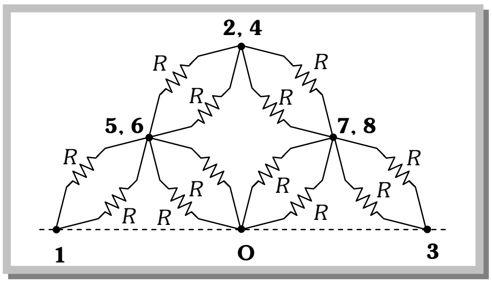
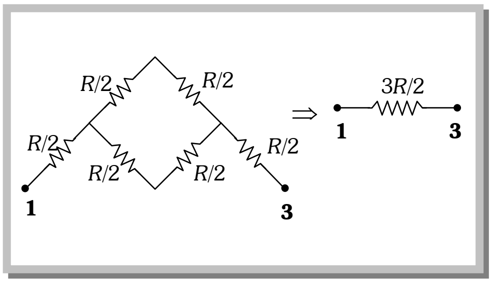
(iii) Points lying on the parallel axis of symmetry can never have same potential.

(iv) The network can be folded about the parallel axis of symmetry, and the overlapping nodes have same potential. Thus as shown in figure, the following points have same potential

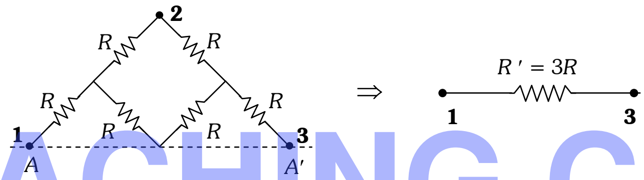
(a) 5 and 6 (b) 2, 0 and 4 (c) 7 and 8



⇒



Note : ≅ Above network may be split up into two equal parts about the parallel axis of symmetry as shown in figure each part has a resistance R', then the equivalent resistance of the network will be $R = \frac{R'}{2}$.

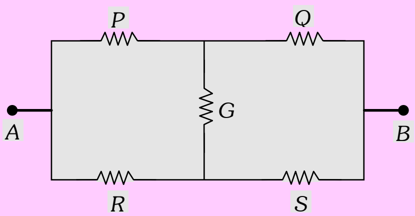


TEACHING CARE

Some Standard Results for Equivalent Resistance.

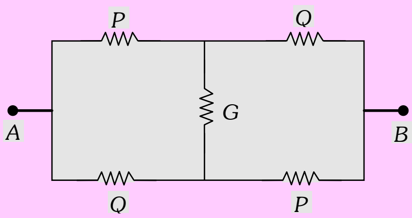
(1) Equivalent resistance between points *A* and *B* in an unbalanced Wheatstone's bridge as shown in the diagram.

(i)



$$R_{AB} = \frac{PQ(R+S)+(P+Q)RS+G(P+Q)(R+S)}{G(P+Q+R+S)+(P+R)(Q+S)}$$

(ii)



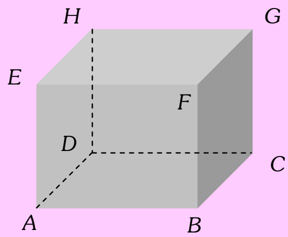
$$R_{AB} = \frac{2PQ+G(P+Q)}{2G+P+Q}$$

(2) A cube each side have resistance *R* then equivalent resistance in different situations

(i) Between *E* and *C* *i.e.* across the diagonal of the cube $R_{EC} = \frac{5}{6}R$

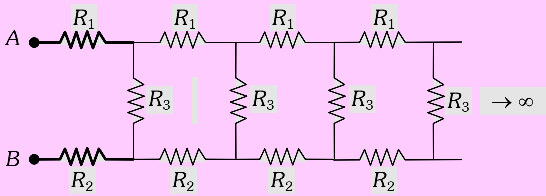
(ii) Between *A* and *B* *i.e.* across one side of the cube $R_{AB} = \frac{7}{12}R$

(iii) Between *A* and *C* *i.e.* across the diagonal of one face of the cube $R_{AC} = \frac{3}{4}R$



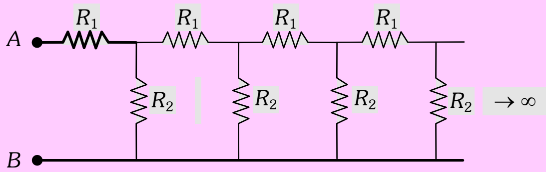
(3) The equivalent resistance of infinite network of resistances

(i)



$$R_{AB} = \frac{1}{2}(R_1 + R_2) + \frac{1}{2}\left[(R_1 + R_2)^2 + 4R_3(R_1 + R_2)\right]^{1/2}$$

(ii)



$$R_{AB} = \frac{1}{2}R_1\left[1 + \sqrt{1 + 4\left(\frac{R_2}{R_1}\right)}\right]$$

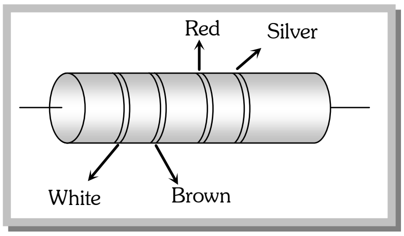
Concepts

- *If n identical resistances are first connected in series and then in parallel, the ratio of the equivalent resistance is given by* $\frac{R_p}{R_s} = \frac{n^2}{1}$.
- *If equivalent resistance of R_1 and R_2 in series and parallel be R_s and R_p respectively then* $R_1 = \frac{1}{2}\left[R_s + \sqrt{R_s^2 - 4R_sR_p}\right]$ *and* $R_2 = \frac{1}{2}\left[R_s - \sqrt{R_s^2 - 4R_sR_p}\right]$.
- *If a wire of resistance R, cut in n equal parts and then these parts are collected to form a bundle then equivalent resistance of combination will be* $\frac{R}{n^2}$.

TEACHING CARE

Example

Example: 27 In the figure a carbon resistor has band of different colours on its body. The resistance of the following body is **[Kerala PET 2002]**



(a) 2.2 $k\Omega$

(b) 3.3 $k\Omega$

(c) 5.6 $k\Omega$

(d) 9.1 $k\Omega$

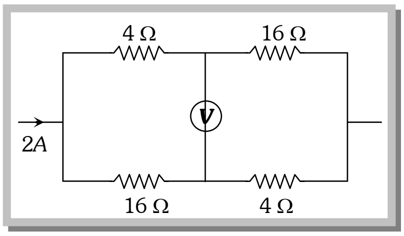
Solution : (d) $R = 91 \times 10^2 \pm 10\% \approx 9.1\ k\Omega$

Example: 28 What is the resistance of a carbon resistance which has bands of colours brown, black and brown **[DCE 1999]**

(a) 100 Ω (b) 1000 Ω (c) 10 Ω (d) 1 Ω

Solution : (a) $R = 10 \times 10^1 \pm 20\% \approx 100\ \Omega$

Example: 29 In the following circuit reading of voltmeter V is **[MP PET 2003]**



(a) 12 V

(b) 8 V

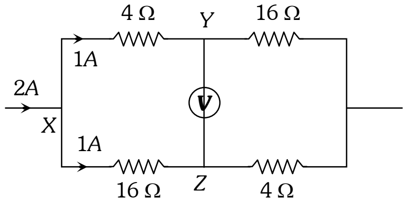
(c) 20 V

(d) 16 V

Solution : (a) P.d. between X and Y is $V_{XY} = V_X - V_Y = 1 \times 4 = 4\,V$ .... (i)

and p.d. between X and Z is $V_{XZ} = V_X - V_Z = 1 \times 16 = 16\,V$ .... (ii)

