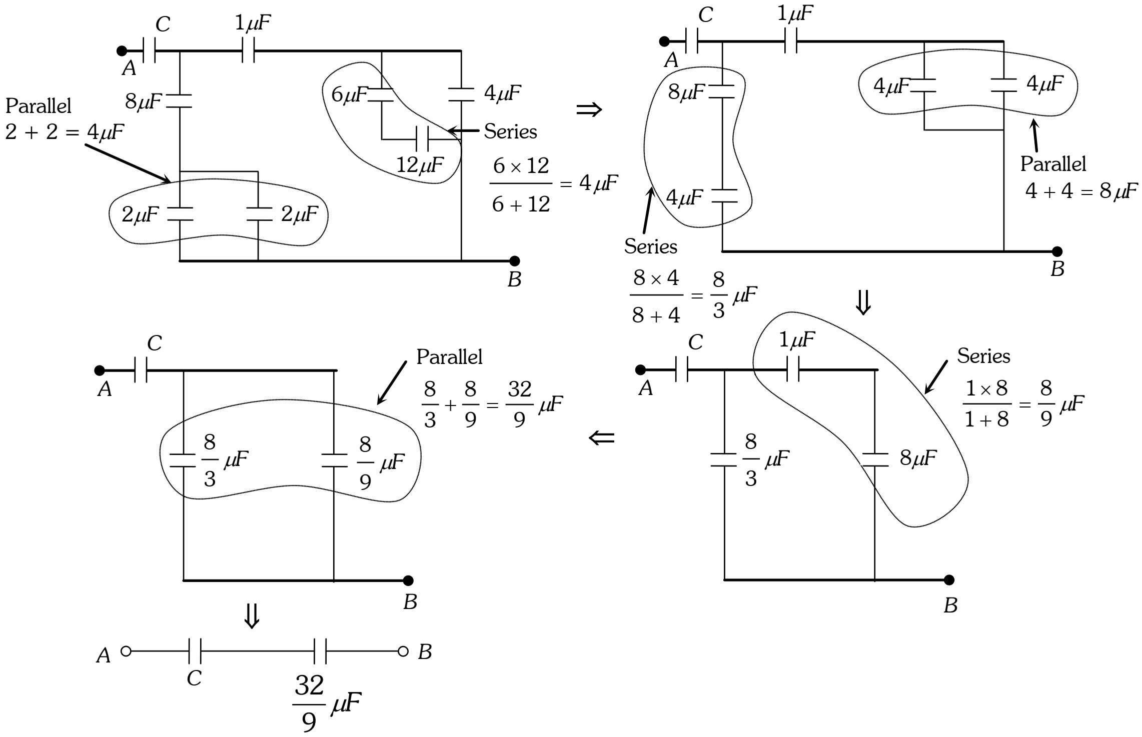
So, $\frac{U_1}{U_2} = \frac{3}{5}$

Example: 134 In the following figure the resultant capacitance between A and B is $1\mu F$. The capacitance C is **[IIT 1977]**



(a) $\frac{32}{11}\mu F$ (b) $\frac{11}{32}\mu F$ (c) $\frac{23}{32}\mu F$ (d) $\frac{32}{23}\mu F$

Solution: (d) Given network can be simplified as follows



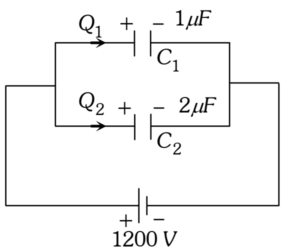
Given that equivalent capacitance between A and B i.e., $C_{AB} = 1\mu F$

But $C_{AB} = \dfrac{C \times \frac{32}{9}}{C + \frac{32}{9}}$ hence $\dfrac{C \times \frac{32}{9}}{C + \frac{32}{9}} = 1 \Rightarrow C = \dfrac{32}{23}\mu F$.

Example: 135 A $1\mu F$ capacitor and a $2\mu F$ capacitor are connected in parallel across a 1200 volts line. The charged capacitors are then disconnected from the line and from each other. These two capacitors are now connected to each other in parallel with terminals of unlike signs together. The charges on the capacitors will now be

(a) $1800\mu C$ each (b) $400\mu C$ and $800\mu C$ (c) $800\mu C$ and $400\mu C$ (d) $800\mu C$ and $800\mu C$

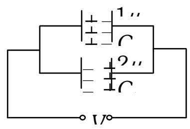
Solution: (b) Initially charge on capacitors can be calculated as follows



$Q_1 = 1 \times 1200 = 1200\ \mu C$ and $Q_2 = 2 \times 1200 = 2400\ \mu C$

Finally when battery is disconnected and unlike plates are connected together then common potential $V' = \dfrac{Q_2 - Q_1}{C_1 + C_2}$

$= \dfrac{2400 - 1200}{1+2} = 400V$



Hence, New charge on C_1 is $1 \times 400 = 400\mu C$

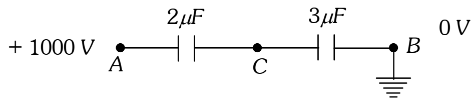
And New charge on C_2 is $2 \times 400 = 800\mu C$.

