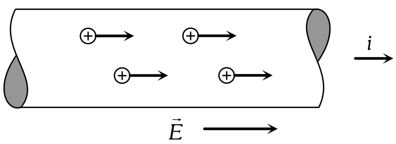
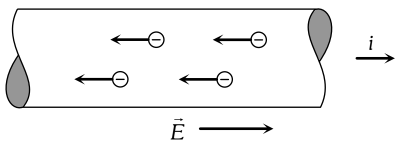
Electric Current.

(1) **Definition :** The time rate of flow of charge through any cross-section is called current. So if through a cross-section, ΔQ charge passes in time Δt then $i_{av} = \frac{\Delta Q}{\Delta t}$ and instantaneous current $i = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$. If flow is uniform then $i = \frac{Q}{t}$. Current is a scalar quantity. It's S.I. unit is *ampere* (*A*) and C.G.S. unit is *emu* and is called *biot* (*Bi*), or *ab ampere*. 1*A* = (1/10) *Bi* (*ab amp.*)

(2) **The direction of current :** The conventional direction of current is taken to be the direction of flow of positive charge, *i.e.* field and is opposite to the direction of flow of negative charge as shown below.

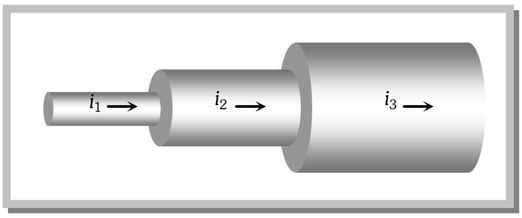




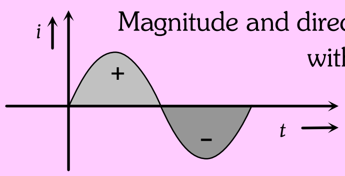
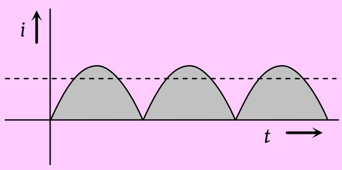
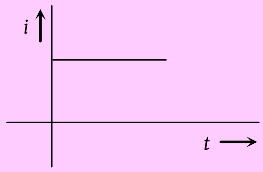
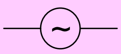
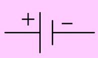
Though conventionally a direction is associated with current (Opposite to the motion of electron), it is not a vector. It is because the current can be added algebraically. Only scalar quantities can be added algebraically not the vector quantities.

(3) **Charge on a current carrying conductor :** In conductor the current is caused by electron (free electron). The no. of electron (negative charge) and proton (positive charge) in a conductor is same. Hence the net charge in a current carrying conductor is zero.

(4) **Current through a conductor of non-uniform cross-section :** For a given conductor current does not change with change in cross-sectional area. In the following figure $i_1 = i_2 = i_3$



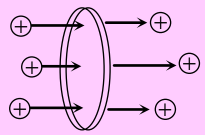
(5) **Types of current :** Electric current is of two type :

Alternating current (ac)	Direct current (dc)
(i) Magnitude and direction both varies with time 	(i) (Pulsating dc)  (Constant dc) 
ac → Rectifier → dc	dc → Inverter → ac
(ii) Shows heating effect only	(ii) Shows heating effect, chemical effect and magnetic effect of current
(iii) It's symbol is 	(iii) It's symbol is 

Note :≅ In our houses ac is supplied at 220*V*, 50*Hz*.

(6) **Current in difference situation :**

(i) **Due to translatory motion of charge**



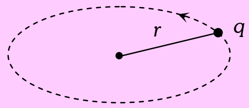
In n particle each having a charge q, pass through a given area in time t then $\boldsymbol{i} = \frac{\boldsymbol{nq}}{\boldsymbol{t}}$

If n particles each having a charge q pass per second per unit area, the current associated with cross-sectional area A is $\boldsymbol{i} = \boldsymbol{nqA}$

If there are n particle per unit volume each having a charge q and moving with velocity v, the current thorough, cross section A is $\boldsymbol{i} = \boldsymbol{nqvA}$

(ii) **Due to rotatory motion of charge**

If a point charge q is moving in a circle of radius r with speed v (frequency ν, angular speed ω and time period T) then corresponding currents $\boldsymbol{i} = \boldsymbol{q\nu} = \frac{\boldsymbol{q}}{\boldsymbol{T}} = \frac{\boldsymbol{qv}}{\boldsymbol{2\pi r}} = \frac{\boldsymbol{q\omega}}{\boldsymbol{2\pi}}$



(iii) **When a voltage V applied across a resistance R** : Current flows through the conductor $i = \frac{V}{R}$ also by definition of power $i = \frac{P}{V}$

(7) **Current carriers :** The charged particles whose flow in a definite direction constitutes the electric current are called current carriers. In different situation current carriers are different.

(i) Solids : In solid conductors like metals current carriers are free electrons.

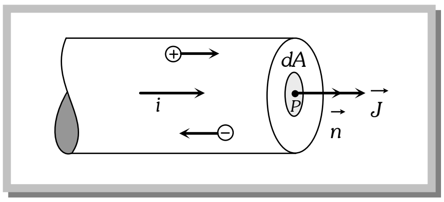
(ii) Liquids : In liquids current carriers are positive and negative ions.

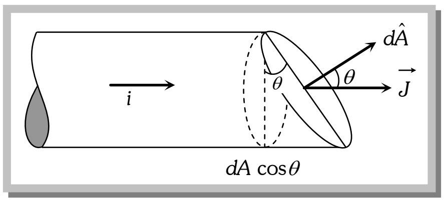
(iii) Gases : In gases current carriers are positive ions and free electrons.

(iv) Semi conductor : In semi conductors current carriers are holes and free electrons.

Current density (J).

In case of flow of charge through a cross-section, current density is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is normal to the direction of charge flow (or current passes) through that point. Current density at point P is given by $\vec{\boldsymbol{J}} = \frac{\boldsymbol{di}}{\boldsymbol{dA}}\vec{\boldsymbol{n}}$





If the cross-sectional area is not normal to the current, the cross-sectional area normal to current in accordance with following figure will be $dA\cos\theta$ and so in this situation:

$$J = \frac{di}{dA\cos\theta} \qquad i.e.\ di = JdA\cos\theta \ \text{ or } \ di = \vec{J}.\vec{dA} \Rightarrow i = \int \vec{J}\cdot\vec{dA}$$

i.e., in terms of current density, current is the flux of current density.

Note : ≅If current density $\vec{J}$ is uniform for a normal cross-section $\vec{A}$ then : $i = \int \vec{J} \cdot \vec{ds} = \vec{J} \cdot \int \vec{ds}$ [as $\vec{J}$ = constant]

or $i = \vec{J} \cdot \vec{A} = JA\cos 0 = JA \Rightarrow \boldsymbol{J} = \frac{\boldsymbol{i}}{\boldsymbol{A}}$ [as $\int \vec{dA} = \vec{A}$ and $\theta = 0°$]

(1) **Unit and dimension :** Current density $\vec{J}$ is a vector quantity having S.I. unit Amp/m^2 and dimension.$[L^{-2}A]$

(2) **Current density in terms of velocity of charge** : In case of uniform flow of charge through a cross-section normal to it as $i = nqvA$ so, $\vec{J} = \frac{i}{A}\vec{n} = (nqv)\vec{n}$ or $\vec{J} = nq\vec{v} = \vec{v}\,(\rho)$ [With $\rho = \frac{\text{charge}}{\text{volume}} = nq$]

i.e., current density at a point is equal to the product of volume charge density with velocity of charge distribution at that point.

(3) **Current density in terms of electric field :** Current density relates with electric field as $\boldsymbol{J} = \sigma \boldsymbol{E} = \frac{\boldsymbol{E}}{\rho}$;

where σ = conductivity and ρ = resistivity or specific resistance of substance.

(i) Direction of current density $\vec{J}$ is same as that of electric field $\vec{E}$.