On solving equations (i) and (ii) we get potential difference between Y and Z *i.e.*, reading of voltmeter is $V_Y - V_Z = 12V$

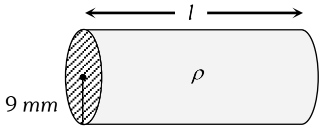


Example: 30 An electric cable contains a single copper wire of radius 9 *mm*. It's resistance is 5 Ω. This cable is replaced by six insulated copper wires, each of radius 3 *mm*. The resultant resistance of cable will be **[CPMT 1988]**

(a) 7.5 Ω (b) 45 Ω (c) 90 Ω (d) 270 Ω

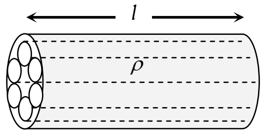
Solution : (a) Initially : Resistance of given cable

$$R = \rho \frac{l}{\pi \times (9 \times 10^{-3})^2} \quad ..... \text{(i)}$$



Finally : Resistance of each insulated copper wire is

$$R' = \rho \frac{l}{\pi \times (3 \times 10^{-3})^2}$$



Hence equivalent resistance of cable

$$R_{eq} = \frac{R'}{6} = \frac{1}{6} \times \left(\rho \frac{l}{\pi \times (3 \times 10^{-3})^2}\right) \quad ..... \text{(ii)}$$

TEACHING CARE

On solving equation (i) and (ii) we get $R_{eq} = 7.5\,\Omega$

Example: 31 Two resistance R_1 and R_2 provides series to parallel equivalents as $\frac{n}{1}$ then the correct relationship is

(a) $\left(\frac{R_1}{R_2}\right)^2 + \left(\frac{R_2}{R_1}\right)^2 = n^2$ (b) $\left(\frac{R_1}{R_2}\right)^{3/2} + \left(\frac{R_2}{R_1}\right)^{3/2} = n^{3/2}$

(c) $\left(\frac{R_1}{R_2}\right) + \left(\frac{R_2}{R_1}\right) = n$ (d) $\left(\frac{R_1}{R_2}\right)^{1/2} + \left(\frac{R_2}{R_1}\right)^{1/2} = n^{1/2}$

Solution : (d) Series resistance $R_S = R_1 + R_2$ and parallel resistance $R_P = \frac{R_1 R_2}{R_1 + R_2} \Rightarrow \frac{R_S}{R_P} = \frac{(R_1 + R_2)^2}{R_1 R_2} = n$

$$\Rightarrow \frac{R_1 + R_2}{\sqrt{R_1 R_2}} = \sqrt{n} \qquad \Rightarrow \frac{\sqrt{R_1^2}}{\sqrt{R_1 R_2}} + \frac{\sqrt{R_2^2}}{\sqrt{R_1 R_2}} = \sqrt{n} \Rightarrow \sqrt{\frac{R_1}{R_2}} + \sqrt{\frac{R_2}{R_1}} = \sqrt{n}$$

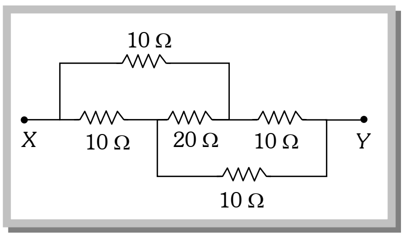
Example: 32 Five resistances are combined according to the figure. The equivalent resistance between the point X and Y will be **[UPSEAT 1999; AMU 1995; CPMT 1986]**

(a) 10 Ω

(b) 22 Ω

(c) 20 Ω

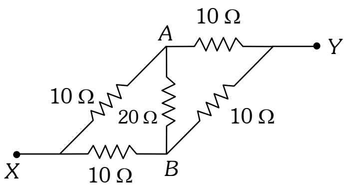
(d) 50 Ω



Solution : (a) The equivalent circuit of above can be drawn as

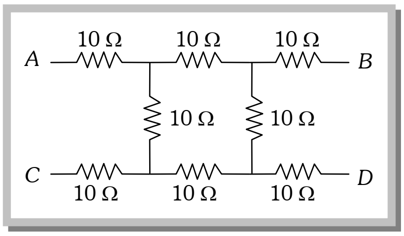
Which is a balanced wheatstone bridge.

So current through *AB* is zero.



So $\frac{1}{R} = \frac{1}{20} + \frac{1}{20} = \frac{1}{10} \Rightarrow R = 10\ \Omega$

***Example*: 33** What will be the equivalent resistance of circuit shown in figure between points *A* and *D* **[CBSE PMT 1996]**



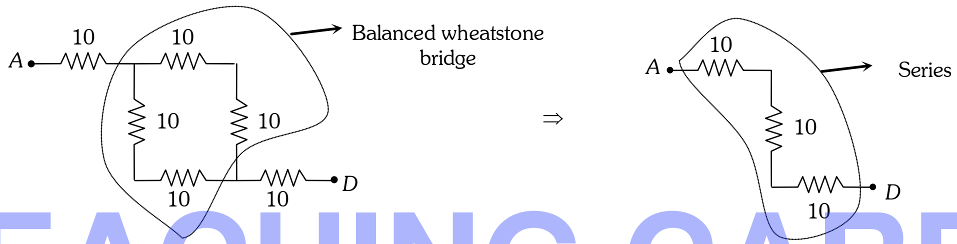
(a) 10 Ω

(b) 20 Ω

(c) 30 Ω

(d) 40 Ω

Solution : (c) The equivalent circuit of above fig between *A* and *D* can be drawn as



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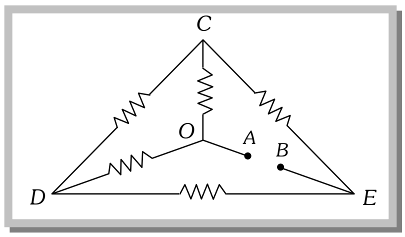
So $R_{eq} = 10 + 10 + 10 = 30\Omega$

***Example*: 34** In the network shown in the figure each of resistance is equal to 2Ω. The resistance between *A* and *B* is **[CBSE PMT 1995]**



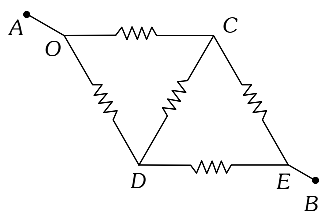
(a) 1 Ω

(b) 2 Ω

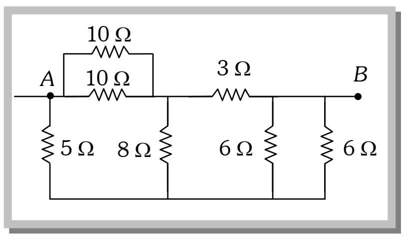
(c) 3 Ω

(d) 4 Ω

Solution : (b) Taking the portion *COD* is figure to outside the triangle (left), the above circuit will be now as resistance of each is 2 Ω the circuit will behaves as a balanced wheatstone bridge and no current flows through *CD*. Hence $R_{AB} = 2\Omega$



***Example*: 35** Seven resistances are connected as shown in figure. The equivalent resistance between *A* and *B* is **[MP PET 2000]**



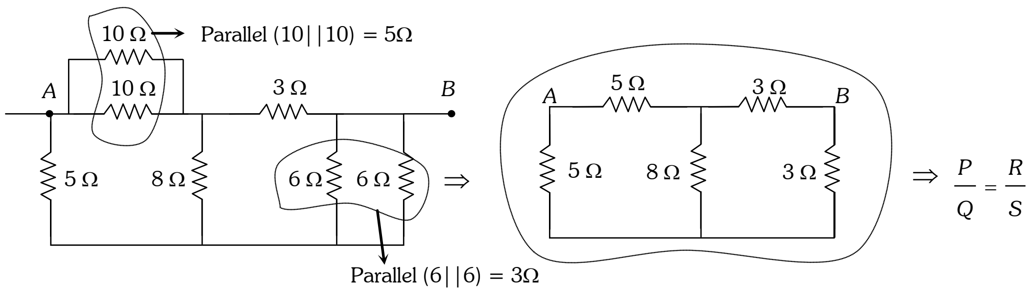
(a) 3 Ω

(b) 4 Ω

(c) 4.5 Ω

(d) 5 Ω

Solution : (b)



So the circuit is a balanced wheatstone bridge.