Example: 136 The two condensers of capacitances $2\mu F$ and $3\mu F$ are in series. The outer plate of the first condenser is at 1000 volts and the outer plate of the second condenser is earthed. The potential of the inner plate of each condenser is

(a) 300 *volts* (b) 500 *volts* (c) 600 *volts* (d) 400 *volts*

Solution: (d) Here, potential difference across the combination is $V_A - V_B = 1000V$

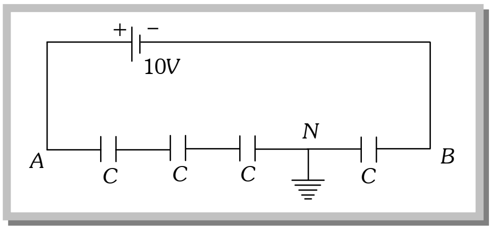
Equivalent capacitance $C_{eq} = \frac{2\times 3}{2+3} = \frac{6}{5}\mu F$



Hence, charge on each capacitor will be $Q = C_{eq} \times (V_A - V_B) = \frac{6}{5}\times 1000 = 1200\mu C$

So potential difference between A and C, $V_A - V_C = \frac{1200}{2} = 600V \Rightarrow 1000 - V_C = 600 \Rightarrow \quad V_c = 400V$

Example: 137 Four identical capacitors are connected in series with a 10*V* battery as shown in the figure. The point *N* is earthed. The potentials of points *A* and *B* are



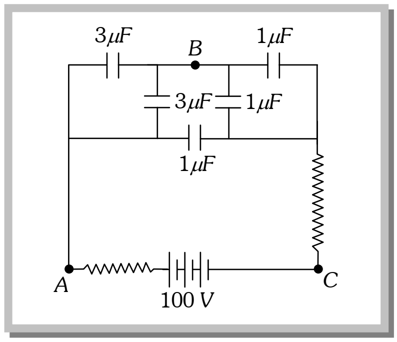
(a) $10V, 0V$ (b) $7.5V - 2.5V$ (c) $5V - 5V$ (d) $7.5V, 2.5V$

Solution: (b) Potential difference across each capacitor will be $\frac{10}{4} = 2.5V$

Hence potential difference between *A* & *N* i.e., $V_A - V_N = 2.5 + 2.5 + 2.5 = 7.5V$

$\Rightarrow \quad V_A - 0 = V_A = 7.5V$ While $V_N - V_B = 2.5 \quad \Rightarrow 0 - V_B = 2.5 \quad \Rightarrow V_B = -2.5V$

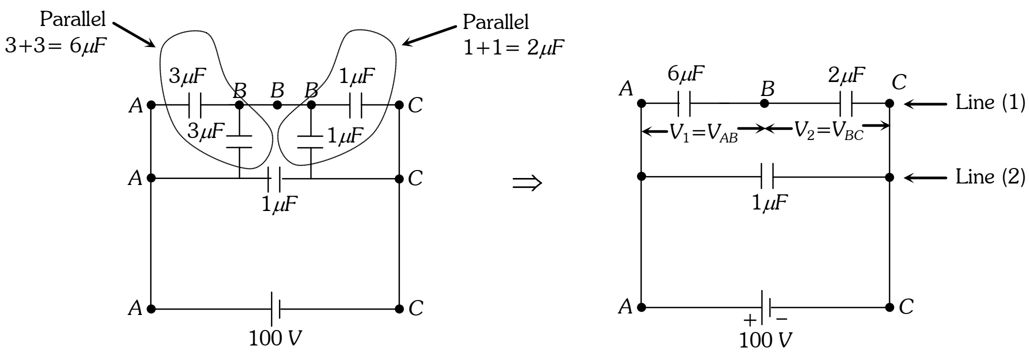
Example: 138 In the figure below, what is the potential difference between the points A and B and between *B* and *C* respectively in steady state **[IIT-JEE 1979]**



(a) 100 *volts* both (b) $V_{AB} = 75$ *volts*, $V_{BC} = 25$ *volts*

(c) $V_{AB} = 25$ *volts*, $V_{BC} = 75$ *volts* (d) $V_{AB} = 50$ *volts* $V_{BC} = 50$ *volts*