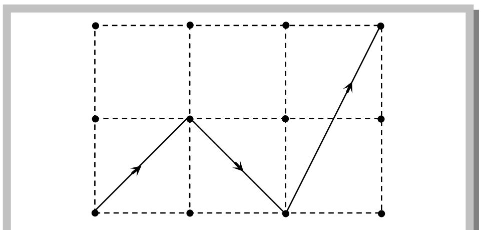
(ii) If electric field is uniform (*i.e.* $\vec{E}$ = constant) current density will be constant [as σ = constant]

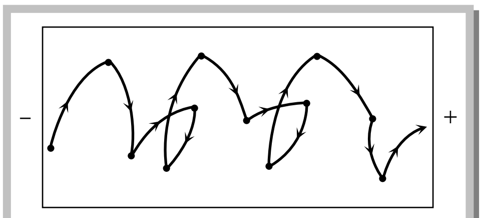
(iii) If electric field is zero (as in electrostatics inside a conductor), current density and hence current will be zero.

Conduction of Current in Metals.

According to modern views, a metal consists of a '*lattice*' of fixed positively charged ions in which billions and billions of free electrons are moving randomly at speed which at room temperature (*i.e.* 300 *K*) in accordance with kinetic theory of gases is given by $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times (1.38 \times 10^{-23}) \times 300}{9.1 \times 10^{-31}}} \simeq 10^5 \, m/s$

The randomly moving free electrons inside the metal collide with the lattice and follow a zig-zag path as shown in figure (A).


(A)

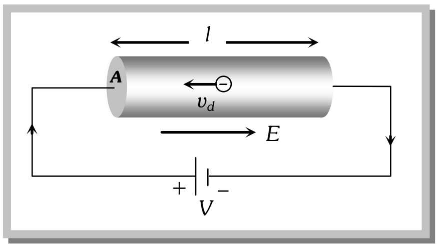


(B)

However, in absence of any electric field due to this random motion, the number of electrons crossing from left to right is equal to the number of electrons crossing from right to left (otherwise metal will not remain equipotential) so the net current through a cross-section is zero.

When an electric field is applied, inside the conductor due to electric force the path of electron in general becomes curved (parabolic) instead of straight lines and electrons drift opposite to the field figure (B). Due to this drift the random motion of electrons get modified and there is a net transfer of electrons across a cross-section resulting in current.

(1) **Drift velocity :** Drift velocity is the average uniform velocity acquired by free electrons inside a metal by the application of an electric field which is responsible for current through it. Drift velocity is very small it is of the order of 10^{-4} *m/s* as compared to thermal speed $(\simeq 10^5\ m/s)$ of electrons at room temperature.



If suppose for a conductor

n = Number of electron per unit volume of the conductor

A = Area of cross-section

V = potential difference across the conductor

E = electric field inside the conductor

i = current, J = current density, ρ = specific resistance, σ = conductivity $\left(\sigma = \frac{1}{\rho}\right)$ then current relates with drift velocity as $\boldsymbol{i = neAv_d}$ we can also write $\boldsymbol{v_d} = \frac{\boldsymbol{i}}{\boldsymbol{neA}} = \frac{\boldsymbol{J}}{\boldsymbol{ne}} = \frac{\boldsymbol{\sigma E}}{\boldsymbol{ne}} = \frac{\boldsymbol{E}}{\boldsymbol{\rho ne}} = \frac{\boldsymbol{V}}{\boldsymbol{\rho l ne}}$.

Note : ≅The direction of drift velocity for electron in a metal is opposite to that of applied electric field (*i.e.* current density $\vec{J}$).

≅ $v_d \propto E$ *i.e.*, greater the electric field, larger will be the drift velocity.

≅ When a steady current flows through a conductor of non-uniform cross-section drift velocity varies inversely with area of cross-section $\left(v_d \propto \frac{1}{A}\right)$

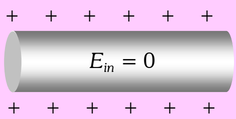
≅ If diameter of a conductor is doubled, then drift velocity of electrons inside it will not change.

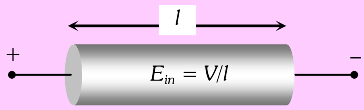
(2) **Relaxation time (τ) :** The time interval between two successive collisions of electrons with the positive ions in the metallic lattice is defined as relaxation time $\tau = \frac{\text{mean free path}}{\text{r.m.s. velocity of electrons}} = \frac{\lambda}{v_{rms}}$ with rise in temperature v_{rms} increases consequently τ decreases.

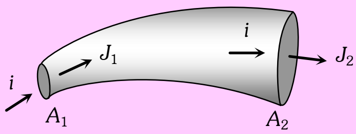
(3) **Mobility :** Drift velocity per unit electric field is called mobility of electron *i.e.* $\mu = \frac{v_d}{E}$. It's unit is $\frac{m^2}{volt - \sec}$.

Concepts

- *Human body, though has a large resistance of the order of kΩ (say 10 kΩ), is very sensitive to minute currents even as low as a few mA. Electrocution, excites and disorders the nervous system of the body and hence one fails to control the activity of the body.*
- *1 ampere of current means the flow of 6.25×10^{18} electrons per second through any cross-section of the conductors.*
- *dc flows uniformly throughout the cross-section of conductor while ac mainly flows through the outer surface area of the conductor. This is known as skin effect.*
- *It is worth noting that electric field inside a charged conductor is zero, but it is non zero inside a current carrying conductor and is given by $E = \frac{V}{l}$ where V = potential difference across the conductor and l = length of the conductor. Electric field out side the current carrying is zero.*







- For a given conductor $JA = i$ = constant so that $J \propto \frac{1}{A}$ i.e., $J_1 A_1 = J_2 A_2$; this is called equation of continuity
- If cross-section is constant, $I \propto J$ i.e. for a given cross-sectional area, greater the current density, larger will be current.
- The drift velocity of electrons is small because of the frequent collisions suffered by electrons.
- The small value of drift velocity produces a large amount of electric current, due to the presence of extremely large number of free electrons in a conductor. The propagation of current is almost at the speed of light and involves electromagnetic process. It is due to this reason that the electric bulb glows immediately when switch is on.
- In the absence of electric field, the paths of electrons between successive collisions are straight line while in presence of electric field the paths are generally curved.
- Free electron density in a metal is given by $n = \frac{N_A x d}{A}$ where N_A = Avogrado number, x = number of free electrons per atom, d = density of metal and A = Atomic weight of metal.

Example

Example: 1 The potential difference applied to an X-ray tube is 5 *KV* and the current through it is 3.2 *mA*. Then the number of electrons striking the target per second is **[IIT-JEE (Screening) 2002]**

(a) 2×10^{16} (b) 5×10^{6} (c) 1×10^{17} (d) 4×10^{15}

Solution : (a) $i = \frac{q}{t} = \frac{ne}{t}$ $\Rightarrow n = \frac{it}{e} = \frac{3.2 \times 10^{-3} \times 1}{1.6 \times 10^{-19}} = 2 \times 10^{16}$

Example: 2 A beam of electrons moving at a speed of 10^6 *m/s* along a line produces a current of 1.6×10^{-6} *A*. The number of electrons in the 1 *metre* of the beam is **[CPMT 2000]**

(a) 10^6 (b) 10^7 (c) 10^{13} (d) 10^{19}

Solution : (b) $i = \frac{q}{t} = \frac{q}{(x/v)} = \frac{qv}{x} = \frac{nev}{x} \Rightarrow n = \frac{ix}{ev} = \frac{1.6 \times 10^{-6} \times 1}{1.6 \times 10^{-19} \times 10^{6}} = 10^7$

Example: 3 In the Bohr's model of hydrogen atom, the electrons moves around the nucleus in a circular orbit of a radius 5×10^{-11} *metre*. It's time period is 1.5×10^{-16} *sec*. The current associated is **[MNR 1992]**

(a) Zero (b) 1.6×10^{-19} *A* (c) 0.17 *A* (d) 1.07×10^{-3} *A*

Solution : (d) $i = \frac{q}{T} = \frac{1.6 \times 10^{-19}}{1.5 \times 10^{-16}} = 1.07 \times 10^{-3} A$

Example: 4 An electron is moving in a circular path of radius 5.1×10^{-11} *m* at a frequency of 6.8×10^{15} revolution/sec. The equivalent current is approximately **[MP PET 2000 Similar to EAMCET (Med.) 2000]**