So current through 8Ω is zero $R_{eq} = (5+3)\,||\,(5+3) = 8\,||\,8 = 4\Omega$

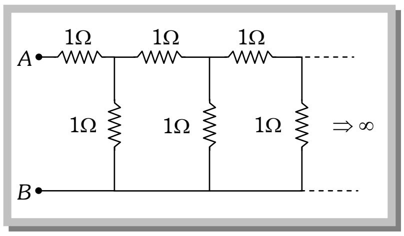
Example: 36 The equivalent resistance between points *A* and *B* of an infinite network of resistance, each of 1 Ω, connected as shown is **[CEE Haryana 1996]**

(a) Infinite

(b) 2 Ω

(c) $\dfrac{1+\sqrt{5}}{2}\Omega$

(d) Zero



Solution : (c) Suppose the effective resistance between *A* and *B* is R_{eq}. Since the network consists of infinite cell. If we exclude one cell from the chain, remaining network have infinite cells *i.e.* effective resistance between *C* and *D* will also R_{eq}

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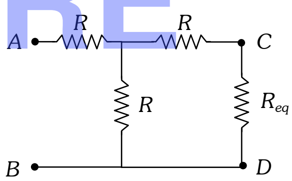
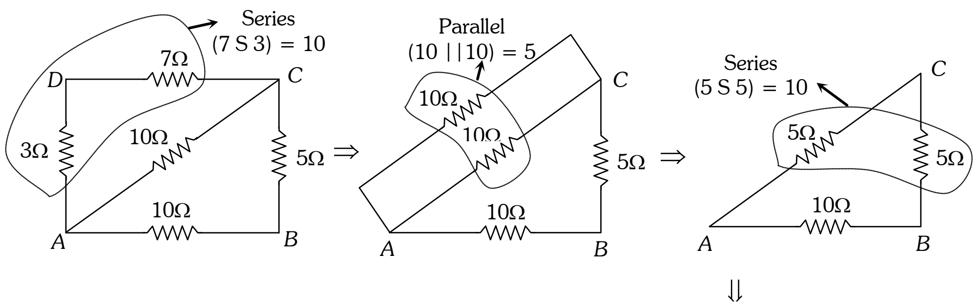


So now $R_{eq} = R_o + (R\,||\,R_{eq}) = R + \dfrac{R\,R_{eq}}{R+R_{eq}} \Rightarrow R_{eq} = \dfrac{1}{2}[1+\sqrt{5}]$

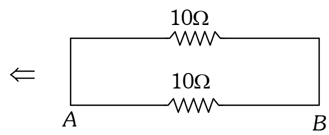
Example: 37 Four resistances 10 Ω, 5 Ω, 7 Ω and 3 Ω are connected so that they form the sides of a rectangle *AB*, *BC*, *CD* and *DA* respectively. Another resistance of 10 Ω is connected across the diagonal *AC*. The equivalent resistance between *A* & *B* is **[EAMCET Med. 2000]**

(a) 2 Ω (b) 5 Ω (c) 7 Ω (d) 10 Ω

Solution : (b)



So $R_{eq} = \dfrac{10\times 10}{10+10} = 5\Omega$



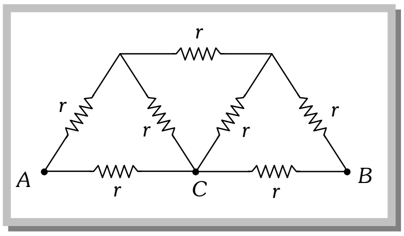
Example: 38 The equivalent resistance between A and B in the circuit shown will be

(a) $\frac{5}{4}r$

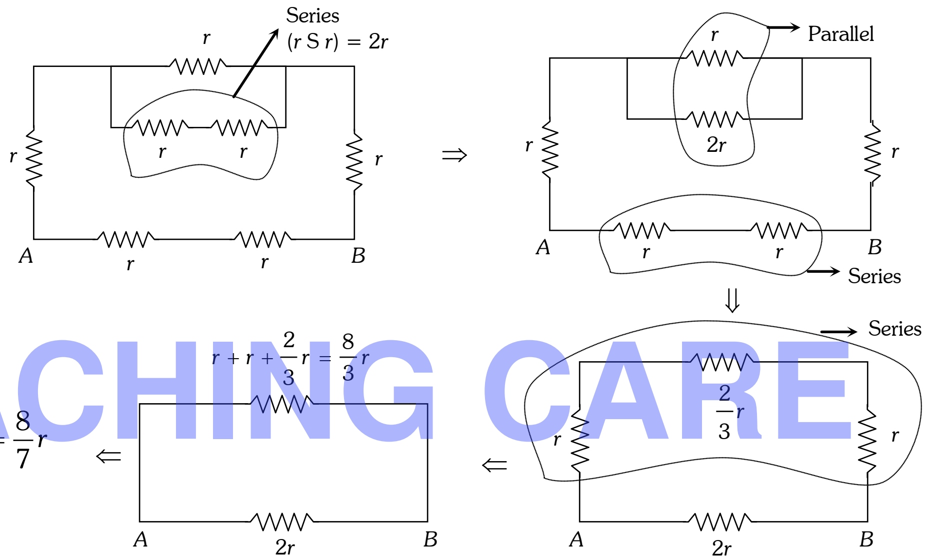
(b) $\frac{6}{5}r$

(c) $\frac{7}{6}r$

(d) $\frac{8}{7}r$



Solution : (d) In the circuit, by means of symmetry the point C is at zero potential. So the equivalent circuit can be drawn as



$R_{eq} = \left(\frac{8r}{3} \| 2r\right) = \frac{8}{7}r$

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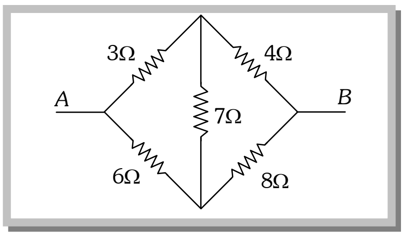
Example: 39 In the given figure, equivalent resistance between A and B will be **[CBSE PMT 2000]**