Solution: (c) In steady state No current flows in the given circuit hence resistances can be eliminated

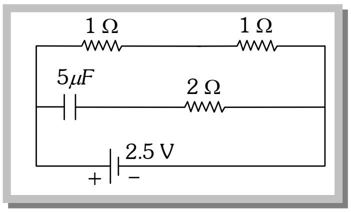


By using the formula to find potential difference in series combination of two capacitor

$$\left(V_1 = \left(\frac{C_2}{C_1 + C_2}\right).V \text{ and } V_2 = \frac{C_1}{C_2 + C_2}V\right)$$

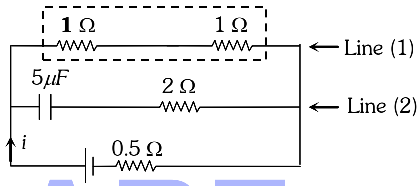
$$V_1 = V_{AB} = \left(\frac{2}{2+6}\right) \times 100 = 25V\ ; \qquad V_2 = V_{BC} = \left(\frac{6}{2+6}\right) \times 100 = 75V.$$

Example: 139 A capacitor of capacitance $5\mu F$ is connected as shown in the figure. The internal resistance of the cell is 0.5Ω. The amount of charge on the capacitor plate is **[MP PET 1997]**



(a) $0\mu C$ (b) $5\mu C$ (c) $10\mu C$ (d) $25\mu C$

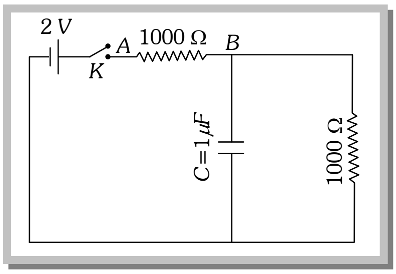
Solution: (c) In steady state current drawn from the battery $i = \dfrac{2.5}{(1+1+0.5)} = 1A$



In steady state capacitor is fully charged hence No current will flow through line (2)

Hence potential difference across line (1) is $V = 1 \times 2 = 2volt$, the same potential difference appears across the capacitor, so charge on capacitor $Q = 5 \times 2 = 10\mu C$

Example: 140 When the key K is pressed at time $t = 0$. Which of the following statements about the current i in the resistor AB of the adjoining circuit is true **[CBSE 1995]**



(a) $i = 2mA$ at all t (b) i oscillates between $1mA$ and $2mA$

(c) $i = 1mA$ at all t (d) At $t = 0$, $i = 2mA$ and with time it goes to $1mA$

Solution: (d) At $t = 0$ whole current passes through capacitance; so effective resistance of circuit is 1000Ω and current $i = \dfrac{2}{1000} = 2 \times 10^{-3} A = 2mA$. After sufficient time, steady state is reached; then there is no current in capacitor branch; so effective resistance of circuit is $1000 + 1000 = 2000\Omega$ and current $i = \dfrac{2}{2000} = 1 \times 10^{-3} A = 1mA$ *i.e.*, current is $2mA$ at $t = 0$ and with time it goes to $1mA$.

Example: 141 The plates of a capacitor are charged to a potential difference of 320 *volts* and are then connected across a resistor. The potential difference across the capacitor decays exponentially with time. After 1 *second* the potential difference between the plates of the capacitor is 240 *volts*, then after 2 and 3 *seconds* the potential difference between the plates will be **[MP PET 1998]**

(a) 200 and 180 *volts* (b) 180 and 135 *volts* (c) 160 and 80 *volts* (d) 140 and 20 *volts*

Solution: (b) During discharging potential difference across the capacitor falls exponentially as $V = V_0 e^{-\lambda t}$ $(\lambda = 1/RC)$

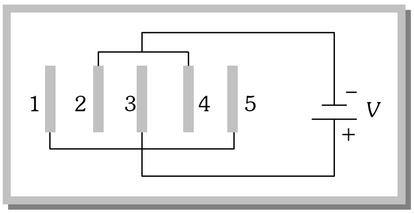
Where V = Instantaneous P.D. and V_0 = max. P.D. across capacitor

After 1 *second* $V_1 = 320\,(e^{-\lambda}) \Rightarrow 240 = 320\,(e^{-\lambda}) \Rightarrow e^{-\lambda} = \frac{3}{4}$

After 2 *seconds* $V_2 = 320\,(e^{-\lambda})^2 \Rightarrow\ 320 \times \left(\frac{3}{4}\right)^2 = 180\,volt$

After 3 *seconds* $V_3 = 320\,(e^{-\lambda})^3 =\ 320 \times \left(\frac{3}{4}\right)^3 = 135\,volt$