(a) $5.1 \times 10^{-3} A$ (b) $6.8 \times 10^{-3} A$ (c) $1.1 \times 10^{-3} A$ (d) $2.2 \times 10^{-3} A$

Solution : (c) $\nu = 6.8 \times 10^{15} \Rightarrow T = \frac{1}{6.8 \times 10^{15}} \sec \Rightarrow i = \frac{Q}{T} = 1.6 \times 10^{-19} \times 6.8 \times 10^{15} = 1.1 \times 10^{-3} A$

Example: 5 A copper wire of length 1*m* and radius 1*mm* is joined in series with an iron wire of length 2*m* and radius 3*mm* and a current is passed through the wire. The ratio of current densities in the copper and iron wire is **[MP PMT 1994]**

(a) 18 : 1 (b) 9 : 1 (c) 6 : 1 (d) 2 : 3

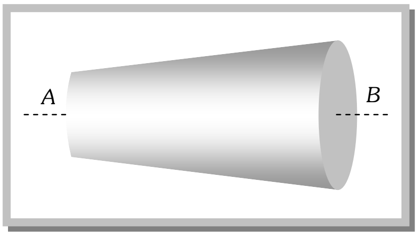
Solution : (b) We know $J = \frac{i}{A}$ when i = constant $J \propto \frac{1}{A} \Rightarrow \frac{J_c}{J_i} = \frac{A_i}{A_c} = \left(\frac{r_i}{r_c}\right)^2 = \left(\frac{3}{1}\right)^2 = \frac{9}{1}$

Example: 6 A conducting wire of cross-sectional area 1 cm^2 has 3×10^{23} m^{-3} charge carriers. If wire carries a current of 24 *mA*, the drift speed of the carrier is **[UPSEAT 2001]**

(a) 5×10^{-6} *m/s* (b) 5×10^{-3} *m/s* (c) 0.5 *m/s* (d) 5×10^{-2} *m/s*

Solution : (b) $v_d = \frac{i}{neA} = \frac{24 \times 10^{-3}}{3 \times 10^{23} \times 1.6 \times 10^{-19} \times 10^{-4}} = 5 \times 10^{-3} m/s$

Example: 7 A wire has a non-uniform cross-sectional area as shown in figure. A steady current *i* flows through it. Which one of the following statement is correct



(a) The drift speed of electron is constant (b) The drift speed increases on moving from *A* to *B*

(c) The drift speed decreases on moving from *A* to *B* (d) The drift speed varies randomly

Solution : (c) For a conductor of non-uniform cross-section $v_d \propto \frac{1}{\text{Area of cross-section}}$

Example: 8 In a wire of circular cross-section with radius *r*, free electrons travel with a drift velocity *v*, when a current *i* flows through the wire. What is the current in another wire of half the radius and of the some material when the drift velocity is 2*v* **[MP PET 1997]**

(a) 2*i* (b) *i* (c) *i*/2 (d) *i*/4

Solution : (c) $i = neAv_d = ne\pi r^2 v$ and $i' = ne\pi\left(\frac{r}{2}\right)^2 .2v = \frac{ne\pi r^2 v}{2} = \frac{i}{2}$

Example: 9 A potential difference of *V* is applied at the ends of a copper wire of length *l* and diameter *d*. On doubling only *d*, drift velocity **[MP PET 1995]**

(a) Becomes two times (b) Becomes half (c) Does not change (d) Becomes one fourth

Solution : (c) Drift velocity doesn't depends upon diameter.

***Example*: 10** A current flows in a wire of circular cross-section with the free electrons travelling with a mean drift velocity v. If an equal current flows in a wire of twice the radius new mean drift velocity is

(a) v (b) $\frac{v}{2}$ (c) $\frac{v}{4}$ (d) None of these

Solution : (c) By using $v_d = \frac{i}{neA} \Rightarrow v_d \propto \frac{1}{A} \Rightarrow v' = \frac{v}{4}$

***Example*: 11** Two wires *A* and *B* of the same material, having radii in the ratio 1 : 2 and carry currents in the ratio 4 : 1. The ratio of drift speeds of electrons in *A* and *B* is

(a) 16 : 1 (b) 1 : 16 (c) 1 : 4 (d) 4 : 1

Solution : (a) As $i = neAv_d \Rightarrow \frac{i_1}{i_2} = \frac{A_1}{A_2} \times \frac{v_{d_1}}{v_{d_2}} = \frac{r_1^2}{r_2^2} \cdot \frac{v_{d_1}}{v_{d_2}} \Rightarrow \frac{v_{d_1}}{v_{d_2}} = \frac{16}{1}$

Tricky example: 1

In a neon discharge tube 2.9×10^{18} Ne^+ ions move to the right each second while 1.2×10^{18} electrons move to the left per second. Electron charge is 1.6×10^{-19} *C*. The current in the discharge tube

[MP PET 1999]

(a) 1 *A* towards right (b) 0.66 *A* towards right (c) 0.66 *A* towards left (d) Zero

Solution: (b) Use following trick to solve such type of problem.

Trick : In a discharge tube positive ions carry q units of charge in t seconds from anode to cathode and negative carriers (electrons) carry the same amount of charge from cathode to anode in t′ second.

The current in the tube is $\boldsymbol{i} = \frac{\boldsymbol{q}}{\boldsymbol{t}} + \frac{\boldsymbol{q'}}{\boldsymbol{t'}}$.

Hence in this question current $i = \frac{2.9 \times 10^{18} \times e}{1} + \frac{1.2 \times 10^{18} \times e}{1} = 0.66A$ towards right.

Tricky example: 2

If the current flowing through copper wire of 1 *mm* diameter is 1.1 *amp*. The drift velocity of electron is (Given density of *Cu* is 9 gm/cm^3, atomic weight of *Cu* is 63 *grams* and one free electron is contributed by each atom)

[J&K CEET 2000]

(a) 0.1 *mm/sec* (b) 0.2 *mm/sec* (c) 0.3 *mm/sec* (d) 0.5 *mm/sec*

Solution: (a) 6.023×10^{23} atoms has mass $= 63 \times 10^{-3}$ *kg*

So no. of atoms per m^3 $= n = \frac{6.023 \times 10^{23}}{63 \times 10^{-3}} \times 9 \times 10^3 = 8.5 \times 10^{28}$

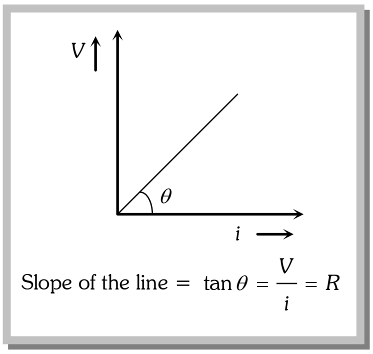
$$v_d = \frac{i}{neA} = \frac{1.1}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times \pi \times (0.5 \times 10^{-3})^2} = 0.1 \times 10^{-3}\, m/\sec = 0.1\, mm/\sec$$

Ohm's Law.

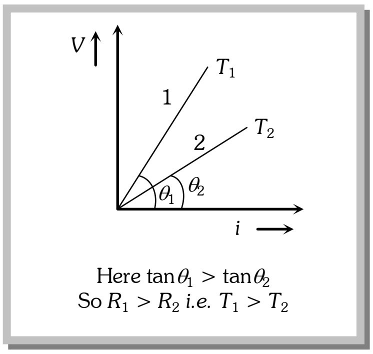
If the physical circumstances of the conductor (length, temperature, mechanical strain etc.) remains constant, then the current flowing through the conductor is directly proportional to the potential difference across it's two ends i.e. $\boldsymbol{i} \propto \boldsymbol{V}$

$\Rightarrow \boldsymbol{V} = \boldsymbol{iR}$ or $\dfrac{V}{i} = R$; where R is a proportionality constant, known as electric resistance.

(1) Ohm's law is not a universal law, the substance which obeys ohm's law are known as ohmic substance for such ohmic substances graph between V and i is a straight line as shown. At different temperatures V-i curves are different.