(a) $\frac{14}{3}\Omega$

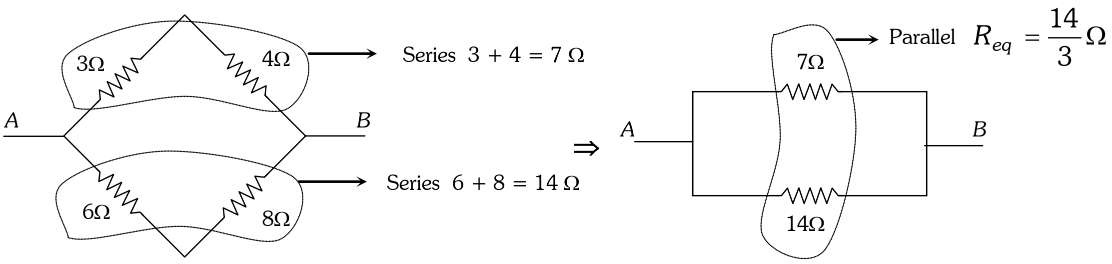
(b) $\frac{3}{14}\Omega$

(c) $\frac{9}{14}\Omega$

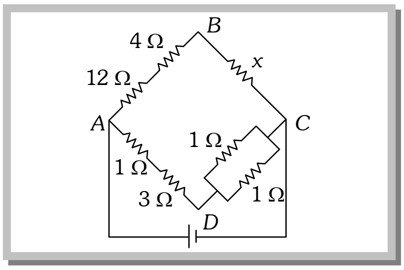
(d) $\frac{14}{9}\Omega$



Solution : (a) Given Wheatstone bridge is balanced because $\frac{P}{Q} = \frac{R}{S}$. Hence the circuit can be redrawn as follows



***Example*: 40** In the combination of resistances shown in the figure the potential difference between B and D is zero, when unknown resistance (x) is **[UPSEAT 1999; CPMT 1986]**



(a) 4 Ω

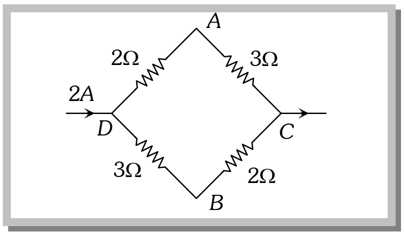
(b) 2 Ω

(c) 3 Ω

(d) The emf of the cell is required

Solution : (b) The potential difference across B, D will be zero, when the circuit will act as a balanced wheatstone bridge and $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{12+4}{x} = \frac{1+3}{1/2} \Rightarrow x = 2\Omega$

***Example*: 41** A current of 2 A flows in a system of conductors as shown. The potential difference $(V_A - V_B)$ will be **[CPMT 1975, 76]**



(a) + 2V

(b) + 1V

(c) – 1 V

(d) – 2 V

Solution : (b) In the given circuit 2A current divides equally at junction D along the paths DAC and DBC (each path carry 1A current).

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Potential difference between D and A, $V_D - V_A = 1 \times 2 = 2\ volt$ …. (i)

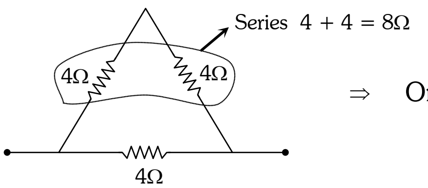
Potential difference between D and B, $V_D - V_B = 1 \times 3 = 3\ volt$ ….. (ii)

On solving (i) and (ii) $V_A - V_B = +1\ volt$

***Example*: 42** Three resistances each of 4 Ω are connected in the form of an equilateral triangle. The effective resistance between two corners is **[CBSE PMT 1993]**

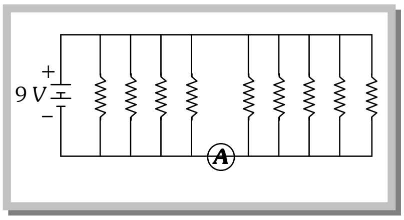
(a) 8 Ω (b) 12 Ω (c) $\frac{3}{8}\Omega$ (d) $\frac{8}{3}\Omega$

Solution : (d)



$\Rightarrow$ On Solving further we get equivalent resistance is $\frac{8}{3}\Omega$

***Example*: 43** If each resistance in the figure is of 9 Ω then reading of ammeter is **[RPMT 2000]**



(a) 5 A

(b) 8 A

(c) 2 A

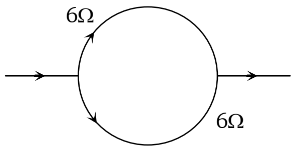
(d) 9 A

Solution : (a) Main current through the battery $i = \frac{9}{1} = 9A$. Current through each resistance will be 1A and only 5 resistances on the right side of ammeter contributes for passing current through the ammeter. So reading of ammeter will be 5A.

Example: 44 A wire has resistance 12 Ω. It is bent in the form of a circle. The effective resistance between the two points on any diameter is equal to **[JIPMER 1999]**

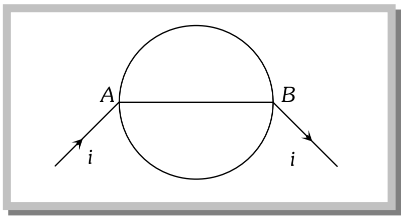
(a) 12 Ω (b) 6 Ω (c) 3 Ω (d) 24 Ω

Solution : (c) Equivalent resistance of the following circuit will be



$$R_{eq} = \frac{6}{2} = 3\Omega$$

Example: 45 A wire of resistance 0.5 Ωm^{-1} is bent into a circle of radius 1 *m*. The same wire is connected across a diameter *AB* as shown in fig. The equivalent resistance is



(a) π *ohm*

(b) $\pi(\pi + 2)$ *ohm*

(c) $\pi / (\pi + 4)$ *ohm*

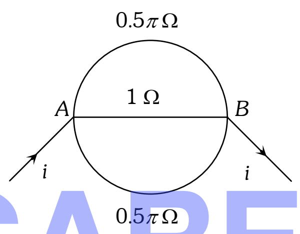
(d) $(\pi + 1)$ *ohm*

Solution : (c) Resistance of upper semicircle = Resistance of lower semicircle

$= 0.5 \times (\pi R) = 0.5\ \pi\Omega$

Resistance of wire *AB* $= 0.5 \times 2 = 1\ \Omega$

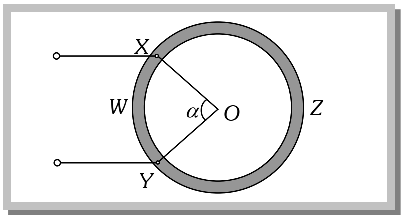
Hence equivalent resistance between *A* and *B*



$$\frac{1}{R_{AB}} = \frac{1}{0.5\pi} + \frac{1}{1} + \frac{1}{0.5\pi} \Rightarrow R_{AB} = \frac{\pi}{(\pi + 4)}\Omega$$

TEACHING CARE

Example: 46 A wire of resistor *R* is bent into a circular ring of radius *r*. Equivalent resistance between two points *X* and *Y* on its circumference, when angle *XOY* is α, can be given by



(a) $\frac{R\alpha}{4\pi^2}(2\pi - \alpha)$

(b) $\frac{R}{2\pi}(2\pi - \alpha)$

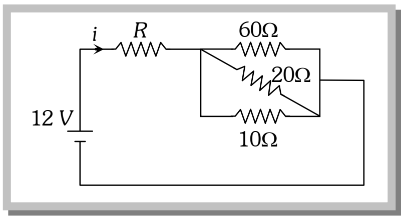
(c) $R\ (2\pi - \alpha)$

(d) $\frac{4\pi}{R\alpha}(2\pi - \alpha)$

Solution : (a) Here $R_{XWY} = \frac{R}{2\pi r} \times (r\alpha) = \frac{R\alpha}{2\pi}$ $\left(\because \alpha = \frac{l}{r}\right)$ and $R_{XZY} = \frac{R}{2\pi r} \times r(2\pi - \alpha) = \frac{R}{2\pi}(2\pi - \alpha)$

$$R_{eq} = \frac{R_{XWY} R_{XZY}}{R_{XWY} + R_{XZY}} = \frac{\frac{R\alpha}{2\pi} \times \frac{R}{2\pi}(2\pi - \alpha)}{\frac{R\alpha}{2\pi} + \frac{R(2\pi - \alpha)}{2\pi}} = \frac{R\alpha}{4\pi^2}(2\pi - \alpha)$$