Example: 142 Five similar condenser plates, each of area A. are placed at equal distance d apart and are connected to a source of e.m.f E as shown in the following diagram. The charge on the plates 1 and 4 will be

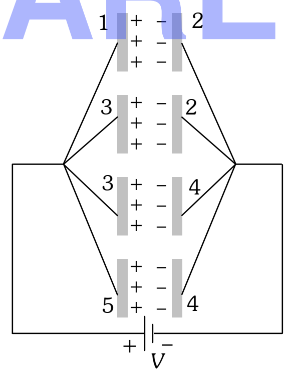


(a) $\frac{\varepsilon_0 A}{d}, \frac{-2\varepsilon_0 A}{d}$ (b) $\frac{\varepsilon_0 AV}{d}, \frac{-2\varepsilon_0 AV}{d}$ (c) $\frac{\varepsilon_0 AV}{d}, \frac{-3\varepsilon_0 AV}{d}$ (d) $\frac{\varepsilon_0 AV}{d}, \frac{-4\varepsilon_0 AV}{d}$

Solution: (b) Here five plates are given, even number of plates are connected together while odd number of plates are connected together so, four capacitors are formed and they are in parallel combination, hence redrawing the figure as shown below.

Capacitance of each

Capacitor is $C = \frac{\varepsilon_0 A}{d}$



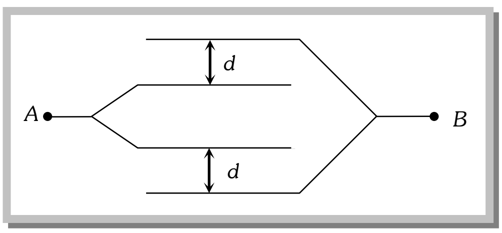
Potential difference across each capacitor is V

So charge on each capacitor $Q = \frac{\varepsilon_0 A}{d} V$

Charge on plate (1) is $+\frac{\varepsilon_0 AV}{d}$

While charge on plate 4 is $-\frac{\varepsilon_0 AV}{d} \times 2 = -\frac{2\varepsilon_0 AV}{d}$.

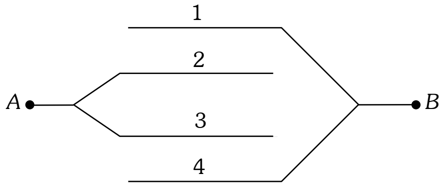
Example: 143 Four plates are arranged as shown in the diagram. If area of each plate is A and the distance between two neighbouring parallel plates is d, then the capacitance of this system between A and B will be



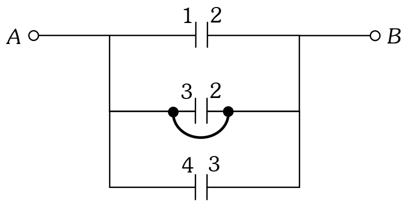
(a) $\frac{4\varepsilon_0 A}{d}$ (b) $\frac{3\varepsilon_0 A}{d}$ (c) $\frac{2\varepsilon_0 A}{d}$ (d) $\frac{\varepsilon_0 A}{d}$

Solution: (c) To solve such type of problem following guidelines should be follows

Guideline 1. Mark the number (1,2,3……..) on the plates

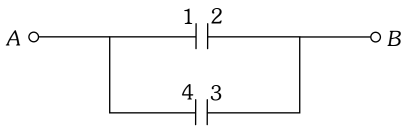


Guideline 2. Rearrange the diagram as shown below



Guideline 3. Since middle capacitor having plates 2, 3 is short circuited so it should be eliminated from the circuit

Hence equivalent capacitance between A and B $C_{AB} = 2\frac{\varepsilon_0 A}{d}$



Tricky example: 17

A capacitor of capacitance $C_1 = 1\mu F$ can withstand maximum voltage $V_1 = 6\ KV$ (*kilo-volt*) and another capacitor of capacitance $C_2 = 3\mu F$ can withstand maximum voltage $V_2 = 4KV$. When the two capacitors are connected in series, the combined system can withstand a maximum voltage of

[MP PET 2001]

(a) $4\ KV$ (b) $6\ KV$ (c) $8\ KV$ (d) $10\ KV$

Solution: (c) We know $Q = CV$

Hence $(Q_1)_{max} = 6\ mC$ while $(Q_2)_{max} = 12\ mC$

However in series charge is same so maximum charge on C_2 will also be $6\ mC$ (and not $12\ mC$) and hence potential difference across C_2 will be $V_2 = \frac{6mC}{3\mu F} = 2KV$ and as in series $V = V_1 + V_2$

So $V_{max} = 6KV + 2KV = 8KV$