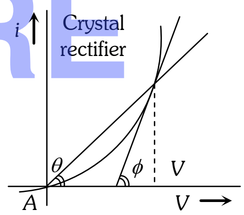


Slope of the line = $\tan\theta = \dfrac{V}{i} = R$



Here $\tan\theta_1 > \tan\theta_2$
So $R_1 > R_2$ i.e. $T_1 > T_2$

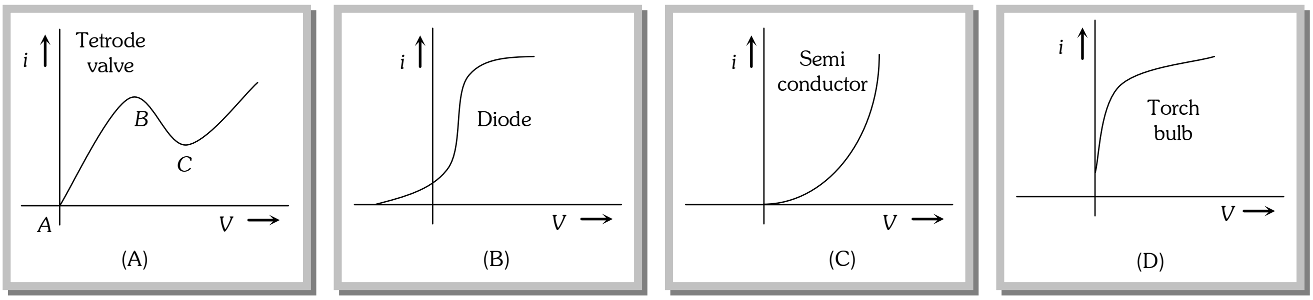
(2) The device or substances which doesn't obey ohm's law e.g. gases, crystal rectifiers, thermoionic valve, transistors etc. are known as non-ohmic or non-linear conductors. For these V-i curve is not linear. In these situation the ratio between voltage and current at a particular voltage is known as static resistance. While the rate of change of voltage to change in current is known as dynamic resistance.



$$R_{st} = \frac{V}{i} = \frac{1}{\tan\theta}$$

while $$R_{dyn} = \frac{\Delta V}{\Delta I} = \frac{1}{\tan\phi}$$

(3) Some other non-ohmic graphs are as follows :



Resistance.

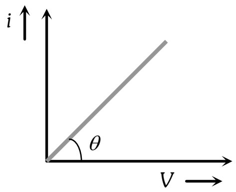
(1) **Definition :** The property of substance by virtue of which it opposes the flow of current through it, is known as the resistance.

(2) **Cause of resistance of a conductor :** It is due to the collisions of free electrons with the ions or atoms of the conductor while drifting towards the positive end of the conductor.

(3) **Formula of resistance :** For a conductor if l = length of a conductor A = Area of cross-section of conductor, n = No. of free electrons per unit volume in conductor, τ = relaxation time then resistance of conductor $\boldsymbol{R} = \rho\frac{\boldsymbol{l}}{\boldsymbol{A}} = \frac{\boldsymbol{m}}{\boldsymbol{ne^2\tau}}.\frac{\boldsymbol{l}}{\boldsymbol{A}}$; where ρ = resistivity of the material of conductor

(4) **Unit and dimension :** It's S.I. unit is *Volt/Amp.* or *Ohm* (Ω). Also 1 *ohm* $= \frac{1 volt}{1 Amp} = \frac{10^8 \, emu \text{ of potential}}{10^{-1} \, emu \text{ of current}} =$ 10^9 *emu* of resistance. It's dimension is $[ML^2T^{-3}A^{-2}]$.

(5) **Conductance (C) :** Reciprocal of resistance is known as conductance. $C = \frac{1}{R}$ It's unit is $\frac{1}{\Omega}$ or Ω^{-1} or "Siemen".

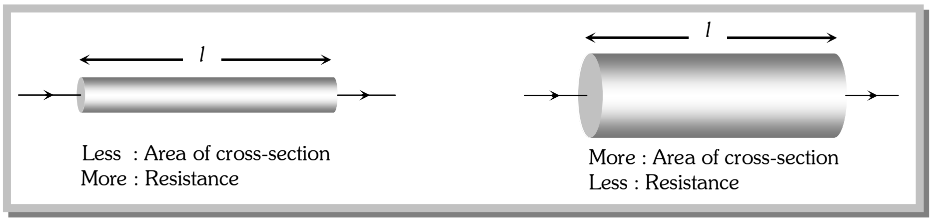


Slope $= \tan\theta = \frac{i}{V} = \frac{1}{R} = C$

(6) **Dependence of resistance :** Resistance of a conductor depends on the following factors.

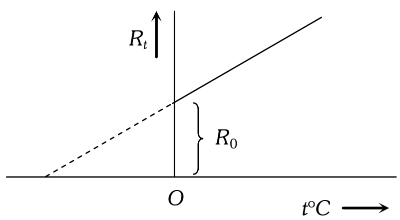
(i) Length of the conductor : Resistance of a conductor is directly proportional to it's length *i.e.* $\boldsymbol{R \propto l}$ *e.g.* a conducting wire having resistance R is cut in n equal parts. So resistance of each part will be $\frac{R}{n}$.

(ii) Area of cross-section of the conductor : Resistance of a conductor is inversely proportional to it's area of cross-section *i.e.* $R \propto \frac{1}{A}$



(iii) Material of the conductor : Resistance of conductor also depends upon the nature of material *i.e.* $R \propto \frac{1}{n}$, for different conductors n is different. Hence R is also different.

(iv) Temperature : We know that $R = \frac{m}{ne^2\tau}.\frac{l}{A} \Rightarrow R \propto \frac{l}{\tau}$ when a metallic conductor is heated, the atom in the metal vibrate with greater amplitude and frequency about their mean positions. Consequently the number of collisions between free electrons and atoms increases. This reduces the relaxation time τ and increases the value of resistance R *i.e.* for a conductor **Resistance $\propto$ temperature**.



If $\quad R_0$ = resistance of conductor at 0°C

$\quad R_t$ = resistance of conductor at t°C

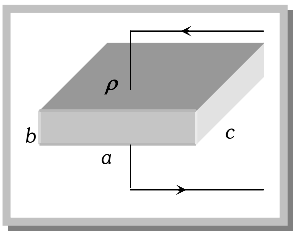
and α, β = temperature co-efficient of resistance (unit $\rightarrow$ per°C)

then $R_t = R_0(1 + \alpha t + \beta t^2)$ for $t > 300$°C and $\boldsymbol{R_t = R_0(1 + \alpha t)}$ for $t \leq 300$°C or $\alpha = \frac{R_t - R_0}{R_0 \times t}$

Note : ≅ If R_1 and R_2 are the resistances at t_1°C and t_2°C respectively then $\frac{R_1}{R_2} = \frac{1+\alpha t_1}{1+\alpha t_2}$.

≅ The value of α is different at different temperature. Temperature coefficient of resistance averaged over the temperature range t_1°C to t_2°C is given by $\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$ which gives $R_2 = R_1\,[1 + \alpha\,(t_2 - t_1)]$. This formula gives an approximate value.

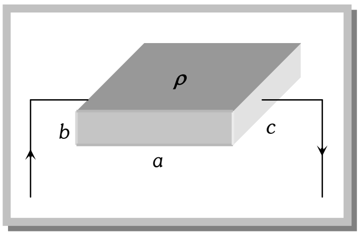
(v) **Resistance according to potential difference :** Resistance of a conducting body is not unique but depends on it's length and area of cross-section *i.e.* how the potential difference is applied. See the following figures



Length = b

Area of cross-section = $a \times c$

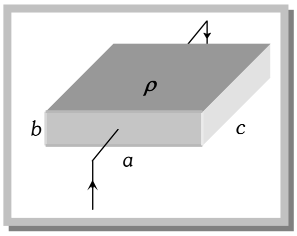
Resistance $R = \rho\left(\frac{b}{a \times c}\right)$



Length = a

Area of cross-section = $b \times c$

Resistance $R = \rho\left(\frac{a}{b \times c}\right)$



Length = c

Area of cross-section = $a \times b$

Resistance $R = \rho\left(\frac{c}{a \times b}\right)$

(7) **Variation of resistance of some electrical material with temperature :**

(i) Metals : For metals their temperature coefficient of resistance $\alpha > 0$. So resistance increases with temperature.