Example: 47 If in the given figure $i = 0.25$ *amp*, then the value *R* will be **[RPET 2000]**



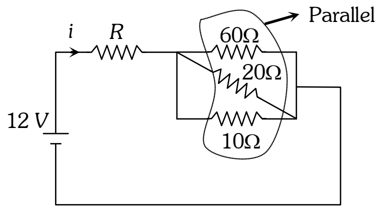
(a) 48 Ω

(b) 12 Ω

(c) 120 Ω

(d) 42 Ω

Solution : (d) $i = 0.25\ amp\ V = 12\ V$ $\quad R_{eq} = \frac{V}{i} = \frac{12}{0.25} = 48\,\Omega$



Now from the circuit $R_{eq} = R + (60\,||\,20\,||\,10)$

$= R + 6$

$\Rightarrow R = R_{eq} - 6 = 48 - 6 = 42\,\Omega$

Example: 48 Two uniform wires A and B are of the same metal and have equal masses. The radius of wire A is twice that of wire B. The total resistance of A and B when connected in parallel is **[MNR 1994]**

(a) $4\,\Omega$ when the resistance of wire A is $4.25\,\Omega$ (b) $5\,\Omega$ when the resistance of wire A is $4\,\Omega$

(c) $4\,\Omega$ when the resistance of wire B is $4.25\,\Omega$ (d) $5\,\Omega$ when the resistance of wire B is $4\,\Omega$

Solution : (a) Density and masses of wire are same so their volumes are same *i.e.* $A_1 l_1 = A_2 l_2$

Ratio of resistances of wires A and B $\frac{R_A}{R_B} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^4$

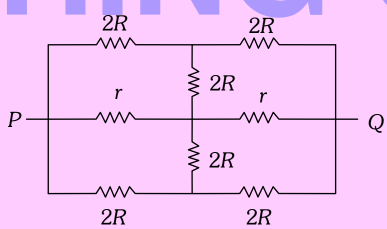
Since $r_1 = 2r_2$ so $\frac{R_A}{R_B} = \frac{1}{16} \Rightarrow R_B = 16\,R_A$

Resistance R_A and R_B are connected in parallel so equivalent resistance $R = \frac{R_A R_B}{R_A + R_B} = \frac{16R_A}{17}$, By checking correctness of equivalent resistance from options, only option (a) is correct.

Tricky Example: 5

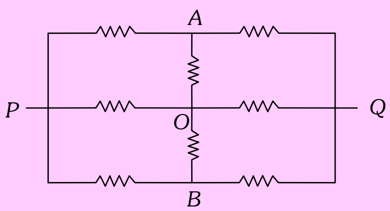
TEACHING CARE

The effective resistance between point P and Q of the electrical circuit shown in the figure is **[IIT-JEE 1991]**



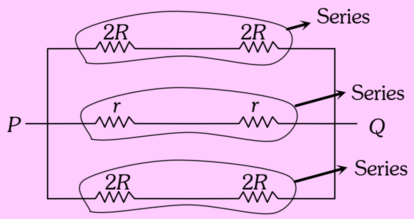
(a) $\frac{2Rr}{R+r}$ (b) $\frac{8R(R+r)}{3R+r}$ (c) $2r + 4R$ (d) $\frac{5R}{2} + 2r$

Solution : (a) The points A, O, B are at same potential. So the figure can be redrawn as follows



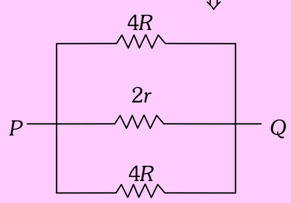
(I)

$\Rightarrow$



(II)

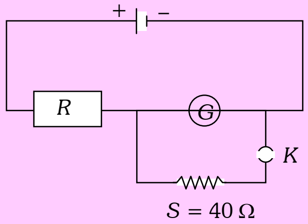
$\Downarrow$



$\Leftarrow$ $4R\,||\,2r\,||\,4R$ $\Leftarrow$ $R_{eq} = \frac{2Rr}{R+r}$

Tricky Example: 6

In the following circuit if key K is pressed then the galvanometer reading becomes half. The resistance of galvanometer is

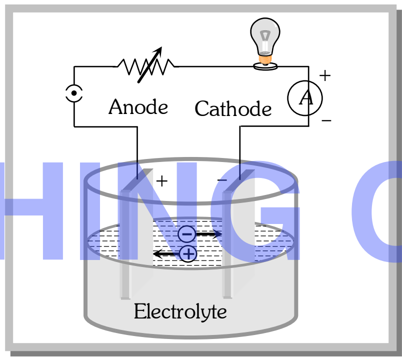


(a) 20 Ω (b) 30 Ω (c) 40 Ω (d) 50 Ω

Solution : (c) Galvanometer reading becomes half means current distributes equally between galvanometer and resistance of 40 Ω. Hence galvanometer resistance must be 40 Ω.

Cell.

The device which converts chemical energy into electrical energy is known as electric cell.



TEACHING CARE

(1) A cell neither creates nor destroys charge but maintains the flow of charge present at various parts of the circuit by supplying energy needed for their organised motion.

(2) Cell is a source of constant emf but not constant current.

(3) Mainly cells are of two types :

(i) Primary cell : Cannot be recharged

(ii) Secondary cell : Can be recharged

(4) The direction of flow of current inside the cell is from negative to positive electrode while outside the cell is form positive to negative electrode.

(5) A cell is said to be ideal, if it has zero internal resistance.

(6) **Emf of cell (E) :** The energy given by the cell in the flow of unit charge in the whole circuit (including the cell) is called it's electromotive force (emf) *i.e.* emf of cell $E = \frac{W}{q}$, It's unit is *volt*

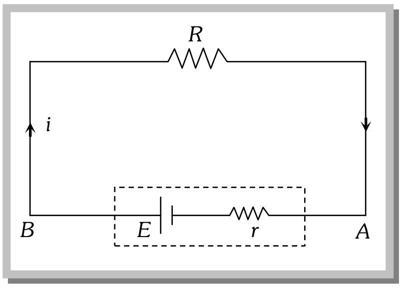
or

The potential difference across the terminals of a cell when it is not given any current is called it's emf.

(7) **Potential difference (V) :** The energy given by the cell in the flow of unit charge in a specific part of electrical circuit (external part) is called potential difference. It's unit is also *volt*

or

The voltage across the terminals of a cell when it is supplying current to external resistance is called potential difference or terminal voltage. Potential difference is equal to the product of current and resistance of that given part *i.e.* $V = iR$.



(8) **Internal resistance (r) :** In case of a cell the opposition of electrolyte to the flow of current through it is called internal resistance of the cell. The internal resistance of a cell depends on the distance between electrodes ($r \propto d$), area of electrodes [$r \propto (1/A)$] and nature, concentration ($r \propto C$) and temperature of electrolyte [$r \propto (1/\text{temp.})$]. Internal resistance is different for different types of cells and even for a given type of cell it varies from to cell.

Cell in Various Position.

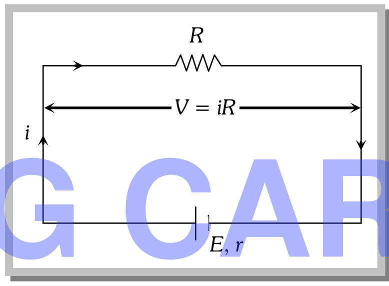
(1) **Closed circuit (when the cell is discharging)**

(i) Current given by the cell $i = \dfrac{E}{R + r}$

(ii) Potential difference across the resistance $V = iR$

(iii) Potential drop inside the cell $= ir$

(iv) Equation of cell $E = V + ir \quad (E > V)$



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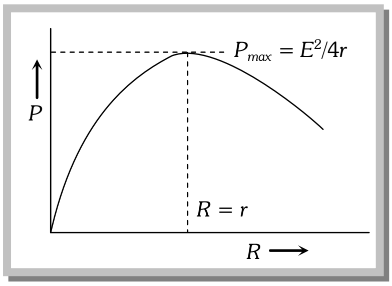
(v) Internal resistance of the cell $r = \left(\dfrac{E}{V} - 1\right) \cdot R$

(vi) Power dissipated in external resistance (load) $P = Vi = i^2 R = \dfrac{V^2}{R} = \left(\dfrac{E}{R + r}\right)^2 .R$

Power delivered will be maximum when $R = r$ so $P_{\max} = \dfrac{E^2}{4r}$.

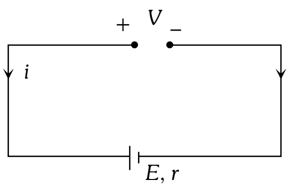


This statement in generalised from is called "*maximum power transfer theorem*".

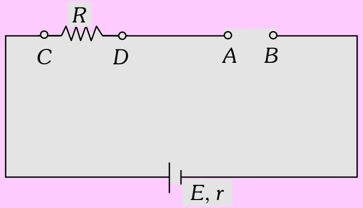
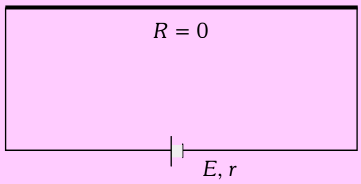
(vii) **Short trick to calculate E and r :** In the closed circuit of a cell having emf E and internal resistance r. If external resistance changes from R_1 to R_2 then current changes from i_1 to i_2 and potential difference changes from V_1 to V_2. By using following relations we can find the value of E and r.

$$\boldsymbol{E = \frac{i_1 i_2}{i_2 - i_1}(R_1 - R_2) \quad r = \left(\frac{i_2 R_2 - i_1 R_1}{i_1 - i_2}\right) = \frac{V_2 - V_1}{i_1 - i_2}}$$

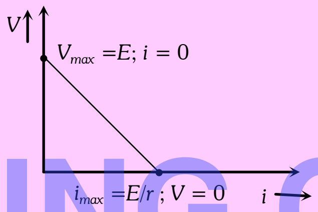
Note : ≅ When the cell is charging *i.e.* current is given to the cell then $E = V - ir$ and $E < V$.



(2) **Open circuit and short circuit**

Open circuit	**Short circuit**
	
(i) Current through the circuit $i = 0$	(i) Maximum current (called short circuit current) flows momentarily $i_{sc} = \frac{E}{r}$
(ii) Potential difference between A and B, $V_{AB} = E$	(ii) Potential difference $V = 0$
(iii) Potential difference between C and D, $V_{CD} = 0$	

Note : ≅ Above information's can be summarized by the following graph



TEACHING CARE

Concepts

- *It is a common misconception that "current in the circuit will be maximum when power consumed by the load is maximum."*

  Actually current $i = E/(R+r)$ *is maximum (= E/r) when R = min = 0 with* $P_L = (E/r)^2 \times 0 = 0\,\text{min}$. *while power consumed by the load* $E^2R/(R+r)^2$ *is maximum (=* $E^2/4r$*) when R = r and* $i = (E/2r) \neq \max(= E/r)$.

- *Emf is independent of the resistance of the circuit and depends upon the nature of electrolyte of the cell while potential difference depends upon the resistance between the two points of the circuit and current flowing through the circuit.*
- *Emf is a cause and potential difference is an effect.*
- *Whenever a cell or battery is present in a branch there must be some resistance (internal or external or both) present in that branch. In practical situation it always happen because we can never have an ideal cell or battery with zero resistance.*

Example

***Example*: 49** A new flashlight cell of emf 1.5 *volts* gives a current of 15 *amps*, when connected directly to an ammeter of resistance 0.04 Ω. The internal resistance of cell is **[MP PET 1994]**

(a) 0.04 Ω (b) 0.06 Ω (c) 0.10 Ω (d) 10 Ω

Solution : (b) By using $i = \frac{E}{R+r} \Rightarrow 15 = \frac{1.5}{0.04 + r} \Rightarrow r = 0.06\,\Omega$

***Example*: 50** For a cell, the terminal potential difference is 2.2 *V* when the circuit is open and reduces to 1.8 *V*, when the cell is connected across a resistance, $R = 5\Omega$. The internal resistance of the cell is **[CBSE PMT 2002]**

(a) $\frac{10}{9}\Omega$ (b) $\frac{9}{10}\Omega$ (c) $\frac{11}{9}\Omega$ (d) $\frac{5}{9}\Omega$

Solution : (a) In open circuit, $E = V = 2.2\,V$, In close circuit, $V = 1.8\,V$, $R = 5\Omega$

So internal resistance, $r = \left(\frac{E}{V} - 1\right)R = \left(\frac{2.2}{1.8} - 1\right) \times 5 \Rightarrow r = \frac{10}{9}\Omega$

***Example*: 51** The internal resistance of a cell of emf 2*V* is 0.1 Ω. It's connected to a resistance of 3.9 Ω. The voltage across the cell will be **[CBSE PMT 1999; AFMC 1999; MP PET 1993; CPMT 1990]**

(a) 0.5 *volt* (b) 1.9 *volt* (c) 1.95 *volt* (d) 2 *volt*

Solution : (c) By using $r = \left(\frac{E}{V} - 1\right)R \Rightarrow 0.1 = \left(\frac{2}{V} - 1\right) \times 3.9 \Rightarrow V = 1.95\,volt$

***Example*: 52** When the resistance of 2 Ω is connected across the terminal of the cell, the current is 0.5 *amp*. When the resistance is increased to 5 Ω, the current is 0.25 *amp*. The emf of the cell is **[MP PMT 2000]**

(a) 1.0 *volt* (b) 1.5 *volt* (c) 2.0 *volt* (d) 2.5 *volt*

Solution : (b) By using $E = \frac{i_1 i_2}{(i_2 - i_1)}(R_1 - R_2) = \frac{0.5 \times 0.25}{(0.25 - 0.5)}(2 - 5) = 1.5\,volt$

***Example*: 53** A primary cell has an emf of 1.5 *volts*, when short-circuited it gives a current of 3 *amperes*. The internal resistance of the cell is **[CPMT 1976, 83]**

TEACHING CARE

(a) 4.5 *ohm* (b) 2 *ohm* (c) 0.5 *ohm* (d) 1/4.5 *ohm*

Solution : (c) $i_{sc} = \frac{E}{r} \Rightarrow 3 = \frac{1.5}{r} \Rightarrow r = 0.5\,\Omega$