Physical explanation : Collision frequency of free electrons with the immobile positive ions increases

(ii) Solid non-metals : For these $\alpha = 0$. So resistance is independence of temperature.

Physical explanation : Complete absence of free electron.

(iii) Semi-conductors : For semi-conductor $\alpha < 0$ *i.e.* resistance decreases with temperature rise.

Physical explanation : Covalent bonds breaks, liberating more free electron and conduction increases.

(iv) Electrolyte : For electrolyte $\alpha < 0$ *i.e.* resistance decreases with temperature rise.

Physical explanation : The degree of ionisation increases and solution becomes less viscous.

(v) Ionised gases : For ionised gases $\alpha < 0$ *i.e.* resistance decreases with temperature rise.

Physical explanation : Degree of ionisation increases.

(vi) Alloys : For alloys α has a small positive values. So with rise in temperature resistance of alloys is almost constant. Further alloy resistances are slightly higher than the pure metals resistance.

Alloys are used to made standard resistances, wires of resistance box, potentiometer wire, meter bridge wire *etc*.

Commonly used alloys are : Constantan, mangnin, Nichrome etc.

(vii) Super conductors : At low temperature, the resistance of certain substances becomes exactly zero. (*e.g.* *Hg* below 4.2 *K* or *Pb* below 7.2 *K*).

These substances are called super conductors and phenomenon super conductivity. The temperature at which resistance becomes zero is called critical temperature and depends upon the nature of substance.

Resistivity or Specific Resistance (ρ).

(1) **Definition :** From $R = \rho \frac{l}{A}$; If l = 1*m*, A = 1 m^2 then $\boldsymbol{R} = \theta$ *i.e.* resistivity is numerically equal to the resistance of a substance having unit area of cross-section and unit length.

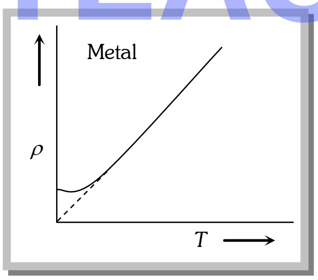
(2) **Unit and dimension :** It's S.I. unit is *ohm* × *m* and dimension is $[ML^3T^{-3}A^{-2}]$

(3) **It's formula** : $\rho = \frac{m}{ne^2\tau}$

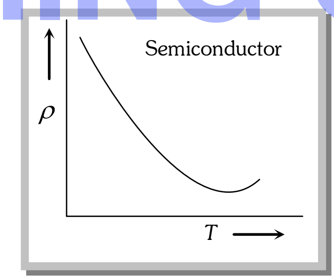
(4) **It's dependence :** Resistivity is the intrinsic property of the substance. It is independent of shape and size of the body (*i.e.* l and A). It depends on the followings :

(i) Nature of the body : For different substances their resistivity also different *e.g.* ρ_{silver} = minimum = 1.6×10^{-8} Ω-*m* and $\rho_{fused\ quartz}$ = maximum ≈ 10^{16} Ω-*m*

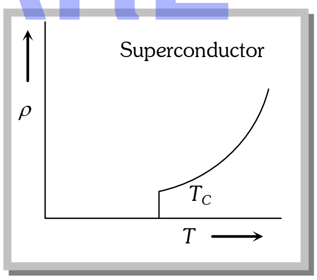
(ii) Temperature : Resistivity depends on the temperature. For metals $\rho_t = \rho_0 (1 + \alpha\Delta t)$ *i.e.* resitivity increases with temperature.



ρ increases with temperature



ρ decreases with temperature



ρ decreases with temperature and becomes zero at a certain temperature

(iii) Impurity and mechanical stress : Resistivity increases with impurity and mechanical stress.

(iv) Effect of magnetic field : Magnetic field increases the resistivity of all metals except iron, cobalt and nickel.

(v) Effect of light : Resistivity of certain substances like selenium, cadmium, sulphides is inversely proportional to intensity of light falling upon them.

(5) **Resistivity of some electrical material :** $\rho_{insulator}$ (Maximum for fused quartz) > ρ_{alloy} > $\rho_{semi\text{-}conductor}$ > $\rho_{conductor}$ (Minimum for silver)

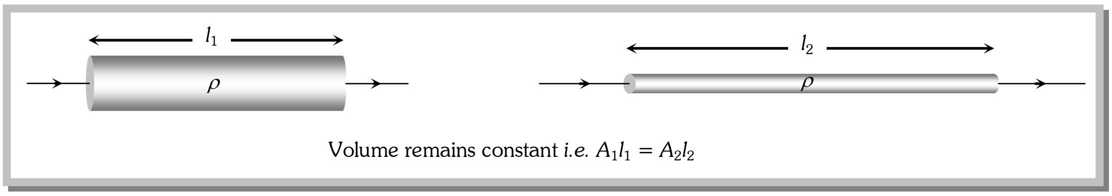
Note : ≅Reciprocal of resistivity is called conductivity (σ) *i.e.* $\sigma = \frac{1}{\rho}$ with unit *mho*/*m* and dimensions $[M^{-1}L^{-3}T^3A^2]$.

Stretching of Wire.

If a conducting wire stretches, it's length increases, area of cross-section decreases so resistance increases but volume remain constant.

Suppose for a conducting wire before stretching it's length = l_1, area of cross-section = A_1, radius = r_1, diameter = d_1, and resistance $R_1 = \rho \frac{l_1}{A_1}$

Before stretching After stretching



After stretching length = l_2, area of cross-section = A_2, radius = r_2, diameter = d_2 and resistance = $R_2 = \rho \frac{l_2}{A_2}$

Ratio of resistances $$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^4 = \left(\frac{d_2}{d_1}\right)^4$$

(1) If length is given then $R \propto l^2 \Rightarrow \frac{R_1}{R_2} = \left(\frac{l_1}{l_2}\right)^2$

(2) If radius is given then $R \propto \frac{1}{r^4} \Rightarrow \frac{R_1}{R_2} = \left(\frac{r_2}{r_1}\right)^4$

Note : ≅ After stretching if length increases by n times then resistance will increase by n^2 times *i.e.* $R_2 = n^2 R_1$. Similarly if radius be reduced to $\frac{1}{n}$ times then area of cross-section decreases $\frac{1}{n^2}$ times so the resistance becomes n^4 times *i.e.* $R_2 = n^4 R_1$.

≅ After stretching if length of a conductor increases by x% then resistance will increases by $2x$ % (valid only if $x < 10\%$)

Various Electrical Conducting Material For Specific Use.

(1) **Filament of electric bulb :** Is made up of tungsten which has high resistivity, high melting point.

(2) **Element of heating devices (such as heater, geyser or press) :** Is made up of nichrome which has high resistivity and high melting point.

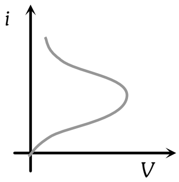
(3) **Resistances of resistance boxes (standard resistances) :** Are made up of manganin, or constantan as these materials have moderate resistivity which is practically independent of temperature so that the specified value of resistance does not alter with minor changes in temperature.

(4) **Fuse-wire :** Is made up of tin-lead alloy (63% tin + 37% lead). It should have low melting point and high resistivity. It is used in series as a safety device in an electric circuit and is designed so as to melt and thereby open

the circuit if the current exceeds a predetermined value due to some fault. The function of a fuse is independent of its length.

Safe current of fuse wire relates with it's radius as $\boldsymbol{i} \propto \boldsymbol{r}^{3/2}$.

(5) **Thermistors** : A thermistor is a heat sensitive resistor usually prepared from oxides of various metals such as nickel, copper, cobalt, iron etc. These compounds are also semi-conductor. For thermistors α is very high which may be positive or negative. The resistance of thermistors changes very rapidly with change of temperature.



Thermistors are used to detect small temperature change and to measure very low temperature.