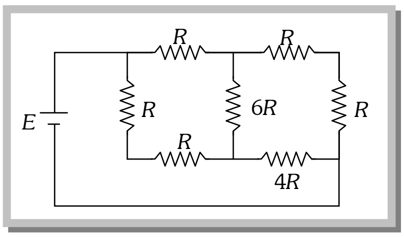
***Example*: 54** A battery of internal resistance 4 Ω is connected to the network of resistances as shown. In order to give the maximum power to the network, the value of *R* (in Ω) should be **[IIT-JEE 1995]**

(a) 4/9

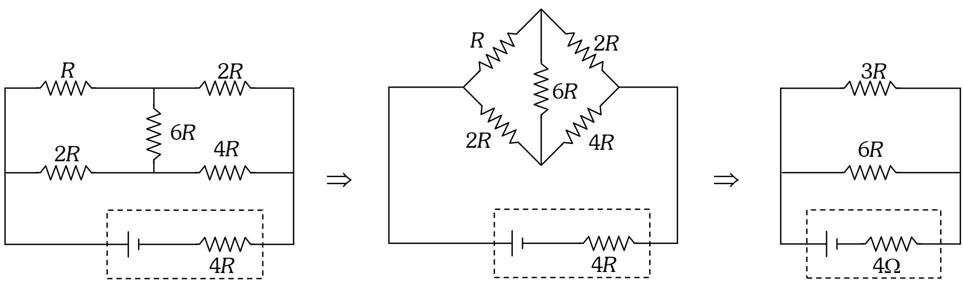
(b) 8/9

(c) 2

(d) 18



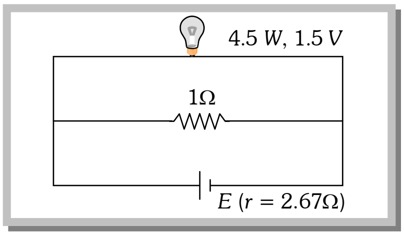
Solution : (c) The equivalent circuit becomes a balanced wheatstone bridge



For maximum power transfer, external resistance should be equal to internal resistance of source

$$\Rightarrow \frac{(R+2R)(2R+4R)}{(R+2R)+(2R+4R)} = 4 \;\; i.e. \;\; \frac{3R \times 6R}{3R+6R} = 4 \;\; \text{or } R = 2\Omega$$

Example: 55 A torch bulb rated as 4.5 *W*, 1.5 *V* is connected as shown in the figure. The emf of the cell needed to make the bulb glow at full intensity is **[MP PMT 1999]**



(a) 4.5 *V*

(b) 1.5 *V*

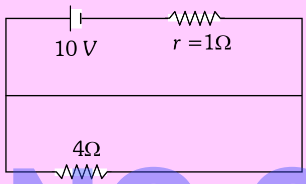
(c) 2.67 *V*

(d) 13.5 *V*

Solution : (d) When bulb glows with full intensity, potential difference across it is 1.5 *V*. So current through the bulb and resistance of 1Ω are 3 *A* and 1.5 *A* respectively. So main current from the cell $i = 3 + 1.5 = 4.5\,A$. By using $E = V + iR \Rightarrow E = 1.5 + 4.5 \times 2.67 = 13.5\,V$.

Tricky Example: 7

Potential difference across the terminals of the battery shown in figure is (r = internal resistance of battery)



(a) 8 *V* (b) 10 *V* (c) 6 *V* (d) Zero

Solution : (d) Battery is short circuited so potential difference is zero.

TEACHING CARE

Grouping of cell.

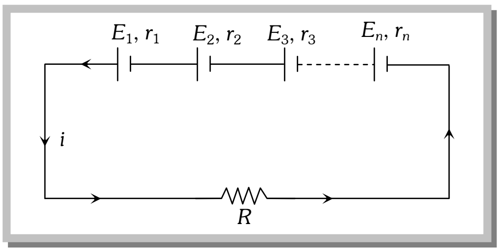
Group of cell is called a battery.

(1) **Series grouping :** In series grouping anode of one cell is connected to cathode of other cell and so on.

(i) ***n* identical cells are connected in series**

(a) Equivalent emf of the combination $E_{eq} = nE$

(b) Equivalent internal resistance $r_{eq} = nr$



(c) Main current = Current from each cell $= i = \dfrac{nE}{R+nr}$

(d) Potential difference across external resistance $V = iR$

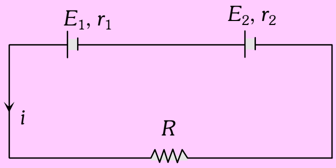
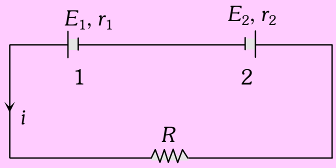
(e) Potential difference across each cell $V' = \dfrac{V}{n}$

(f) Power dissipated in the circuit $P = \left(\dfrac{nE}{R+nr}\right)^2 .R$

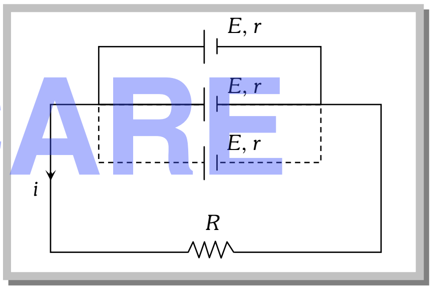
(g) Condition for maximum power $R = nr$ and $P_{\max} = n\left(\dfrac{E^2}{4r}\right)$

(h) This type of combination is used when $nr << R$.

(ii) If non-identical cell are connected in series

Cells are connected in right order	Cells are wrongly connected
	$(E_1 > E_2)$ 
(a) Equivalent emf $E_{eq} = E_1 + E_2$	(a) Equivalent emf $E_{eq} = E_1 - E_2$
(b) Current $i = \frac{E_{eq}}{R + r_{eq}}$	(b) Current $i = \frac{E_1 - E_2}{R + r_{eq}}$
(c) Potential difference across each cell $V_1 = E_1 - ir_1$ and $V_2 = E_2 - ir_2$	(c) in the above circuit cell 1 is discharging so it's equation is $E_1 = V_1 + ir_1 \Rightarrow \boldsymbol{V_1 = E_1 - ir_1}$ and cell 2 is charging so it's equation $E_2 = V_2 - ir_2 \Rightarrow \boldsymbol{V_2 = E_2 + ir_2}$

(2) **Parallel grouping :** In parallel grouping all anodes are connected at one point and all cathode are connected together at other point.