Concepts

- *In the absence of radiation loss, the time in which a fuse will melt does not depends on it's length but varies with radius as* $\boldsymbol{t} \propto \boldsymbol{r}^4$.
- *If length (l) and mass (m) of a conducting wire is given then* $\boldsymbol{R} \propto \dfrac{\boldsymbol{l}^2}{\boldsymbol{m}}$.
- *Macroscopic form of Ohm's law is* $R = \dfrac{V}{i}$, *while it's microscopic form is* $J = \sigma E$.

Example

Example: 12 Two wires of resistance R_1 and R_2 have temperature co-efficient of resistance α_1 and α_2 respectively. These are joined in series. The effective temperature co-efficient of resistance is **[MP PET 2003]**

(a) $\dfrac{\alpha_1 + \alpha_2}{2}$ (b) $\sqrt{\alpha_1 \alpha_2}$ (c) $\dfrac{\alpha_1 R_1 + \alpha_2 R_2}{R_1 + R_2}$ (d) $\dfrac{\sqrt{R_1 R_2 \alpha_1 \alpha_2}}{\sqrt{R_1^2 + R_2^2}}$

Solution : (c) Suppose at $t°C$ resistances of the two wires becomes R_{1t} and R_{2t} respectively and equivalent resistance becomes R_t. In series grouping $R_t = R_{1t} + R_{2t}$, also $R_{1t} = R_1(1 + \alpha_1 t)$ and $R_{2t} = R_2(1 + \alpha_2 t)$

$$R_t = R_1(1 + \alpha_1 t) + R_2(1 + \alpha_2 t) = (R_1 + R_2) + (R_1\alpha_1 + R_2\alpha_2)t = (R_1 + R_2)\left[1 + \frac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2} t\right].$$

Hence effective temperature co-efficient is $\dfrac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2}$.

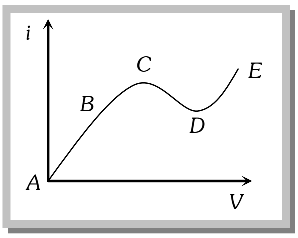
Example: 13 From the graph between current *i* & voltage *V* shown, identity the portion corresponding to negative resistance **[CBSE PMT 1997]**

(a) *DE*

(b) *CD*

(c) *BC*

(d) *AB*



Solution : (b) $R = \frac{\Delta V}{\Delta I}$, in the graph *CD* has only negative slope. So in this portion *R* is negative.

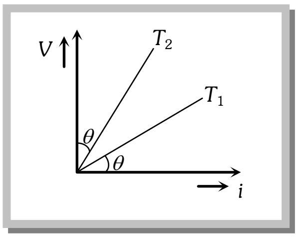
***Example*: 14** A wire of length *L* and resistance *R* is streched to get the radius of cross-section halfed. What is new resistance

[NCERT 1974; CPMT 1994; AIIMS 1997; KCET 1999; Haryana PMT 2000; UPSEAT 2001]

(a) 5 *R* (b) 8 *R* (c) 4 *R* (d) 16 *R*

Solution : (d) By using $\frac{R_1}{R_2} = \left(\frac{r_2}{r_1}\right)^4 \Rightarrow \frac{R}{R'} = \left(\frac{r/2}{r}\right)^4 \Rightarrow R' = 16R$

***Example*: 15** The *V-i* graph for a conductor at temperature T_1 and T_2 are as shown in the figure. $(T_2 - T_1)$ is proportional to



(a) $\cos 2\theta$

(b) $\sin\theta$

(c) $\cot 2\theta$

(d) $\tan\theta$

Solution : (c) As we know, for conductors resistance ∝ Temperature.

From figure $R_1 \propto T_1 \Rightarrow \tan\theta \propto T_1 \Rightarrow \tan\theta = kT_1$ .... (i) (k = constant)

and $R_2 \propto T_2 \Rightarrow \tan(90° - \theta) \propto T_2 \Rightarrow \cot\theta = kT_2$ ....(ii)

From equation (i) and (ii) $k(T_2 - T_1) = (\cot\theta - \tan\theta)$

$$(T_2 - T_1) = \left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right) = \frac{(\cos^2\theta - \sin^2\theta)}{\sin\theta\cos\theta} = \frac{\cos 2\theta}{\sin\theta\cos\theta} = 2\cot 2\theta \Rightarrow (T_2 - T_1) \propto \cot 2\theta$$

***Example*: 16** The resistance of a wire at 20°*C* is 20 Ω and at 500°*C* is 60Ω. At which temperature resistance will be 25Ω

[UPSEAT 1999]

(a) 50°*C* (b) 60°*C* (c) 70°*C* (d) 80°*C*

Solution : (d) By using $\frac{R_1}{R_2} = \frac{(1+\alpha t_1)}{(1+\alpha t_2)} \Rightarrow \frac{20}{60} = \frac{1+20\alpha}{1+500\alpha} \Rightarrow \alpha = \frac{1}{220}$

Again by using the same formula for 20Ω and 25Ω $\Rightarrow \frac{20}{25} = \frac{\left(1+\frac{1}{220}\times 20\right)}{\left(1+\frac{1}{220}\times t\right)} \Rightarrow t = 80°C$

***Example*: 17** The specific resistance of manganin is 50×10^{-8} Ω*m*. The resistance of a manganin cube having length 50 *cm* is

[MP PMT 1978]

(a) 10^{-6} Ω (b) 2.5×10^{-5} Ω (c) 10^{-8} Ω (d) 5×10^{-4} Ω

Solution : (a) $R = \rho\frac{l}{A} = \frac{50\times 10^{-8}\times 50\times 10^{-2}}{(50\times 10^{-2})^2} = 10^{-6}\,\Omega$

***Example*: 18** A rod of certain metal is 1 *m* long and 0.6 *cm* in diameter. It's resistance is 3×10^{-3}Ω. A disc of the same metal is 1 *mm* thick and 2 *cm* in diameter, what is the resistance between it's circular faces.

(a) 1.35×10^{-6}Ω (b) 2.7×10^{-7} Ω (c) 4.05×10^{-6}Ω (d) 8.1×10^{-6} Ω

Solution : (b) By using $R = \rho.\frac{l}{A}; \frac{R_{disc}}{R_{rod}} = \frac{l_{disc}}{l_{rod}} \times \frac{A_{rod}}{A_{disc}} \Rightarrow \frac{R_{disc}}{3\times 10^{-3}} = \frac{10^{-3}}{1} \times \frac{\pi(0.3\times 10^{-2})^2}{\pi(10^{-2})^2} \Rightarrow R_{disc} = 2.7 \times 10^{-7}\Omega$.

***Example*: 19** An aluminium rod of length 3.14 m is of square cross-section $3.14 \times 3.14\ mm^2$. What should be the radius of 1 m long another rod of same material to have equal resistance

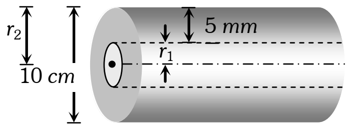
(a) 2 mm (b) 4 mm (c) 1 mm (d) 6 mm

Solution : (c) By using $R = \rho.\frac{l}{A} \Rightarrow l \propto A \Rightarrow \frac{3.14}{1} = \frac{3.14 \times 3.14 \times 10^{-6}}{\pi \times r^2} \Rightarrow r = 10^{-3}\ m = 1\ mm$

***Example*: 20** Length of a hollow tube is 5m, it's outer diameter is 10 cm and thickness of it's wall is 5 mm. If resistivity of the material of the tube is $1.7 \times 10^{-8}\ \Omega \times m$ then resistance of tube will be

(a) $5.6 \times 10^{-5}\ \Omega$ (b) $2 \times 10^{-5}\ \Omega$ (c) $4 \times 10^{-5}\ \Omega$ (d) None of these

Solution : (a) By using $R = \rho.\frac{l}{A}$; here $A = \pi(r_2^2 - r_1^2)$



Outer radius $r_2 = 5cm$

Inner radius $r_1 = 5 - 0.5 = 4.5\ cm$

So $R = 1.7 \times 10^{-8} \times \frac{5}{\pi\{(5 \times 10^{-2})^2 - (4.5 \times 10^{-2})^2\}} = 5.6 \times 10^{-5}\ \Omega$