(i) If *n* identical cells are connected in parallel

TEACHING CARE

(a) Equivalent emf $E_{eq} = E$

(b) Equivalent internal resistance $R_{eq} = r/n$

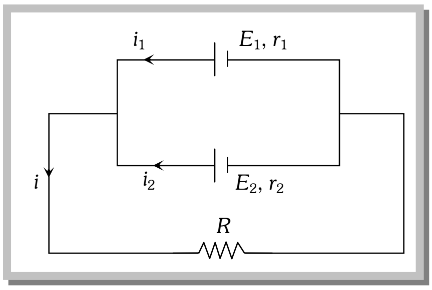
(c) Main current $i = \frac{E}{R + r/n}$

(d) P.d. across external resistance = p.d. across each cell = $V = iR$

(e) Current from each cell $i' = \frac{i}{n}$ (f) Power dissipated in the circuit $P = \left(\frac{E}{R + r/n}\right)^2 . R$

(g) Condition for max power $R = r/n$ and $P_{max} = n\left(\frac{E^2}{4r}\right)$ (h) This type of combination is used when $nr >> R$

(ii) If non-identical cells are connected in parallel : If cells are connected with right polarity as shown below then



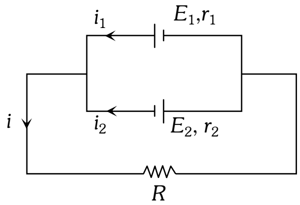
(a) Equivalent emf $E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$

(b) Main current $i = \frac{E_{eq}}{r + R_{eq}}$

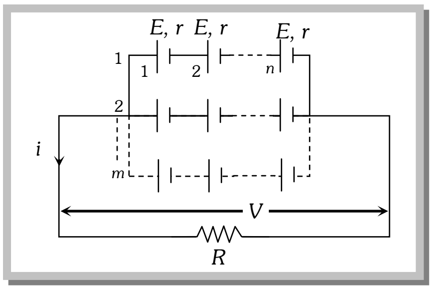
(c) Current from each cell $i_1 = \frac{E_1 - iR}{r_1}$ and $i_2 = \frac{E_2 - iR}{r_2}$

Note : ≅In this combination if cell's are connected with reversed polarity as shown in figure then :

Equivalent emf $\boldsymbol{E_{eq} = \frac{E_1 r_2 - E_2 r_1}{r_1 + r_2}}$



(3) **Mixed Grouping :** If n identical cell's are connected in a row and such m row's are connected in parallel as shown.



(i) Equivalent emf of the combination $E_{eq} = nE$

(ii) Equivalent internal resistance of the combination $r_{eq} = \dfrac{nr}{m}$

(iii) Main current flowing through the load $i = \dfrac{nE}{R + \dfrac{nr}{m}} = \dfrac{mnE}{mR + nr}$

(iv) Potential difference across load $V = iR$

(v) Potential difference across each cell $V' = \dfrac{V}{n}$

(vi) Current from each cell $i' = \dfrac{i}{n}$

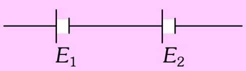
(vii) Condition for maximum power $R = \dfrac{nr}{m}$ and $P_{\max} = (mn)\dfrac{E^2}{4r}$

(viii) Total number of cell $= mn$

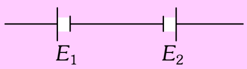
TEACHING CARE

Concepts

- *In series grouping of cell's their emf's are additive or subtractive while their internal resistances are always additive. If dissimilar plates of cells are connected together their emf's are added to each other while if their similar plates are connected together their emf's are subtractive.*



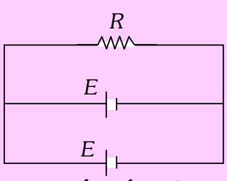
$E_{eq} = E_1 + E_2$ & $r_{eq} = r_1 + r_2$



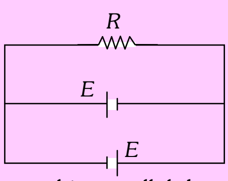
$E_{eq} = E_1 - E_2\ (E_1 > E_2)$ & $r_{eq} = r_1 + r_2$

- *In series grouping of identical cells. If one cell is wrongly connected then it will cancel out the effect of two cells e.g. If in the combination of n identical cells (each having emf E and internal resistance r) if x cell are wrongly connected then equivalent emf* $E_{eq} = (n - 2x)E$ *and equivalent internal resistance* $r_{eq} = nr$.

- *In parallel grouping of two identical cell having no internal resistance*

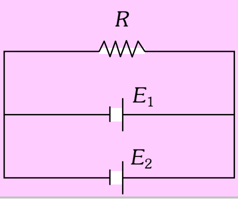


$E_{eq} = E$



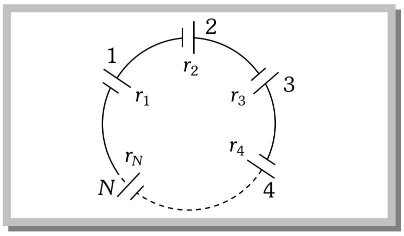
$E_{eq} = 0$

- *When two cell's of different emf and no internal resistance are connected in parallel then equivalent emf is indeterminate, note that connecting a wire with a cell but with no resistance is equivalent to short circuiting. Therefore the total current that will be flowing will be infinity.*



Example

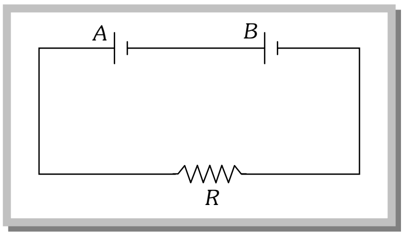
Example: 56 A group of N cells whose emf varies directly with the internal resistance as per the equation $E_N = 1.5\ r_N$ are connected as shown in the following figure. The current i in the circuit is **[KCET 2003]**



(a) 0.51 *amp*
(b) 5.1 *amp*
(c) 0.15 *amp*
(d) 1.5 *amp*

Solution : (d) $i = \frac{E_{eq}}{r_{eq}} = \frac{1.5r_1 + 1.5r_2 + 1.5r_3 + ...}{r_1 + r_2 + r_3 + ...} = 1.5\ amp$

Example: 57 Two batteries A and B each of emf 2 *volt* are connected in series to external resistance $R = 1\ \Omega$. Internal resistance of A is 1.9 Ω and that of B is 0.9 Ω, what is the potential difference between the terminals of battery A **[MP PET 2001]**



(a) 2 V
(b) 3.8 V
(c) 0
(d) None of these

Solution : (c) $i = \frac{E_1 + E_2}{R + r_1 + r_2} = \frac{2+2}{1+1.9+0.9} = \frac{4}{3.8}$. Hence $V_A = E_A - ir_A = 2 - \frac{4}{3.8}1.9 = 0$

TEACHING CARE

Example: 58 In a mixed grouping of identical cells 5 rows are connected in parallel by each row contains 10 cell. This combination send a current i through an external resistance of 20 Ω. If the emf and internal resistance of each cell is 1.5 *volt* and 1 Ω respectively then the value of i is **[KCET 2000]**

(a) 0.14 (b) 0.25 (c) 0.75 (d) 0.68

Solution : (d) No. of cells in a row $n = 10$; No. of such rows $m = 5$

$$i = \frac{nE}{\left(R + \frac{nr}{m}\right)} = \frac{10 \times 1.5}{\left(20 + \frac{10 \times 1}{5}\right)} = \frac{15}{22} = 0.68\ amp$$

Example: 59 To get maximum current in a resistance of 3 Ω one can use n rows of m cells connected in parallel. If the total no. of cells is 24 and the internal resistance of a cell is 0.5 then

(a) $m = 12, n = 2$ (b) $m = 8, n = 4$ (c) $m = 2, n = 12$ (d) $m = 6, n = 4$

Solution : (a) In this question $R = 3\Omega$, $mn = 24$, $r = 0.5\Omega$ and $R = \frac{mr}{n}$. On putting the values we get $n = 2$ and $m = 12$.

Example: 60 100 cells each of emf 5V and internal resistance 1 Ω are to be arranged so as to produce maximum current in a 25 Ω resistance. Each row contains equal number of cells. The number of rows should be **[MP PMT 1997]**

(a) 2 (b) 4 (c) 5 (d) 100

Solution : (a) Total no. of cells, $= mn = 100$ …….. (i)

Current will be maximum when $R = \frac{nr}{m}$; $25 = \frac{n \times 1}{m} \Rightarrow n = 25\,m$ …….. (ii)

From equation (i) and (ii) $n = 50$ and $m = 2$