***Example*: 21** If a copper wire is stretched to make it 0.1% longer, the percentage increase in resistance will be

[MP PMT 1996, 2000; UPSEAT 1998; MNR 1990]

(a) 0.2 (b) 2 (c) 1 (d) 0.1

Solution : (a) In case of streching $R \propto l^2$ So $\frac{\Delta R}{R} = 2\frac{\Delta l}{l} = 2 \times 0.1 = 0.2$

***Example*: 22** The temperature co-efficient of resistance of a wire is 0.00125/°C. At 300 K. It's resistance is 1Ω. The resistance of the wire will be 2Ω at **[MP PMT 2001; IIT 1980]**

(a) 1154 K (b) 1127 K (c) 600 K (d) 1400 K

Solution: (b) By using $R_t = R_o\,(1 + \alpha\Delta t) \Rightarrow \frac{R_1}{R_2} = \frac{1 + \alpha t_1}{1 + \alpha t_2}$ So $\frac{1}{2} = \frac{1 + (300 - 273)\alpha}{1 + \alpha t_2} \Rightarrow t_2 = 854°C = 1127\ K$

***Example*: 23** Equal potentials are applied on an iron and copper wire of same length. In order to have same current flow in the wire, the ratio $\left(\frac{r_{iron}}{r_{copper}}\right)$ of their radii must be [Given that specific resistance of iron = $1.0 \times 10^{-7}\ \Omega m$ and that of copper = $1.7 \times 10^{-8}\ \Omega m$] **[MP PMT 2000]**

(a) About 1.2 (b) About 2.4 (c) About 3.6 (d) About 4.8

Solution: (b) V = constant., i = constant. So R = constant

$\Rightarrow \frac{P_i l_i}{A_i} = \frac{\rho_{Cu} l_{Cu}}{A_{Cu}} \Rightarrow \frac{\rho_i l_i}{r_i^2} = \frac{\rho_{Cu} l_{Cu}}{r_{Cu}^2}$ $\Rightarrow \frac{r_i}{r_{Cu}} = \sqrt{\frac{\rho_i}{\rho_{Cu}}} = \sqrt{\frac{1.0 \times 10^{-7}}{1.7 \times 10^{-8}}} = \sqrt{\frac{100}{17}} \approx 2.4$

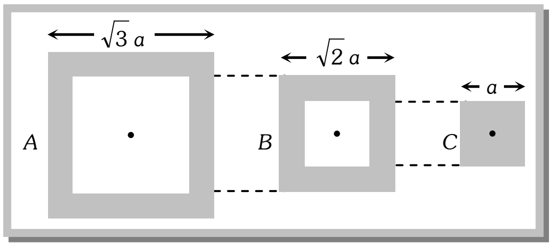
***Example*: 24** Masses of three wires are in the ratio 1 : 3 : 5 and their lengths are in the ratio 5 : 3 : 1. The ratio of their electrical resistance is **[AFMC 2000]**

(a) 1 : 3 : 5 (b) 5 : 3 : 1 (c) 1 : 15 : 125 (d) 125 : 15 : 1

Solution: (d) $R = \rho\frac{l}{A} = \rho\frac{l^2}{V} = \rho\frac{l^2}{m}\sigma$ $\left(\because \sigma = \frac{m}{V}\right)$

$$R_1 : R_2 : R_3 = \frac{l_1^2}{m_1} : \frac{l_2^2}{m_2} : \frac{l_3^2}{m_3} = 25 : \frac{9}{3} : \frac{1}{5} = 125 : 15 : 1$$

Example: 25 Following figure shows cross-sections through three long conductors of the same length and material, with square cross-section of edge lengths as shown. Conductor B will fit snugly within conductor A, and conductor C will fit snugly within conductor B. Relationship between their end to end resistance is



(a) $R_A = R_B = R_C$

(b) $R_A > R_B > R_C$

(c) $R_A < R_B < R$

(d) Information is not sufficient

Solution : (a) All the conductors have equal lengths. Area of cross-section of A is $\left\{(\sqrt{3}\,a)^2 - (\sqrt{2}\,a)^2\right\} = a^2$

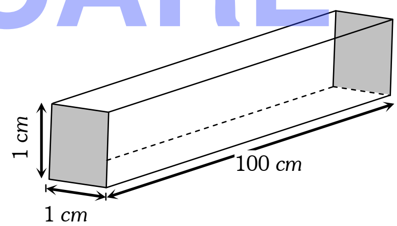
Similarly area of cross-section of B = Area of cross-section of $C = a^2$

Hence according to formula $R = \rho \frac{l}{A}$; resistances of all the conductors are equal *i.e.* $R_A = R_B = R_C$

Example: 26 Dimensions of a block are 1 *cm* × 1 *cm* × 100 *cm*. If specific resistance of its material is 3×10^{-7} *ohm-m*, then the resistance between it's opposite rectangular faces is **[MP PET 1993]**

(a) 3×10^{-9} *ohm* (b) 3×10^{-7} *ohm* (c) 3×10^{-5} *ohm* (d) 3×10^{-3} *ohm*

Solution: (b) Length $l = 1\ cm = 10^{-2}\ m$



Area of cross-section $A = 1\ cm \times 100\ cm$

$= 100\ cm^2 = 10^{-2}\ m^2$

Resistance $R = 3 \times 10^{-7} \times \frac{10^{-2}}{10^{-2}} = 3 \times 10^{-7}\ \Omega$

Note :≅ In the above question for calculating equivalent resistance between two opposite square faces.

$$l = 100\ cm = 1\ m,\ A = 1\ cm^2 = 10^{-4}\ m^2, \text{ so resistance } R = 3 \times 10^{-7} \times \frac{1}{10^{-4}} = 3 \times 10^{-3}\ \Omega$$

Tricky example: 3

Two rods A and B of same material and length have their electric resistances are in ratio 1 : 2. When both the rods are dipped in water, the correct statement will be **[RPMT 1997]**

(a) A has more loss of weight (b) B has more loss of weight

(c) Both have same loss of weight (d) Loss of weight will be in the ratio 1 : 2

Solution: (a) $R = \rho \frac{L}{A} \Rightarrow \frac{R_1}{R_2} = \frac{A_2}{A_1}$ (ρ, L constant) $\Rightarrow \frac{A_1}{A_2} = \frac{R_2}{R_1} = 2$

Now when a body dipped in water, loss of weight = $V\sigma_L g = AL\sigma_L g$

So $\frac{(\text{Loss of weight})_1}{(\text{Loss of weight})_2} = \frac{A_1}{A_2} = 2$; So A has more loss of weight

Tricky example: 4

The V-i graph for a conductor makes an angle θ with V-axis. Here V denotes the voltage and i denotes current. The resistance of conductor is given by

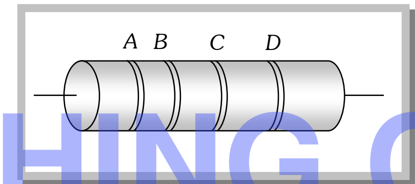
(a) $\sin\theta$ (b) $\cos\theta$ (c) $\tan\theta$ (d) $\cot\theta$

Solution: (d) At an instant approach the student will choose $\tan\theta$ will be the right answer. But it is to be seen here the curve makes the angle θ with the V-axis. So it makes an angle $(90 - \theta)$ with the i-axis. So resistance $=$ slope $= \tan(90 - \theta) = \cot\theta$.

Colour Coding of Resistance.

The resistance, having high values are used in different electrical and electronic circuits. They are generally made up of carbon, like 1 $k\Omega$, 2 $k\Omega$, 5 $k\Omega$ *etc*. To know the value of resistance colour code is used. These code are printed in form of set of rings or strips. By reading the values of colour bands, we can estimate the value of resistance.

The carbon resistance has normally four coloured rings or strips say A, B, C and D as shown in following figure.



Colour band A and B indicate the first two significant figures of resistance in *ohm*, while the C band gives the decimal multiplier *i.e.* the number of zeros that follows the two significant figures A and B.

Last band (D band) indicates the tolerance in percent about the indicated value or in other ward it represents the percentage accuracy of the indicated value.

The tolerance in the case of gold is ± 5% and in silver is ± 10%. If only three bands are marked on carbon resistance, then it indicate a tolerance of 20%.

The following table gives the colour code for carbon resistance.

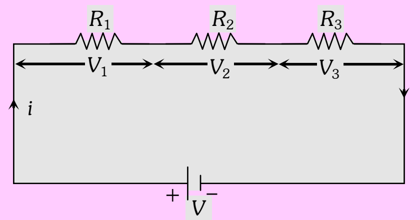
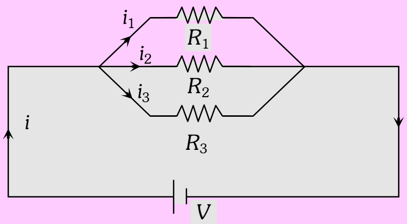
Letters as an aid to memory	Colour	Figure (A, B)	Multiplier (C)	Colour	Tolerance (D)
B	Black	0	10^0	Gold	5%
B	Brown	1	10^1	Silver	10%
R	Red	2	10^2	No-colour	20%
O	Orange	3	10^3		
Y	Yellow	4	10^4		
G	Green	5	10^5		
B	Blue	6	10^6		

V	Violet	7	10^7
G	Grey	8	10^8
W	White	9	10^9

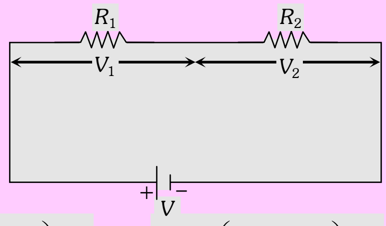
Note : ≅ To remember the sequence of colour code following sentence should kept in memory.

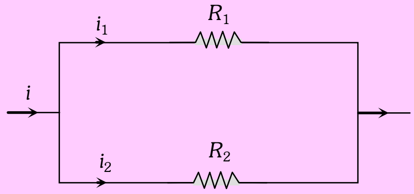
***B B R O Y G**reat **B**ritain **V**ery **G**ood **W**ife.*

Grouping of Resistance.

Series	Parallel
(1) 	(1) 
(2) Same current flows through each resistance but potential difference distributes in the ratio of resistance *i.e.* $V \propto R$ Power consumed are in the ratio of their resistance *i.e.* $P \propto R \Rightarrow P_1 : P_2 : P_3 = R_1 : R_2 : R_3$	(2) Same potential difference appeared across each resistance but current distributes in the reverse ratio of their resistance *i.e.* $i \propto \frac{1}{R}$ Power consumed are in the reverse ratio of resistance *i.e.* $P \propto \frac{1}{R} \Rightarrow P_1 : P_2 : P_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$
(3) $R_{eq} = R_1 + R_2 + R_3$ equivalent resistance is greater than the maximum value of resistance in the combination.	(3) $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ or $R_{eq} = (R_1^{-1} + R_2^{-1} + R_3^{-1})^{-1}$ or $R_{eq} = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_2R_1}$ equivalent resistance is smaller than the minimum value of resistance in the combination.
(4) For two resistance in series $R_{eq} = R_1 + R_2$	(4) For two resistance in parallel $R_{eq} = \frac{R_1R_2}{R_1 + R_2} = \frac{\text{Multiplication}}{\text{Addition}}$
(5) Potential difference across any resistance $V' = \left(\frac{R'}{R_{eq}}\right) \cdot V$ Where R' = Resistance across which potential difference is to be calculated, R_{eq} = equivalent resistance of that line in which R' is connected, V = p.d. across that line in which R' is connected	(5) Current through any resistance $i' = i \times \left[\frac{\text{Resistance of opposite branch}}{\text{Total resistance}}\right]$ Where i' = required current (branch current) i = main current

e.g.





$V_1 = \left(\frac{R_1}{R_1 + R_2}\right)\cdot V$ and $V_2 = \left(\frac{R_2}{R_1 + R_2}\right)\cdot V$

$i_1 = i\left(\frac{R_2}{R_1 + R_2}\right)$ and $i_2 = i\left(\frac{R_1}{R_1 + R_2}\right)$

(6) If *n* identical resistance are connected in series

$R_{eq} = nR$ and p.d. across each resistance $V' = \frac{V}{n}$

(6) In *n* identical resistance are connected in parallel

$R_{eq} = \frac{R}{n}$ and current through each resistance $i' = \frac{i}{n}$

Note : ≅In case of resistances in series, if one resistance gets open, the current in the whole circuit become zero and the circuit stops working. Which don't happen in case of parallel gouging.

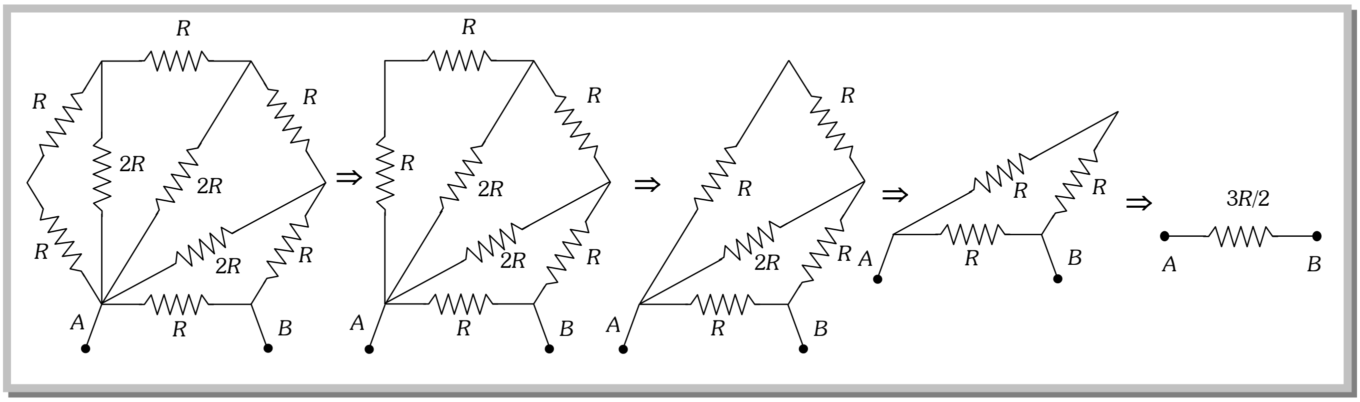
≅ Decoration of lightning in festivals is an example of series grouping whereas all household appliances connected in parallel grouping.

≅ Using *n* conductors of equal resistance, the number of possible combinations is 2^{n-1}.

≅ If the resistance of *n* conductors are totally different, then the number of possible combinations will be 2^n.

Methods of Determining Equivalent Resistance For Some Difficult Networks.

(1) **Method of successive reduction :** It is the most common technique to determine the equivalent resistance. So far, we have been using this method to find out the equivalent resistances. This method is applicable only when we are able to identify resistances in series or in parallel. The method is based on the simplification of the circuit by successive reduction of the series and parallel combinations. For example to calculate the equivalent resistance between the point *A* and *B*, the network shown below successively reduced.



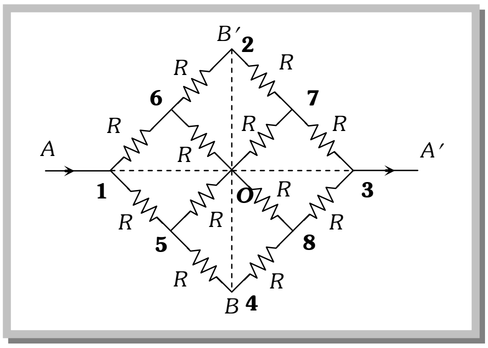
(2) **Method of equipotential points :** This method is based on identifying the points of same potential and joining them. The basic rule to identify the points of same potential is the symmetry of the network.

(i) In a given network there may be two axes of symmetry.

(a) Parallel axis of symmetry, that is, along the direction of current flow.

(b) Perpendicular axis of symmetry, that is perpendicular to the direction of flow of current.

For example in the network shown below the axis AA' is the parallel axis of symmetry, and the axis BB' is the perpendicular axis of symmetry.



(ii) Points lying on the perpendicular axis of symmetry may have same potential. In the given network, point 2, 0 and 4 are at the same potential.